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Modified Geometric Brownian Motion For Inflation Modelling

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ABSTRACT: Accurate inflation forecasting is vital for effective economic planning, monetary policy formulation, and investment decision-making, especially in developing economies like Nigeria. Traditional models such as ARIMA and standard Geometric Brownian Motion (GBM) often assume constant parameters and may fail to capture the dynamic and volatile nature of inflation. This study introduces a novel Dynamic Geometric Brownian Motion (DGBM) model that incorporates rolling window estimates of drift and volatility to account for structural changes and macroeconomic shocks over time. Monthly inflation rate data from January 2003 to December 2024, obtained from the Central Bank of Nigeria, was used to compare the forecasting performance of three models: ARIMA, Traditional GBM, and the proposed Dynamic GBM. Model accuracy was evaluated using metrics such as Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), Mean Absolute Percentage Error (MAPE), and R-squared (R^2). Diagnostic tests including the Augmented Dickey-Fuller, Shapiro-Wilk, and Ljung-Box were conducted to validate model assumptions. The results revealed that while the traditional GBM model performed poorly due to its rigid assumptions, the Dynamic GBM significantly outperformed both ARIMA and standard GBM models in terms of forecast accuracy and adaptability. The DGBM model achieved an R^2 of 0.966, demonstrating its strong predictive power. The study recommends the integration of the Dynamic GBM model into macroeconomic forecasting tools for more responsive and reliable inflation prediction.

Keywords: Inflation, Forecasting, Geometric Brownian Motion, ARIMA, Rolling window.

I. INTRODUCTION

Inflation is the rate at which the general price level of goods and services rises over a specific period, thereby leading to decrease in the purchasing power of money. This reduction in purchasing power is often cited as the most significant adverse effect of inflation. It is commonly measured by changes in the Consumer Price Index (CPI), which reflects the cost of living.

On the positive side, inflation may boost investor returns by raising profit margins and can benefit lenders in some ways. It may also encourage or promote investment under certain economic conditions. On the contrary, inflation can reduce the real value of savings and cause hardship for individuals on fixed earnings. It also disrupts a country's balance of payments and poses significant challenges for economic planning and monetary stability (Kelukume& Salami, 2014; Nathaniel & Emmanuel, 2018).

One of the most pressing macroeconomic challenges for policymakers and central banks is the effective monitoring and accurate forecasting of inflation.

Given the importance of inflation forecasting in shaping monetary policy and investment strategies, robust modeling techniques are essential. This is particularly relevant in the Nigerian scenero. Traditional econometric models often fall short in capturing the stochastic behavior of inflation, particularly in unstable economic environments. While the Geometric Brownian Motion (GBM) model has found extensive application in modeling asset prices and exchange rates, its use in inflation modeling has been limited.

The standard GBM assumes constant drift and volatility—assumptions that may not hold in the face of dynamic macroeconomic conditions. To overcome this limitation, this study introduces a modified approach: the Dynamic GBM Model. This model recalibrates drift and volatility using a rolling window technique to better capture evolving inflationary trends. By comparing the ARIMA, Traditional GBM and Dynamic GBM models, the study aims to identify the more effective approach for modeling and understanding inflation dynamics in Nigeria.

II. LITERATURE REVIEW

2.1 Theoretical Review

2.1.1 Stochastic Process Theory

Stochastic processes describe how variables evolve randomly over time. GBM is based on the following stochastic process principles:

- **Brownian Motion**: Introduced by Louis Bachelier (1900) and popularized by Einstein (1905), Brownian motion describes random fluctuations in variables.
- **Ito's Lemma**: Governs the stochastic differential equations that define GBM
- **Markov Property:** States that future values depend only on the present state, not past history.

These principles form the foundation of GBM modeling in economic applications.

2.1.2 Macroeconomic Inflation Theories

Several economic theories explain inflation dynamics:

1. Social Theory

Social theories of inflation emphasize the broader societal and economic interactions that influence inflationary trends. Two key theories under this framework are:

A. Expectations-Augmented Phillips Curve Theory

The Expectations-Augmented Phillips Curve, introduced by Milton Friedman and Edmund Phelps (1968), extends the traditional Phillips Curve by incorporating inflation expectations. It suggests that inflation is influenced not only by unemployment and economic slack but also by the expectations of consumers and businesses.

Mathematically, the theory is expressed as:

$\pi_t = \pi_t^e + \beta(U_t - U_n)$

Where:

- π_t is the actual inflation rate,
- π_t^e is the expected inflation rate,
- U_t is the actual unemployment rate,
- U_n is the natural rate of unemployment
- β is a sensitivity parameter

This theory aligns with Geometric Brownian Motion (GBM) because both models incorporate random shocks and adaptive expectations in explaining inflation behavior. The stochastic component of GBM ($\sigma I_t dW_t$) captures the unpredictable changes in inflation that result from shifts in expectations and economic shocks.

B. Monetary Theory of Inflation

The Monetarist Theory, popularized by Milton Friedman in the 1970s, states that inflation is fundamentally a monetary phenomenon, meaning that it occurs when the growth rate of money supply outpaces economic output. This relationship is captured by the equation:

$MV = PQ$

Where:

- M is the money supply,
- V is the velocity of money,
- P is the price level
- Q is the real output

Under this framework, if M grows faster than Q, inflation (P) will increase. The GBM model indirectly accommodates this theory by modeling inflation as a multiplicative stochastic process, where both money supply growth and external shocks influence price dynamics.

2. Behavioral Theory

Behavioral theories explain inflation by examining how individuals and firms form expectations and respond to economic conditions. These theories are relevant to GBM because they provide insight into how inflation volatility and randomness emerge from human decision-making.

A. Adaptive Expectations Theory

The Adaptive Expectations Hypothesis, proposed by Cagan (1956), suggests that people form inflation expectations based on past inflation rates. This means that if inflation was high in the past, individuals expect it to remain high in the future. The expectation is updated as new information becomes available, following the equation:

$\pi_t^e = \pi_{t-1} + \lambda(\pi_{t-1} - \pi_{t-2})$

where:

- π_t^e is the expected inflation rate,
- π_{t-1} is the past observed inflation rate,
- λ is the adjustment coefficient.

GBM aligns with this theory as it assumes that inflation follows a continuous path with adjustments based on past trends. The drift term (μ) in GBM can be interpreted as the long-term expected inflation rate, while the stochastic component ($\sigma I_t dW_t$) accounts for unexpected deviations.

B. Rational Expectations Theory

The Rational Expectations Hypothesis, developed by John Muth (1961) and expanded by Robert Lucas (1972), argues that individuals use all available information, including current policies and future projections, to form their expectations of inflation. Unlike adaptive expectations, which rely on past data, rational expectations assume that people anticipate inflation based on economic fundamentals.

Mathematically, rational expectations can be represented as:

$E[\pi_t|I_t]= \pi_t$

Where:

- I_t is the available information.
- π_t is the actual inflation.

Under this theory, inflation follows a stochastic process with informed expectations, which closely resembles the behavior modeled by GBM. The stochastic term in GBM accounts for uncertainty, while the drift term represents expected trends based on rational forecasting.

Understanding these theories helps justify the stochastic modeling of inflation dynamics.

2.2 Empirical Review

2.2.1 Overview of Inflation Forecasting Models

Numerous studies have explored a wide array of inflation forecasting techniques, including econometric, time series, and stochastic models. For instance, In the Nigerian context, Feridun and Adebiyi (2005) investigated the role of monetary aggregates in inflation forecasting using monthly data from January 1986 to April 1998. Applying ARIMA techniques and evaluating forecast accuracy with Mean Absolute Percentage Error (MAPE), they established that variables such as money supply and interest rates significantly enhance inflation predictions.

Comparative model evaluations have also been conducted internationally. Stock and Watson (2007) compared ARIMA, Vector Autoregression (VAR), and Phillips Curve models, concluding that no single model universally outperforms others across different economies.

Ang, Bekaert, and Wei (2007) found that inflation expectation models (e.g., surveys) often yield more accurate short-term forecasts than purely statistical approaches.

Koop and Korobilis (2012) introduced Bayesian time-varying parameter models, showing that non-constant parameter models tend to outperform static models.

Chukwuemeka, Emmanuel, and Michael (2013) applied Fourier series and periodogram analysis to model Nigeria's monthly inflation data from January 2003 to December 2011. Their approach yielded accurate forecasts over a 13-month horizon, which closely matched observed inflation figures.

Wanjoya and Waitiitu (2016) analyzed monthly Consumer Price Index (CPI) data from the Central Bank of Rwanda, covering the period from February 1995 to December 2015 (251 observations). They employed data from 1995:2 to 2013:12 to fit a parsimonious ARIMA model and used 2014:1 to 2015:12 for validation. Their findings identified ARIMA(4,1,6) as the most suitable model for predicting future CPI values.

Similarly, Nyoni (2018) modeled and forecasted Zimbabwe's inflation using monthly data from July 2009 to July 2018. Comparing Autoregressive (AR) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models, the study concluded that GARCH models offer superior predictive power for inflation forecasting.

Awa and Terna (2018) adopted ARIMA models to predict Nigeria's 12-month average inflation using data from January 2006 to December 2017. The ARIMA(1,2,1) model passed all diagnostic checks and demonstrated strong forecasting performance for the year 2017.

2.2.2 Applications of Geometric Brownian Motion (GBM) in Finance and Economics

GBM has been extensively applied in finance and macroeconomics due to its strength in modeling stochastic processes and uncertainty. It is particularly useful in asset pricing, currency exchange forecasting, and stock market dynamics.

The foundational work by Black and Scholes (1973) introduced GBM in option pricing models. Merton (1976) extended its application to corporate finance and economic growth.

Jorion (1996) applied GBM to exchange rate modeling, evaluating both its strengths and limitations. Since inflation shares characteristics with financial assets—such as trend-driven growth and volatility—GBM presents a theoretically sound framework for inflation modeling.

Marathe and Ryan (2005) investigated whether specific time series followed GBM behavior and addressed the importance of eliminating seasonality—since GBM does not inherently model cyclical variations. Their analysis revealed that while some industries conformed to GBM assumptions, others did not.

Irma, Kristin Nova, and Primadina (2008) applied GBM to forecast Indonesian stock prices

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during the COVID-19 outbreak. They analyzed returns, tested for normality, and calculated forecast errors. Their results indicated that the Mean Absolute Percentage Error (MAPE) mostly hovered around 10%, suggesting strong predictive performance.

Abidin and Jaffar (2014) focused on forecasting future stock prices for small-sized companies listed on Bursa Malaysia using GBM. Their study emphasized the importance of data input horizon and found GBM effective across different time frames.

Isaac (2017) evaluated GBM's applicability in forecasting stock prices on the Ghana Stock Exchange, using weekly closing prices of 10 top-performing companies between January 2008 and July 2015. Statistical tests supported GBM's reliability in capturing stock price behavior.

In Nigeria, Imoni and Muhammad (2020) applied the GBM model to forecast stock prices on the Nigerian Stock Exchange using daily data from companies such as Nestle Foods and Dangote Cement. Their findings confirmed that GBM accurately captured short-term price movements.

TopcuGuloksuz (2021) extended the random walk theory to GBM for modeling Walmart's stock prices from March 2019 to March 2020. The model yielded accurate predictions, affirming its practicality.

Peng and Simon (2024) compared GBM predictions with historical Dow Jones Industrial Average (DJIA) data. Using Python simulations for the periods 1900–2000 and 2000–2015, they assessed GBM's predictive validity and emphasized the model's stochastic and memoryless nature. They also discussed limitations such as sensitivity to volatility and external economic factors.

These applications affirm GBM's flexibility and robustness in modeling time series data with stochastic features. However, it's potential in inflation forecasting remains unexplored—an opportunity this study aims to address.

III. METHODOLOGY

The analysis of inflation rate dynamics were performed using the ARIIMA, Traditional Geometric Brownian Motion and the newly developed dynamic Geometric Brownian Motion model. The study considers monthly inflation rate data obtained from CBN official website (statistics.cbn.gov.ng/data-browser) from January 2003 to December 2022 for training and January 2023 to December 2024 for testing. The analysis is conducted using python statistical package.

3.1 Model Specifications

1. **SARIMA (SEASONAL AUTOREGRESSIVE INTEGRATED MOVING AVERAGE)**

The ARIMA (Autoregressive Integrated Moving Average) is a time series model that attributes patterns of a given time series based on its past values using linear regression. The model uses three different terms in one equation. The first specification is the "p" or "AR" term which is the number of lagged variables to be used as predictors. The "q" or "MA" term which is the number of lagged forecast errors that is to be included in the model. The "d" term stands for the order of differencing that is required to make the time series stationary. ARIMA modelling assumes that there is correlation between a time series data and its own lagged data.

A kth-order autoregressive process expresses a dependent variable as a function of past values of the dependent variable, this is expressed as:

$$Y_t = c + \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \emptyset_3 Y_{t-3} + \dots\dots\dots + \emptyset_k Y_{t-k} + \varepsilon_t$$

where

- Y_t is the variable being forecasted at time t
- c is a constant
- $Y_{t-1}, Y_{t-2}, \; Y_{t-3} \dots\dots \; Y_{t-k}$ is the response variable at time lag t-1, t-2, t-3 ……. t-k respectively
- $\emptyset_{1,}\emptyset_2\emptyset_3 \dots\dots..\emptyset_k$ are coefficients to be estimated.
- ε_t is the error term at time t

A pth-order moving-average process expresses a dependent variable Y_t as a function of the past values of the p error terms. This is expressed as:

$$Y_t = \mu + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \theta_3 \varepsilon_{t-3} + \dots\dots\dots + \theta_p \varepsilon_{t-k}$$

where:

- Y_t is the variable being forecasted at time t
- μ is the constant mean of the process
- $\theta_{1,}\theta_2\theta_3 \dots\dots..\theta_k$ are coefficients to be estimated.
- ε_t is the error term at time t
- $\varepsilon_{t-1}, \varepsilon_{t-2}, \; \varepsilon_{t-3} \dots\dots \; \varepsilon_{t-k}$ are the errors in previous time periods that are incorporated in the response Y_t

The ARIMA model can be specified as

$$Y_t = \beta_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} +$$

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$\phi_3 Y_{t-3}$ +........+$\phi_k Y_{t-k}$+ε_t+$\theta_1\varepsilon_{t-1}$ + $\theta_2\varepsilon_{t-2}\theta_3\varepsilon_{t-3}$ +........ + $\theta_p\varepsilon_{t-k}$

A SARIMA model extends ARIMA to capture seasonal patterns in time series data. A SARIMA model is written as: SARIMA(p,d,q)(P,D,Q)$_s$. The general SARIMA model equation is given as

$$\Phi_P(B^S)\phi_p(B)(1-B)^d(1-B^s)^D y_t = \Theta_Q(B^s)\theta_q(B)\varepsilon_t \quad (1)$$

2. GEOMETRIC BROWNIAN MOTION (GBM)

GBM is a continuous-time stochastic process where the logarithm of a randomly varying quantity follows a Brownian motion with drift. GBM is a stochastic process where the underlying quantity changes randomly over time, but its logarithmic transformation follows a Brownian motion with drift and volatility. A stochastic process I_t is said to follow a GBM if it satisfies the following stochastic differential equation (SDE):

$dI_t = \mu I_t dt + \sigma I_t dW_t$

Where:

- I_t= inflation rate at time t
- μ = drift term (long-term average growth rate of inflation)
- σ = volatility (degree of fluctuation or uncertainty in inflation)
- dW_t = standard Brownian motion (Random shocks affecting inflation)

According to Azubuike S.A and C. Anayamoabi (2021). The solution to the GBM SDE is define by:

$S_t=S_{t-1}\exp((\mu-\frac{1}{2}\sigma^2)\Delta t + \sigma W_t)$

where$W_t=\sqrt{\Delta t}\ .\ Z_tandZ_t \sim N(0,1)$

Therefore for the inflation rate modelling:

$$I_t=I_{t-1}\exp((\mu-\frac{1}{2}\sigma^2)\ \Delta t + \sigma\sqrt{\Delta t}\ .\ Z_t) \quad (2)$$

Where:

I_t is the inflation rate at time t

I_{t-1} is the previous inflation rate before time t

Δt is the time increment (e.g., 1 month)

$Z_t$is a random value from a standard normal random variable (Wiener process)

The drift parameter (μ) is used to model deterministic trends while the volatility parameter (σ) models unpredictable events occurring during the motion.

A. PARAMETER ESTIMATION

i. Estimating Drift (μ)

The drift term is estimated using the average of the log returns of inflation:

$$\mu = \frac{1}{T}\sum_{t=1}^{T} r_t \quad (3)$$

wherer_t = $\ln(\frac{I_{t+1}}{I_t})$ is the log return of inflation over time.

ii. Estimating Volatility (σ)

Volatility is calculated as the standard deviation of log returns:

$$\sigma = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(r_t-\mu)^2} \quad (4)$$

3. DYNAMIC GEOMETRIC BROWNIAN MOTION MODEL

To improve traditional GBM, I introduced time-varying parameters. I therefore define the stochastic differential equation of the dynamic Geometric Brownian motion as:

$dI_t = \mu_t I_t dt + \sigma_t I_t dW_t$

The exact solution to the DGBM Stochastic differential equation is obtained as.

$$I_{t+\Delta t} = I_t\exp((\mu_t - \frac{1}{2}\sigma_t{}^2)\Delta t + \sigma_t\sqrt{\Delta t}\ .\ Z_t) \quad (5)$$

The solution follows trivially as that of the standard geometric Brownian motion.

STEPS TO IMPLEMENT THE DYNAMIC GEOMETRIC BROWNIAN MOTION

To apply the Dynamic Geometric Brownian Motion:

1. Choose a rolling window size N
2. Estimate Drift (μ_t) at each time step
   i. Compute log returns: $r_t = \ln(\frac{I_{t+1}}{I_t})$
   ii. Compute rolling mean of returns over W periods: $\widehat{\mu_t}=\frac{1}{W}\sum_{i=t-W+1}^{t} r_i$
3. Estimate volatility (σ_t) at each time step
   i. Compute rolling standard deviation of log returns: $\widehat{\sigma_t}=\sqrt{\frac{1}{W-1}\sum_{i=t-W+1}^{t}(r_i-\widehat{\mu_t})^2}$
4. We then solve the dynamic GBM equation numerically using the updated μ_t and σ_t at each step

DISTINCTION BETWEEN THE GBM AND THE DYNAMIC GBM METHOD.

The traditional GBM simulation splits the dataset into two fixed sets-training set and test set. The training set is used to estimate GBM parameters (μ *and* σ). The test set is used to compare model-generated paths with actual data. The GBM model is trained on historical data and then projected into the future for validation.

The dynamic GBM model continuously updates the training data as new observations arrive instead of a fixed train-test split. A rolling window of a fixed size moves through the dataset, recalculating μ *and* σ at each step.

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3.2 MODEL VALIDATION

We evaluated the performance of each of the model using the:Mean Absolute Error (MAE), Root Mean Squared Error (RMSE). Mean Absolute Percentage Error (MAPE), Coefficient of Determination (R^2), Standard error of the residuals

3.3 STATISTICAL TESTS

3.3.1 Test for Stationarity

To determine whether the log returns series is stationary (i.e has constant mean and variance over time)

Test Used: Augmented Dickey-Fuller (ADF) Test

The ADF test, corresponding to modelling a random walk pattern with drift around a stochastic trend is expressed as:

$$yt = \alpha + \rho y_{t-1} + \sum_{i=1}^{p-1} \delta_i \Delta y_{t-i} + \beta_t + \varepsilon_t \qquad (6)$$

Null Hypothesis (H_0): The time series has a unit root (non-stationary).

Alternative Hypothesis (H_1): The time series does not have a unit root (stationary).

Decision Rule: If the ADF test statistic is less than the critical value and the p-value is below 0.05, reject H_0.

3.3.2 Test for Normality

To assess whether the log returns are normally distributed.

Tests Used: Shapiro-Wilk Test

Null Hypothesis (H_0): The log returns are normally distributed.

Alternative Hypothesis (H_1): The log returns are not normally distributed.

Test statistic:

$$W = \frac{(\sum_{i=1}^{n} a_i x_i)^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2} \qquad (7)$$

Where:

- W is the test statistic
- n is the number of observations in the dataset
- $x_1, x_2, \dots \dots \dots, x_n$ are the ordered sample data points (from smallest to largest)
- $\bar{x}$ is the sample mean of the data
- $a_1, a_2, \dots \dots \dots, a_n$ are the constants derived from the expected values of order statistics for a normal distribution. These constants are based on sample size n and are tabulated for various sample sizes.

Decision Rule: If p-value < 0.05, reject H_0.

3.3.3 Test for Autocorrelation (Independence Test)

To check if past values of returns have predictive power over future values.

Tests Used: Ljung-Box Test

Null Hypothesis (H_0): No autocorrelation at specified lags.

Alternative Hypothesis (H_1): Presence of autocorrelation.

Decision Rule: If p-value < 0.05, there is significant autocorrelation.

IV. RESULTS AND DISCUSSION

4.0 SARIMA MODEL

4.1.1 MODEL SELECTION

The presence of trend in the inflation rate scatter plot shown in figure 4.1 admits the existence of non-stationarity. Also the ACF plot (figure 4.2) shows a slow, gradual decline in autocorrelation with increasing lags, with significant values extending beyond lag 10. This is a clear indicator of non-stationarity. The PACF plot also displays a significant spike at lag 1, followed by a gradual decline with few significant spikes. This again suggests non-stationarity. The result of the stationarity test shown in Table 4.1A confirms the same.

The existence of non-stationarity is addressed by differencing approach. The result of the stationarity test after first differencing shown in Table 4.1B indicate that first differencing is appropriate for the ARIMA model. The ACF and PACF plot of the first difference of the data shown in figure 4.3 suggest an ARIMA (2, 1, 2) model.

The ACF plot (figure 4.2) does not exhibit clear, repeating spikes at seasonal lags which would be indicative of a seasonal pattern. The autocorrelations show a smooth decline without distinct periodic peak, suggesting no strong seasonal component is immediately apparent. Similarly, the PACF plot does not show significant spikes at seasonal lags, further supporting the absence of a clear seasonal effect. The evidence of seasonality is weak based on these plots.

Perhaps there could be a latent annual seasonal pattern not initially apparent in the ACF and PACF first differenced time series. We therefore introduced a seasonal difference (D=1) with a 12 month period (s=12) in order to accommodate this. This brought about a market improvement in fit. (see table 4.1D)

In order to get the model with the best performance, three additional different others for the non-seasonal component of the

SARIMA(p,1,q)$(0,1,0)_{12}$ model are applied to the data. The results of the AIC and BIC values for these four SARIMA models are presented in table 4.1C. It is clear from this table that SARIMA (2,1,2)$(0,1,0)_{12}$ with minimum values for both AIC and BIC model selection criteria has the best performance. Consequently it is adopted for the analysis of the data. Its mathematical representation is given by:

$$(1-\phi_1 B-\phi_2 B^2)(1-B)(1-B^{12})y_t = (1+\theta_1 B+\theta_2 B^2)\varepsilon_t \quad (8)$$

where:

- y_t: original time series
- B: backshift operator, $B^k y_t = y_{t-k}$
- $\emptyset_1 \emptyset_2$ are autoregressive coefficients,
- $\theta_1 \theta_2$ are moving average coefficients
- ε_t is a white noise error term

The results are shown in table 4.1D

4.1.2 MODEL DIAGNOSTIC CHECKS

The residual plots are shown in figure 4.6. The histogram is used to provide a visual impression indicative of the presence of normality distribution in the residual plot while Shapiro-wilk test is used to ascertain the same. The test of the significance of autocorrelation coefficients is also performed. The results of these are presented in table 4.1G

4.1.3 MODEL FORECAST

The SARIMA(2,1,2)$(0,1,0)_{12}$ model was first applied to the inflation data for January 2003 to December 2022 as training set. The model parameters estimated were then used to forecast for January 2023 to December 2024. The plot of the forecasted values are shown in figure 4.5, while the forecasted inflation rate for the period January 2023 to December 2024 are presented in table 4.1F.

Figure 4.1: The trend of the original inflation rate series (January 2003 – December 2022)

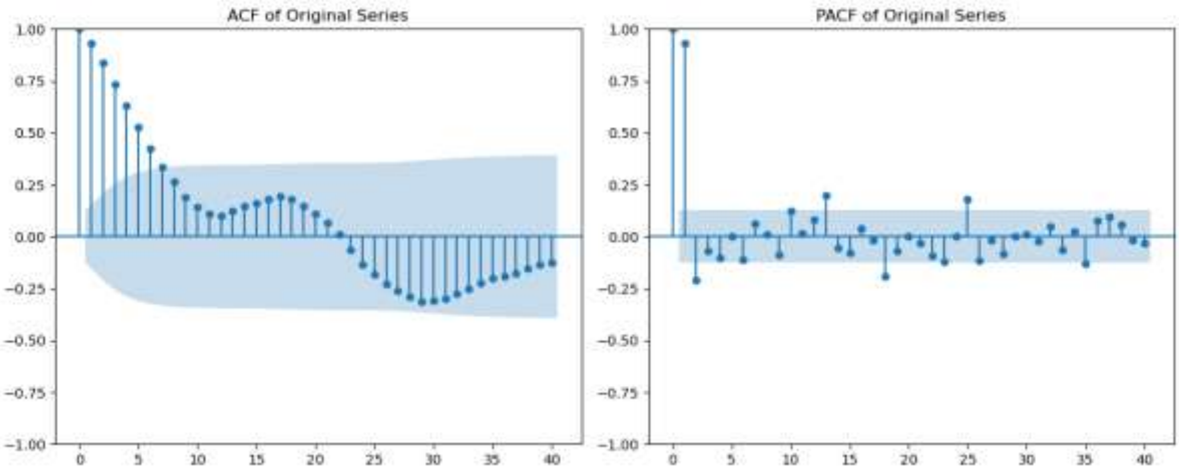

Figure 4.2: The ACF and PACF of the original inflation rate series

Figure 4.3: A plot showing the differenced inflation rate series

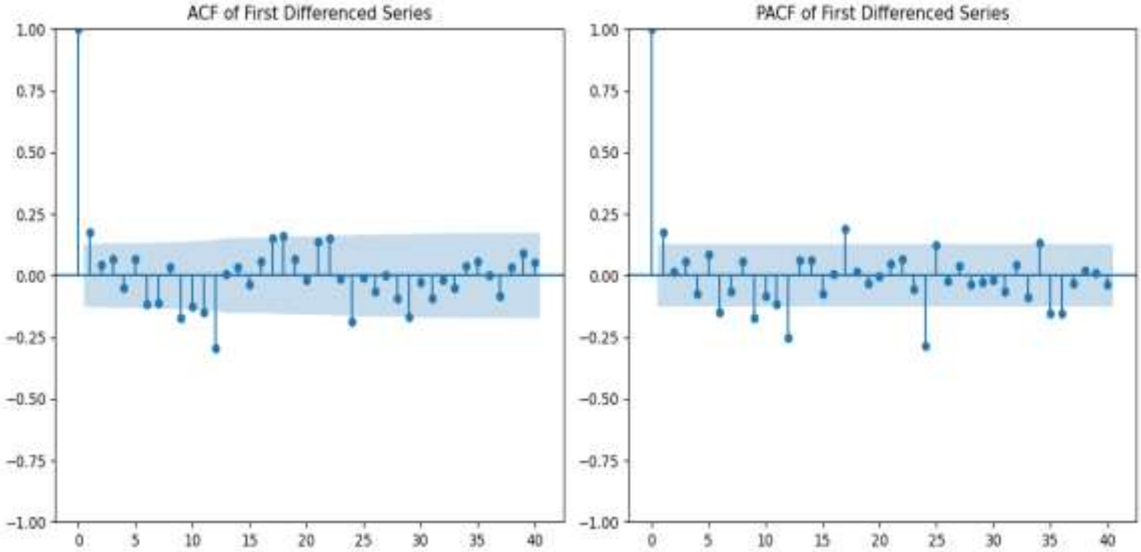

Figure 4.4: The ACF and PACF of the differenced inflation rate series.

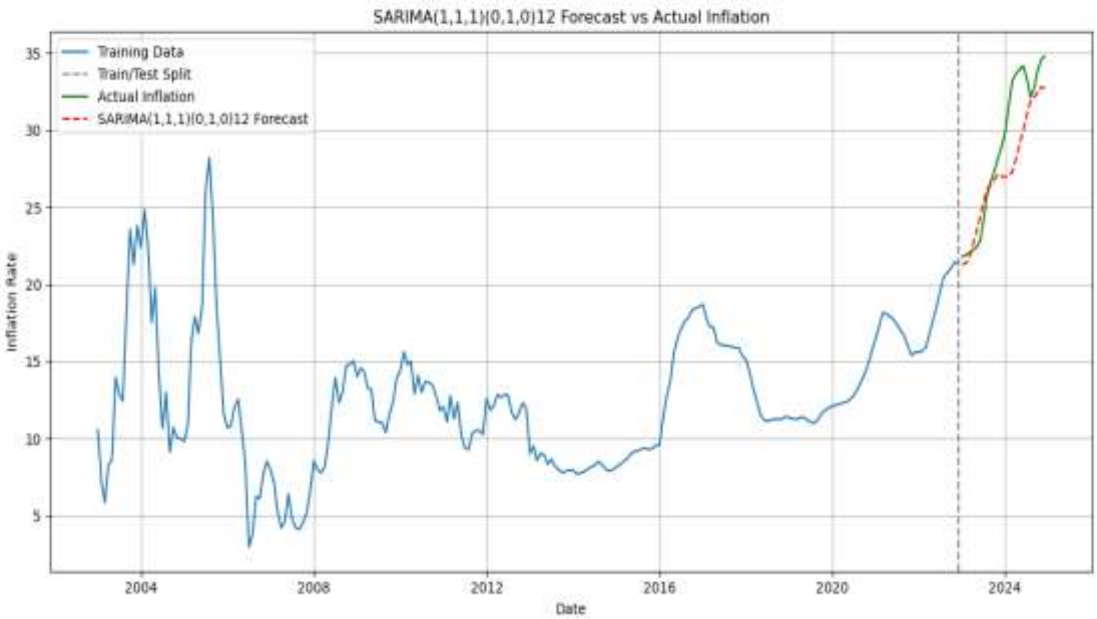

Figure 4.5: The plot of the actual vs SARIMA forecasted inflation rate (January 2023 – December 2024)

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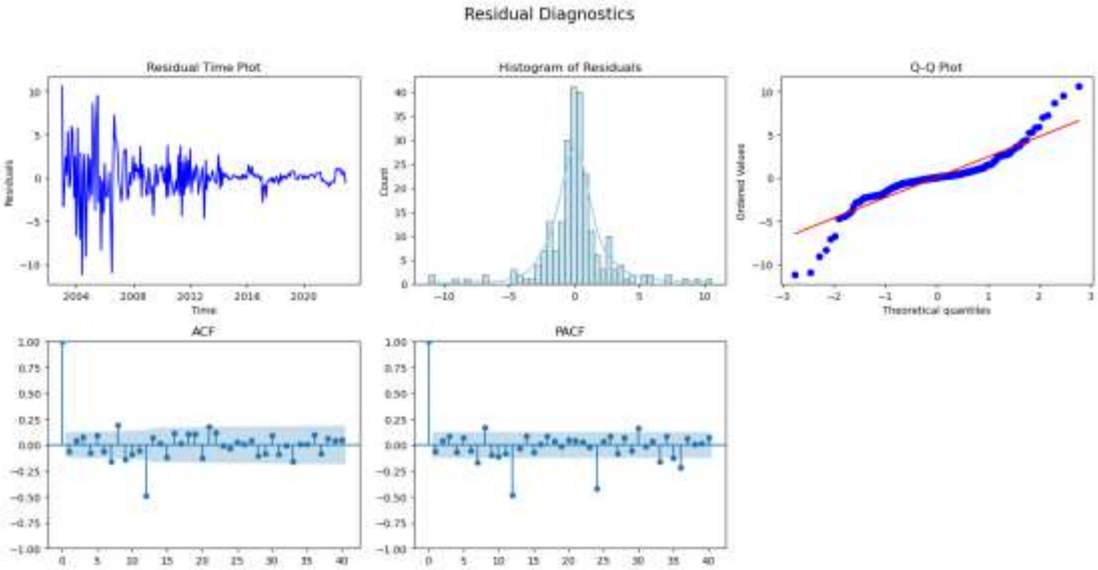

Figure 4.6: Plot of the residuals of the SARIMA model

Table 4.1A: STATIONARITY TEST ON THE ORIGNAL INFLATION RATE

Stationarity Test	ADF Test
ADF Statistic	-1.5674
p-value	0.4999

Table 4.1B: STATIONARITY TEST AFTER FIRST DIFFERENCING

Stationarity Test	ADF Test
ADF Statistic	-8.2332
p-value	0.0000

TABLE 4.1C: AIC AND BIC VALUES FOR FOUR TENTATIVE SARIMA MODELS

SARIMA MODEL	AIC	BIC
(1,1,2)(0,1,0)12	1048.591	1062.291
(2,1,0)(0,1,0)12	1046.912	1057.187
(2,1,1)(0,1,0)12	1048.649	1062.349
(2,1,2)(0,1,0)12	1040.049	1057.174

TABLE 4.1D: RESULTS AND PARAMETER ESTIMATE OF THE SARIMA MODEL

Log Likelihood	-515.024
AIC	1040.049
BIC	1057.174
HQIC	1046.959
Prob(H) (two-sided):	0.00
Heteroskedasticity (H)	0.03
Jarque-Bera (JB)	455.70
Prob(JB)	0.00
Skew	-0.64
Kurtosis	9.82
MSE	6.4338
RMSE	2.5365
MAE	1.8620
R squared	0.7201

TABLE 4.1E: TABLE OF COEFFICIENTS

	Coef	Std	Z	P-value
Ar.l1	-0.3258	0.183	-1.783	0.075
ar.l2	0.4640	0.153	3.042	0.002
ma.L1	0.5931	0.208	2.858	0.004
ma.l2	-0.3969	0.196	-2.021	0.043
$sIgma^2$	5.4310	0.332	16.353	0.000

TABLE 4.1F:FORECAST OF THE INFLATION RATE OF THE SARIMA MODEL (JAN 2023 - DEC 2024)

Date	Forecasted Inflation Rate	Actual Inflation Rate
Jan 2023	21.288942	21.82
Feb 2023	21.451921	21.91
March 2023	21.641627	22.04
April 2023	22.580724	22.22
May 2023	23.443927	22.41
June 2023	24.360801	22.79
July 2023	25.379609	24.08
August 2023	26.278985	25.8
Sept 2023	26.512837	26.72
Oct 2023	26.847091	27.33
Nov 2023	27.214953	28.2
Dec 2023	27.095522	28.92
Jan 2024	27.035387	29.9
Feb 2024	27.206229	31.7
March 2024	27.389161	33.2
April 2024	28.334114	33.69
May 2024	29.192265	33.95
June 2024	30.113503	34.19
July 2024	31.128545	33.4
August 2024	32.031173	32.15
Sept 2024	32.262218	32.7
Oct 2024	32.598895	33.88
Nov 2024	32.964665	34.6
Dec 2024	32.847041	34.8

Table 4.1G: TEST ON THE RESIDUALS OF THE SARIMA MODEL

Type of Test	Test Statistic	p-value
Shapiro-Wilk Test	0.8595	0.0000
Ljung-Box Test	30.106773	0.000823

4.1.4 DISCUSSION OF RESULTS

The SARIMA model fitted to the inflation data from January 2003 to December 2022 reveals several notable insights. The model incorporates both non-seasonal and seasonal dynamics through autoregressive (AR) and moving average (MA) components, along with appropriate differencing to achieve stationarity.

The application of non-seasonal (d=1) and seasonal (D=1, s=12) differencing has proven crucial, transforming the originally non-stationary series into a form suitable for modelling and for resolving the persistent flattened outputs observed in earlier ARIMA attempts.

The results (table 4.1E) show that the non-seasonal AR terms AR(1) is found to be marginally insignificant, with p-value of 0.075, suggesting a weak autoregressive effect at lag 1 indicating that immediate past values at lag 1 have no influence on the present. The AR(2) was found to be highly significant with p-value of 0.002 and the coefficient is 0.4640, suggesting a high degree of persistence in inflation rate movements.

The non-seasonal MA terms MA(1) and MA(2) were found to be statistically significant, as indicated by p-values less than 0.05. This suggests that past forecast errors contribute meaningfully to improving model performance in this case.

The final equation of the SARIMA model is given as: $(1-0.4640B^2)(1-B)y_t = (1+0.5931B-0.3969B^2)\varepsilon_t$

However, residual diagnostics raise important concerns. The Shapiro-Wilk test (table 4.1G) yielded a p-value of 0.0000, indicating that the residuals are not normally distributed. This non-normality was further confirmed by the Jarque-Bera test (table 4.1D), suggesting heavy tails and potential outliers. The heteroskedasticity test yields a statistic of 0.03 with a p-value of 0.00 which shows that the residuals do not have constant variance, violating another key assumption of the SARIMA framework.

It is observed from fig 4.4 that some of the autocorrelations and partial autocorrelations are statistically significant. Also the Ljung-Box test (table 4.1G) indicate the presence of significant autocorrelations in the residuals. Consequently the correlograms of both the autocorrelation and the partial autocorrelation give the impression that the estimated residuals are not purely random, that is not stationary.

In summary, while the SARIMA(2,1,2)(0,1,0)$_{12}$ model captures important trend and seasonal features of inflation dynamics, the diagnostic tests reveal violations of the assumptions of normality, independence, homoscedasticity and stationarity in the residuals. These issues suggest that the model may benefit from adjustments such as exploring other models like dynamic models to better handle volatility. This is the primary objective of this study.

4.2 GEOMETRIC BROWNIAN MOTION

4.2.1 GBM MODEL

The histogram and the scatter plot (fig 4.7) of the log returns of inflation rate was first plotted and tested for normality using the Q-Q plot to ascertain the fulfilment of the normality requirement for the application of a GBM model. The results shown in figure 4.7 clearly indicate that the normality assumption is tenable. The GBM model was then applied to the inflation data from January 2003 to December 2022 as a training set to estimate the parameters μ (drift) and σ(volatility) of the GBM model using the model representation given by equation:

$$I_t = I_{t-1}\exp((\mu - \tfrac{1}{2}\sigma^2)\,\Delta t + \sigma\sqrt{\Delta t}\,.\,Z_t) \qquad (9)$$

The results are shown in table 4.2. This model is then used to obtain forecasted values of inflation rates for the next 24 months (January 2023 to December 2024). The actual and forecasted values of the inflation rate for this period are shown in figure 4.8.

4.2.2 DYNAMIC GBM MODEL.

The dynamic GBM model fitted to the data as explained earlier is represented by

$$I_{t+1} = I_t\exp((\mu_t - \tfrac{1}{2}{\sigma_t}^2)\Delta t + \sigma_t\sqrt{\Delta t}\,.\,Z_t) \qquad (10)$$

Clearly this accommodates the Markov dependence of inflation rates. That is, future evolution of the variable It depends only on its present value and not on the past as stated in the assumption (5) of GBM model.

Consequently the application of this model to data necessitates the implementation of a dynamic algorithm which is briefly described below.

A 240 months rolling window are used. That is, every predicted value is computed based on a 240 months inflation rate data.

We start by using the data for first 240 months (that is January 20023 to December 2022) as our first rolling window. This is used to estimate μ and σ for time t=0, that is $\hat{\mu}_0$ and $\hat{\sigma}_0$. Our I_0 is the inflation rate for December 2022. Our predicted inflation rate for January 2023, $\hat{I}_1 = \hat{I}_0\exp(\hat{\mu}_0 - \frac{1}{2}{\hat{\sigma}_0}^2)\Delta t + \hat{\sigma}_0\sqrt{\Delta t}\,.\,Z_0)$

The first (January 2003) observed value of inflation rate from the 240 month rolling window is removed to obtain the rolling window for the next estimation. $\hat{\mu}_1$ and $\hat{\sigma}_1$ are then estimated from this new rolling window and the estimated value for February 2023 $\hat{I}_2$ is then computed using $\hat{I}_2 = \hat{I}_1\exp(\hat{\mu}_1 - \frac{1}{2}{\hat{\sigma}_1}^2)\Delta t + \hat{\sigma}_1\sqrt{\Delta t}\,.\,Z_1)$

The process is repeated to generate 24 rolling predictions for each of the months from January 2023 to December 2024. The results are shown in table 4.3A and figure 4.9

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TABLE 4.2A: ESTIMATED PARAMETERS OF THE GBM

Parameter	Value
Drift (μ)	0.002932
Volatility (σ)	0.138315

TABLE 4.2B: PERFORMANCE METRICS OF THE GBM

Mean Squared Error (MSE)	66.705851
Root Mean Squared Error (RMSE)	8.167365
Mean Absolute Error (MAE)	6.836863
Mean Absolute Percentage Error (MAPE)	21.73%
Standard Error of Residuals (SER)	8.5305

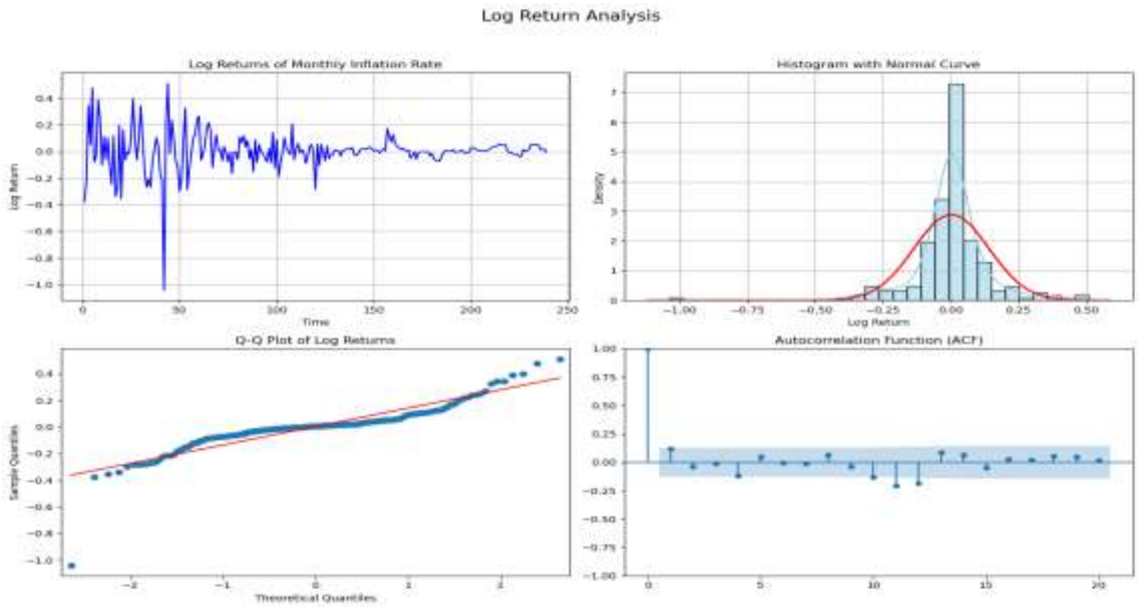

Figure 4.7: Plots showing the analysis on the log returns of the inflation rate

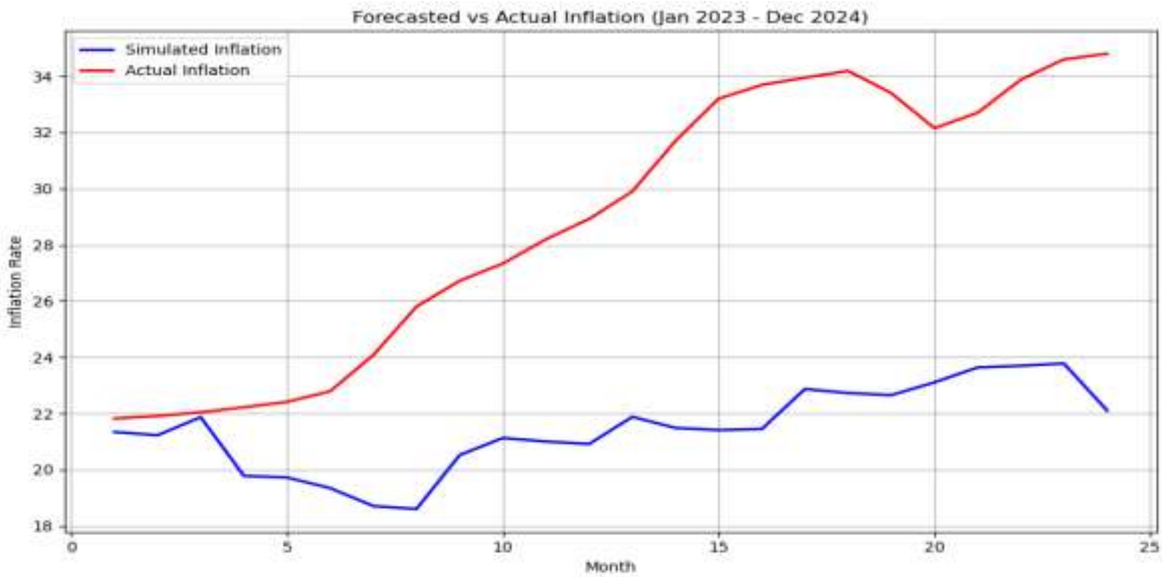

Figure 4.8: GBM Forecasted vs actual inflation rate for January 2023 – December 2024.

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4.3: THE DYNAMIC GBM APPROACH

Table 4.3A: ESTIMATED PARAMETERS OF THE DYNAMIC GBM APPROACH

Month	Estimated Drift	Estimated volatility	Forecasted Inflation rate	Actual Inflation rate
1	0.002932	0.138315	21.34	21.82
2	0.003012	0.138032	21.82	21.91
3	0.003017	0.137746	21.91	22.04
4	0.003029	0.137461	22.04	22.22
5	0.003050	0.137178	22.22	22.41
6	0.003072	0.136897	22.41	22.79
7	0.003128	0.136620	22.79	24.08
8	0.003339	0.136382	24.08	25.8
9	0.003605	0.136170	25.80	26.72
10	0.003732	0.135910	26.72	27.33
11	0.003808	0.135642	27.33	28.2
12	0.003918	0.135381	28.20	28.92
13	0.004002	0.135118	28.92	29.9
14	0.004119	0.134862	29.90	31.7
15	0.004334	0.134639	31.70	33.2
16	0.004499	0.134399	33.20	33.69
17	0.004538	0.134137	33.69	33.95
18	0.004551	0.133874	33.95	34.19
19	0.004560	0.133614	34.19	33.4
20	0.004452	0.133366	33.40	32.15
21	0.004288	0.133134	32.15	32.7
22	0.004336	0.132880	32.70	33.88
23	0.004456	0.132640	33.88	34.6
24	0.004519	0.132390	34.60	34.8

Table 4.3B: MODEL PERFORMANCE OF THE DYNAMIC GBM

Performance metrics	**Values**
Mean Absolute Error	0.7308
Mean Squared Error	0.7849
Root Mean Squared Error	0.8860
Mean Absolute Percentage Error	2.51%
R-squared	0.9659
Standard error	0.9259

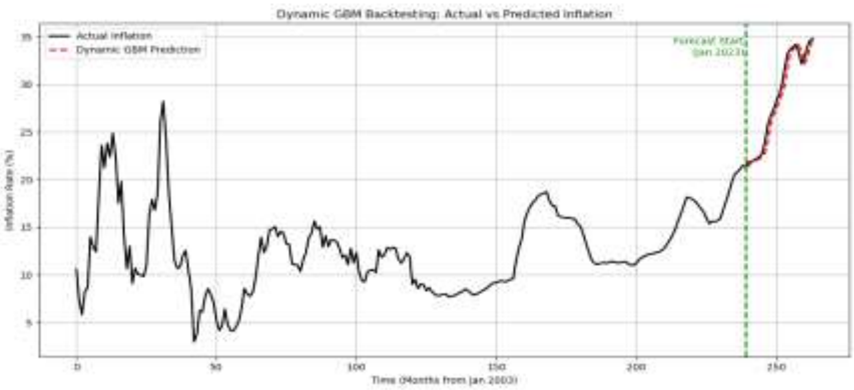

Figure 4.9: Dynamic GBM forecasted vs Actual inflation rate for January 2023 – December 2024

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4.2.3 DISCUSSION OF RESULTS

The results (figure 4.8) for the forecasted values for the GBM model shows that the model is a poor fit to inflation rate data. This prompted a recourse to the application of another model that could bring about an improvement in fit. The GBM model was then modified for this purpose. The modification entails allowing not only inflation rate but also the drift and volatility parameters to be Markov dependent. The results presented in table 4.3B and figure 4.9 shows that a marked improvement in fit is achieved with a high score of value for R^2 =0.967.

Because of the success achieved in the application of the modified GBM model to inflation data, an algorithm for implementation to any similar data for inflation forecasting is developed below.

4.3 GENERALIZED ALGORITHM FOR THE DYNAMIC GEOMETRIC BROWNIAN MOTION

Using a rolling window of size n; forecast the next period using a Geometric Brownian motion**.** At each time t, estimate drift and volatility using the most recent n values, forecast forward for next period using those parameters, and use that value as the forecasted value t+1. Then move to t+1, and repeat the process.

Inputs:

- $S = [S_1, S_2, S_3, \ldots\ldots\ldots\ldots, S_T]$: historical observed data
- n: rolling window size for parameter estimation
- $\Delta t = 1$: time increment (e.g 1 for monthly data)
- $Z \sim N(0,1)$standard random normal shocks

Algorithm:

For each time point where $t \geq n$ and $t < T$:

1. Form the rolling window of size n ending at time t:

$W = [S_{t-n+1}, S_{t-n+2}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, S_t]$

2. Compute log returns from the window:

$R_i = \ln\left(\frac{W_i}{W_{i-1}}\right)$, for $i = t-n+2, \ldots\ldots\ldots\ldots\ldots\ldots t$

3. Estimate drift and volatility parameters:

$$\mu_t = \frac{1}{n}\sum_{i=0}^{n-1} R_i$$

$$\sigma_t = \sqrt{\frac{1}{n-1}\sum_{i=0}^{n-1}(R_i - \mu_t)^2}$$

4. Forecast forward, using GBM formula:

$$\hat{S}_{t+1} = \hat{S}_t \exp\left[\left(\mu_t - \frac{1}{2}\sigma_t^2\right)\Delta t + \sigma_t\sqrt{\Delta t}.Z_t\right]$$

5. Store $\hat{S}_{t+1}$ as the forecasted value for time $t+1$

6. Advance to next time point $t = t+1$, and repeat steps 1-5

Output:

A sequence of forecasted values: $\{\hat{S}_{n+1}, \hat{S}_{n+2}, \ldots\ldots, \hat{S}_T\}$

Summary:

- Drift and volatility are re-estimated at each time step using a fixed-length rolling window.
- GBM is simulated only one step ahead and that forecast is recorded
- The process shifts forward by one time point and repeats.

V. CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

This research demonstrates that inflation modeling using Geometric Brownian Motion can be greatly enhanced through the introduction of dynamic and recursive forecasting structures. The Dynamic Recursive GBM Model, provides a powerful tool for forecasting inflation over multiple time horizons while adapting to evolving macroeconomic conditions.

By incorporating rolling window parameter estimation and recursive updating of forecasted values, the model overcomes the limitations of fixed-parameter assumption in SARIMA and traditional GBM models and therefore captures the nonlinear and stochastic nature of inflation more effectively. The comparative evaluation with SARIMA and traditional GBM confirms that the dynamic model offers superior flexibility, statistical validity, and forecasting accuracy.

5.2 Recommendations

1. In developing countries like Nigeria, significant discrepancies often exist between policy formulation, implementation and actual outcomes. These gaps are frequently due to inconsistencies in policy execution and the difficulty monetary authorities face in monitoring and predicting future inflation trends with precision. This poses a pressing macroeconomic challenges for policy makers and the central bank. The performance metrics of the dynamic GBM model, particularly its high R^2show that it is highly reliable for inflation rate forecasting. It is therefore recommended that the CBN and other macroeconomic agencies should consider

adopting it as more responsive and adaptive forecasting model.

2. The dynamic GBM model should be embedded into real-time forecasting platforms where the estimation window and forecast path are updated monthly to provide timely and data-driven inflation outlooks.

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