International Journal of Advances in Engineering and Management (IJAEM)
Volume 6, Issue 10 Oct. 2024, pp: 414-419 www.ijaem.net ISSN: 2395-5252

Matrix Transformations of Some Generalized Sequence Spaces into L_∞

Dr. Pradip Kumar Jha[1] and Dr. Vishwa Prakash Jha[2]
[1] *Univ. Dept. of Maths., T.M.B.U., Bhagalpur*
[2]*National Institute of Technology Tiruchirappalli, Tamil Nadu, India*

Date of Submission: 20-10-2024 Date of Acceptance: 30-10-2024

ABSTRACT
Wewillrevealthenecessaryaswellassufficientconditionsinthispaperforaninfinite matrix $A=(a_{nk})$,n,k=1,2,...toconvertcertaingeneralized sequencespacesintol_∞.

Keywords: Matrixtransformations,Sequencespaces, Sufficientcondition,etc.

I. INTRODUCTION

Inthispaper,weshallstudytheinclusiontheoremsonmatrixtransformations of some generalized sequence spaces and the convolutionproductsofsummabilitymatricescharacterizedinconnectionwiththegeneralized sequence spaces. Given below briefly deal with the requiredknown properties of the sequence spaces and the other available resultsneededforourinvestigations. Amongthemostsignificantstudiesinthetheoryfocuses ondetermining the conditions that must be met by a matrix for it to change onesequencespaceintothesamespaceoranotherspace. Inthematrixtransformation'sclassicaltheory,issuesof thiskindareoftenreferredtoas "MatrixTransformationsofSequenceSpaces".Thebasicissueisstillunsolvedforany2 arbitrarysequencespacesalthoughsolutionswereprovidedforseveralspecificexamples. Certaingeneralizedsequencespacesshouldbetransformedintospace l_∞.Inthisparagraph,wewilldiscussthedifferenttypesof matrices.

a. $(w(p),l_\infty)$theorem……1.1
b. $(c(p),l_\infty)$theorem……1.2

1.2 w(p)andc(p)matrixTransformationsintol_∞

Theorem 1.1: When$0<p_k\le 1$,$A\in(w(p),l_\infty)$ifandonly if theorem(1.1) mustexistanintegerMgreaterthan 1,then

$$\sup_n \sum_{r=0}^{\infty} \max_r \left\{ \left(2^{r} M^{-1}\right)^{1/p_k} (a_{nk}) \right\} < \infty \qquad (1.1)$$

Wheremaxindicatesthe maximumover $2^r \le k < 2^{r+1}$. ***Proof.***

Forsufficiency let(1.1)hold $x=(x_k) \in w(p)$..then

$$2^{-r}\sum_r |x_k - L|^{p_k} \to 0 \text{ as } r\to\infty \text{ for only } L$$

Here

$\sum_r$ indicatesforthesummationsover $2^r \le k < 2^{r=1}$.

HencethereexistsanintegerRgreaterthan0

sothat

$$2^{-r}\sum_r |x_k - L|^{p_k} < 1/2M \;\&\; 2^{-r}\max(l, L) < 1/2M \text{ for each } r \text{ greater than } R.$$

Hence,

International Journal of Advances in Engineering and Management (IJAEM)
Volume 6, Issue 10 Oct. 2024, pp: 414-419 www.ijaem.net ISSN: 2395-5252

$2^{-r}M(x_k)^{p_k}<1$

Sothat, $0<p_k\leq 1$, wehave

$$2^{-r/p_k}M^{1/p_k}|x_k|<2^{-r}M|x_k^{p_k}|.$$

Thenforevery $r>R$, weget

$$\sum|a_{nk}x_k|\leq \max_r\left\{\left(2^rM^{-1}\right)^{1/p_k}|a_{nk}|\right\}M.g(x)$$

Here $g(x)=\sup_r 2^{-r}\left\{\sum_r|x_k|^{p_k}\right\}$.

Hence for $r>R$, the sum $\sum_{k=2^R}^{\infty}a_{nk}x_k$ isbounded.Now consider

$$\sum_{k=1}^{2^R-1}a_{nk}x_k \tag{1.2}$$

SinceRisfixedand(1.1)impliesisboundedforn(fixed),itimpliesthatacertainsum(1.2)isconstrained.

Therefore, $(y)=\sum_n\left(\sum_k|a_{nk}x_k|\right)\in l_\infty$ that $A\in(w(p),l_\infty)$.

Fortherequirement, let $A\in(w(p),l_\infty)$. Asthe (a_{nk})matrixappliesto allmembersof $w(p)$, $(a_{nk})_{k\geq}\in w^\beta(p)$ forevery $n\geq 1$ hence $\sum_k a_{nk}x_k$ convergesforevery $x=(x_k)\in w(p)$.

Themetricisnowdeterminedusing $g(x)=\sup_r 2\left\{^r\sum_r|x_k|^{p_k}\right\}$. Here $\sum_r$ indicatesthe"summation"over $2^r\leq k<2^{r+1}$,examinesthe topology $w(p)$. Therefore,thecoordinatefunctionalsarecontinuous,asimpliedby thedefinitionof $g(x)$,and

So, $$A_{n,k}(x)=\sum_{k=1}^{k}a_{nk}x_k$$

International Journal of Advances in Engineering and Management (IJAEM)
Volume 6, Issue 10 Oct. 2024, pp: 414-419 www.ijaem.net ISSN: 2395-5252

acomponentofw*(p).Consideringntobeconstant,weareprovidedthat thistendstohavealimit $A_n(x)$foreach,$x\in w(p)$. Therefore,accordingto theuniformboundedconceptthereexistsapositiveintegerδ_n,$G_n$suchthat if$g(x)<\delta_n$,then $A_n(x)<G_n$.If$g(x)<\delta_n$ $|r$(risapositiveinteger),this expressionmaybeformedarbitrarilysmallbyselecting$r$:hence,wemay make $A_n(x)$itcompletelysmall by choosing$g(x)$ Appropriately small.

Therefore $A_n\in w^*(p)$.

Therefore,w(p)indicatesafulllinear"metricspace"inthe $\sup_n |A_n(x)|<\infty$andg(x)metric,on$w(p)$,thereexistsaccordingtothe uniformboundedruleanumberGthatisindependentof$x$&n,aswellasanumber$\delta\leq$ 1so that

$$|A_n(x)|\leq G \qquad (1.3)$$

Foreach $x\in S[\theta,\delta]$ andeachnhere $S[\theta,\delta]$ representsthe"closedsphere" in$w(p)$withradiusδandcenter$\theta=(0,0,...)$.Suppose

Misanintegergreaterthanone,then

$$M^{-1}<\delta \qquad (1.4)$$

Expressing $A(r,k)=(2^r\delta)^{1p_k}$ $|a_{nk}|$ $for 2^r\leq k<2^{r-1}$andlet $k(r)$ issuch that"max,$A(r,k(r))$", usingnasafixedvalue,determineforany s,$x=(x_k)\in w(p)$by

$$x_k=0 \text{ for } k\geq 2^{s-1}, x(r)=(2^r\delta)^{1p_k} \operatorname{sgn} a_{n,k}(r),$$

$$x_k=0,(k\neq k(r)) \text{ for } 0\leq r<s.$$

Therefore,$g(x)\leq\delta$&$x=(x_k)\in S[\theta,\delta](w(p))$.So(1.3)&(1.4)provides

$$\sum_{r=0}^{s}\max_r\left\{\left(2^r M^{-1}\right)^{1p_k} |a_{nk}|\right\}\leq G.$$

International Journal of Advances in Engineering and Management (IJAEM)
Volume 6, Issue 10 Oct. 2024, pp: 414-419 www.ijaem.net ISSN: 2395-5252

Thisappliestoanys,thus itfollowsthat

$$\sum_{r=0}^{\infty} \max_r \left\{ \left(2^r M^{-1}\right)^{1/p_k} \left|a_{nk}\right| \right\} \leq G.$$

Thisisvalidforevery n;thus,wehave(1.1).

Theorem1.2. Suppose $p=(p_k)\in l_\infty$.then $A\in\left(c(p),l_\infty\right)$ifand onlyif

(2.1)there isan"absolute constant"Mgreaterthan 1 then

$$\sup_n \sum_k \left|a_{nk}\right| M^{-1/p_k} < \infty \qquad (1.5)$$

and

$$\sup_n \sum_k \left|a_{nk}\right| < \infty \qquad (1.6)$$

Proof. let(1.5)and (1.6)hold and $x(x_k)\in c(p)$. for the sufficiency,then

$$\left|x_k - L\right|^{p_k} \to 0 \text{ as } k \to \infty \text{ forsome L.}$$

Put$x_k - L = x'_k$. Then$x' = (x'_k) \in c_0(p)$ and

we have

$$\left|An(x)\right| = \left|\sum_k a_{nk}x_k\right| \leq \left|A_n(x')\right| + |L| \left|\sum_k a_{nk}\right| \qquad (1.7)$$

where$A_n(x') = \sum a_{nk}x'_k$.

Sinceholds,bythetheoremofinthispaper,wehavesuchtha $(A_n(x'))\in l_\infty$
t

$$\sup_n \left|A_n(x')\right| < \infty \qquad (1.8)$$

Now using(1.8)&(1.6),weobtainfrom(1.7)

$(A_n(x'))\in l_\infty$so that $A\in\left(c(p),l_\infty\right)$.

Fortherequirement,let $A \in (c(p), l_\infty)$.Now $c_0(p) \subset c(p)$ and $c_0 \subset c(p)$ so that $(c(p), l_\infty) \subset (c_0(p), l_\infty)$ and $(c(p), l_\infty) \subset (c_0, l_\infty)$. Hence $A \in (c_o(p), l_\infty)$ and $A \in (c_o, l_\infty)$. Therefore, thenecessitiesof (1.5) and (1.6)follow fromthetheoremofthispaper.

Corollary $A \in (c_o, l_\infty)$ifand only if

$$\sup_n \sum_k |a_{nk}| < \infty$$

Proof. The theorem leadsto the proofby assuming $p_k = 1$ forall$k$..

II. CONCLUSION:

Several of K.ChandrasekharaRao's findings are expanded in the aforementioned theorem, and further findings are provided about the closure qualities of the convolution of the classes of matrices used in the inclusion the orems of generalized sequence spaces.

REFERENCES:

[1]. Altay,B.,Basar,F.andMursaleen,M.,Onthe Eulersequencespaceswhich include the spaces ℓ_p and ℓ_∞, Inform. Sci., 176(10) (2006)1450-1462.

[2]. Akhmedov, A. M. and Basar, F., The fine spectra of the Cesárooperator C1 over the sequence space bvp ($1 \le p < \infty$), Math. J.OkayamaUniv., 50 (2008)135-147.

[3]. Aydın, C. and Basar, F., Some new sequence spaces which includethespaces$\ell_p$andℓ_∞,DemonstratioMath.,38(3)(2005)641-656.
Alotaibi A, Mursaleen M, Alamri BAS, Mohiuddine SA. Compactoperators on some Fibonacci differennce sequence spaces. Journal ofInequalitiesand Applications2015;2015:203.

[4]. Altay B, Basar F. The matrix domain and the fine spectrum of thedifferennceoperatorΔonthesequencespacel$_p$,(0<p<1).CommunicationsinMathematicalAnalysis2007;2(2):1-11.

[5]. Basarir M, Kara E.E. On compact operators on the RieszB(m) -differenncesequencespaces.IranianJournal ofScienceandTechnology.Transaction A, Science 2011;35:279-285.

[6]. BasarirM,KaraE.E.Onsomedifferennceseq uencespacesofweightedmeansandcompact operators.AnnalsofFunctionalAnalysis2011;2:114-129.

[7]. BasarirM,KaraE.E.OntheB-differenncesequencespacederivedbygeneralizedweightedmeanandcompactoperators.JournalofMathematicalAnalysisand Applications2012;391:67-81.

[8]. CandanM.Domainofthedoublesequentialbandmatrixinthespacesofconvergentandnull sequences.AdvancesinDifferennceEquations2014;2014:163.

[9]. Candan M. A new sequence space isomorphic to the space (p) andcompactoperators.JournalofMathematicalandComputationalScience 2014;4 (2):306-334.

[10]. Hazar GC, Sarigol MA. Absolute Cesaro series spaces and matrixoperators. ActaApplicandaeMathematicae 2018; 36 154 (1): 153-165.

[11]. Ilkhan M. Matrix domain of a regular matrix derived by Euler totientfunctioninthespacesc$_0$andc.MediterraneanJournalofMathematics2020;17 (1):27.

[12]. IlkhanM.Anewconservativematrixderived byCatalannumbersandits matrix domain in the spaces c and c$_0$: Linear and MultilinearAlgebra2020;68 (2):417-434.

[13]. KaraEE,IlkhanM.SomepropertiesofgeneralizedFibonaccisequence spaces. Linear and Multilinear Algebra 2016; 8 64 (11):2208-2223.

[14]. Kara MI, Kara EE. Matrix transformations and compact operators onCatalansequencespaces.JournalofMathematicalAnalysisandApplications2021;498(1):Articleno:124925.

[15]. Meng J, Mei L. A generalized fractional differennce operator and itsapplications. Linear and Multilinear Algebra 16 2020; 68 (9): 1848-1860.

[16]. Meng J, Mei L. The matrix domain and the spectra of a generalizeddifferennceoperator.Journalof

International Journal of Advances in Engineering and Management (IJAEM)
Volume 6, Issue 10 Oct. 2024, pp: 414-419 www.ijaem.net ISSN: 2395-5252

MathematicalAnalysisandApplications2019;470(2):1095-1107.
[17]. MursaleenM,NomanAK.CompactnessbytheHausdorffmeasureofnoncompactness.NonlinearAnalysis2010; 24 (8):2541-2557.
[18]. H. Dutta, Characterization of certain matrix classes involvinggeneralizeddifferencesummability spaces,Appl.Sci.,11(2009),60-67.141
[19]. K.RajandS. Pandoh,Onsome ZweierI-convergentdifferencesequencespaces,J.Appl.Funct. Anal.,10(2015),143-163.
[20]. K.Rajand S.K. Sharma, Doublesequencespacesovern-normedspaces,Arch. Math., 50(2014), 7-18.
[21]. B. C. Tripathy and R. Goswami, Vector valued multiple sequencesspacesdefinedbyOrliczfunction, Bol.Soc.Paran.Mat.,33(2015),69-81