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Optimization of Prior and Posterior Algorithm using Pearson's Product Moment Correlation Analysis

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ABSTRACT
The study aims to analyze the prior and posterior algorithm using Pearson's Product Moment Correlation from Bayesian model iteration. The objective is to ascertain the magnitude of difference between alternative courses of action with the degree of association indicators available for decision making under the situation of certainty and uncertainty in Anambra-Imo river basin. The methodology involves the use of correlation and regression analysis with posterior probability as the dependent variable (y) against the prior probability as the independent variable (x) to confirm their relationship at first, second and third iterations of bayesian decision model at the river basin. The results show that the values obtained from the computed coefficient of correlation and graphical estimation from the regression function were consistent. The computed values of r and the estimated values of (r) obtained are greater than the critical values of r at 0.05 and 0.01 level of significance and degree of freedom (d_f) = 18. respectively for the first, second and third iterations. This confirms that there is a strong positive linear correlation and genuine relationship between the posterior and prior probabilities used in the analysis of the Bayesian Decision model theory.
Keywords: prior, posterior, probabilities, correlation, regression function.

I. INTRODUCTION

The prior and posterior probabilities outcomes are the process of optimization of river basin resources utilization in a multi-purpose/multi-objective river basin development planning and management using Bayesian model optimization techniques. The various purposes of irrigation, agriculture, hydro-electric power generation, water supply, navigation/water transport, drainage/dredging, flood control, recreation/tourism, erosion control, plantation/forestry and reservoir/gullies are for water resources capital development projects. The consideration of the net benefits (objectives) of economic efficiency federal economic redistribution, regional economic redistribution, state economic redistribution, local economic redistribution, social well-being, youth empowerment, environmental quality improvement,. Gender equality and security prior posterior probabilities were analyzed using Anambra-Imo river basin development authority in Nigeria. The hypothesis expresses this relationship between the posterior and prior probabilities in the Bayesian decision model analysis.

II. LITERATURE REVIEW

Ezenweani (2012) identified that inability of management of river basin to control the whole basin and lack of baseline data with inadequate monitoring are some of the problems that hinders River basin development planning and management. Klare (2001) also said that politics to determine who is to be employed, what is on the agenda and how river basin development planning and management proceeds also affects them.

The multipurpose/ multi-objective river basin development project planning and management are multi-disciplinary and may involve a lot of complex situations. Borrow (1998) stated that River basin development planning and management is the process of identifying the best way in which a river and its tributaries may be used to meet competing demands while maintaining river health. It includes the allocation of scarce water resources between different users and purposes, choosing between environmental objectives and competing human needs and choosing between competing food risk

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management requirements (Molle, 2006). The increasing complexity of many of the river basins occasioned by increasing development and population pressure, have resulted many serious crisis related to floods, degradation of water quality, acute water shortage and degradation of ecological health. The various approaches to river basin planning is ultimately playing significant roles to the adaptation of the local circumstances.

The multipurpose/ multi-objective river basin development project planning and management will help to determine levels of development to be apportioned to various purposes for water resources projects. The considerationof economic efficiency, federal economic redistribution, regional economic redistribution, state economic redistribution, local economic redistribution, social well-being, environmental quality improvement, youth employment, gender equality and security are becoming more relevant due to some political, ecological and health concern of the people.

The definition of terms in Bayesian Decision Theory (BDT) was based on Sharma (2008). Bayesian Decision theory involves decision making under risk which is a probabilistic decision situation. In this concept more than one state of nature exists and the decision maker has sufficient information to assign probability values to the likely occurrence of each of these states. When the probability distribution of the states on nature is known, the best decision is to select that course of action which has the largest expected payoff value. The expected (average) payoff of an alternative is the sum of all possible payoffs of that alternative weighed by the probabilities of these payoffs occurring.

Although BDT can be subjective but its subjectivity can be employed as a powerful attribute which considers experts' unbiased opinion as input into the policy iteration algorithm to produce an optimum solution or decision. Bayesian Decision Theory (BDT) can be used for data mining, and Bayesian Decision network for decision making with little or no data.

Bayesian theory describes the magnitude of the difference between alternative actions and provides a variety of estimates for consideration. The decision problem which involves the use of prior probabilities is often called "no data" problems and those involving posterior probabilities are called "data" problems. The increase in expected income as a result of using data is referred to as "value of the data", value of added information, or value of the observation.

The optimal Bayes strategy is generally referred to as one which maximizes the expected monetary value. Bayes' theorem is a mathematical techniques used for updating probabilities on additional information. The policy iteration algorithm of Bayesian Decision Model or Payoff matrix can handle number of "state of nature" and alternative course of action infinitely.

2.1 Bayesian Theory Analysis

This is concerned with the method of computing posterior probability from prior probabilities using Bayes' theorem. An initial probability statement to evaluate expected payoff is called a prior probability distribution. The one which has been revised in the light of new information is called a posterior probability distribution. What is a posterior to one sequence of state of nature becomes the prior on others which are yet to happen. A further analysis of problems using these probabilities with respect to new expected payoffs with additional information is called prior-posterior analysis. The general terms of Bayes' theorem can be stated as follows:-

Let A_1, A_2, … A_n be mutually exclusive and collectively exhaustive outcomes. Their probabilities $P(A_1)$, $P(A_2)$, … $P(A_n)$ are known when there is an experimental outcome, B for which the conditional probabilities $P(B/A_1)$, $P(B/A_2)$, … $P(B/A_n)$ are also known given the information that outcome B has occurred, the revised conditional probabilities of outcomes A_i, i.e. $P(A_i/B)$, i = 1, 2, …, n are determined by using the following conditional probability relationship:

$$P(A_i/B) = \frac{P(A_i \text{ and } B)}{P(B)}$$

$$= \frac{P(A_i \text{ n } B)}{P(B)} \qquad (1)$$

where $P(B) = P(A_1 \text{ n } B) + P(A_2 \text{ n } B) + \cdots + P(A_i \text{ n } B)$.

Since each joint probability can be expressed as the product of a known marginal (prior) and conditional probability, $P(A_i \text{ n } B) = P(A_i) \times P(B/A_i)$

$$\text{Thus } P(A_i/B) = \frac{P(A_i)P(B/A_i)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + \cdots \_ + P(A_n)P(B/A_n)}$$

The Bayesian Analysis involves the computation of Expected Monetary Value (EMV), Expected Opportunity Loss (EOL), Expected Value of Perfect Information (EVPI), Expected Profit with Perfect Information (EPPI) and Expected Value of Sample Information (EVSI).

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Expected Monetary Value (EMV):

The Expected Monetary Value (EMV) or Expected Utility is the most widely used criterion for evaluating various courses of action (alternatives) under risk. The expected monetary value (EMV) for a given course of action is the weighted sum of possible payoffs for each alternative. It is obtained by adding up the payoffs for each course of action multiplied by the probabilities associated with each state of nature. The expected (or mean) value is the long-run average value that would result if the decision were repeated a large number of times. Mathematically, EMV is stated as follows:

$$\mathrm{EMV}\,(\text{Course of action}, \mathrm{S_i}) = \sum_{i-1}^{m} \mathrm{P_{ij}\,P_i} \qquad (2)$$

Where m = number of possible states of nature
P_i = probability of occurrence of state of nature N_i
P_{ij} = Payoff associated with state of nature, V_i and course of action, S_j.
Calculating EMV involves the following steps:

(i). Construct a payoff matrix using all possible courses of action and states of nature
(ii). Enter the conditional payoff values associated with each possible combination of course of action and states of nature along with the probabilities of the occurrence of each course of action.
(iii). Calculate the EMV for each course of action by multiplying the conditional payoffs by the associated probabilities and add these weighted values for each course of action.
(iv). Then select the course of action that yields the optimal EMV*.

Expected Opportunity Loss (EOL):

Expected Opportunity Loss (EOL) is an alternative approach to maximizing the expected monetary value (EMV) by minimizing the expected opportunity loss (EOL). This is also called expected value of regret. Expected Opportunity Loss means the difference between the highest profit (or payoff) for a state of nature and the actual profit obtained for the particular course of action taken. In fact EOL is the amount of payoff that is lost by not selecting the course of action that has the highest payoff for the state of nature that actually occurs. The course of action due to which EOL is minimum, is recommended. The Expected Opportunity Loss as an alternative decision making under risk is synonymous with EMV criterion so any two of the method are applied to reach a decision. Mathematically,

$$\mathrm{EOL}\,(\text{state of nature}, \mathrm{N_i}) = \sum_{i=1}^{m} \mathrm{E_{ij}\,P_i} \qquad (3)$$

E_{ij} = opportunity loss due to state of nature, $N_i$and course of action, S_j.
P_i = probability of occurrence of state of nature, N_i.
The following steps are involved in the computation of Expected Opportunity Loss (EOL):

(i). Prepare a conditional profit table for each course of action and state of nature combination along with the associated probabilities.
(ii). Calculate the conditional opportunity loss (COL) values for each state of nature by subtracting each payoff from the maximum payoff for that outcome.
(iii). Calculate EOL for each course of action by multiplying the probability of each state of nature with the COL value and add-up the values.
(iv). Select a course of action for which the Expected Opportunity Loss (EOL) is minimum.

Expected Value of Perfect Information (EVPI):
For a decision maker under risk, perfect (complete and accurate) information about the occurrence of various states of nature, will make him to select a course of action that yields the desired payoff for whatever states of nature that actually occurs. EMV or EOL criterion helps the decision maker to select a particular course of action that optimizes the expected payoff without any additional information. The Expected Value of Perfect Information (EVPI) is the maximum amount of money the decision maker has to pay to get this additional information about the occurrence of various states of nature before a decision is made.
Mathematically,

$$EVPI = (Expected\ Profit\ with\ Perfect\ Information) - Expected\ profit\ without\ Perfect\ Information$$

$$\therefore EVPI = \sum_{i=1}^{m} P_{ij}\ max(P_{ij}) - EMV \qquad (4)$$

where;
P_{ij} = best payoff when action, Sj is taken in the presence of state of nature, N_i.
P_i = probability of state of nature, N_i;
EMV* = maximum expected monetary value.
∴ EVPI = EPPI – EMV*.

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Expected Profit with Perfect Information (EPPI) is determined or calculated by summing up the multiplication of prior probabilities on each states of nature by the largest values on each courses of action.

Expected Value of Sample Information (EVSI) is obtained by multiplying posterior EOLs with their probabilities. This represents the money which the decision maker has to pay for hiring the services of a consultant.

Courses of Action (actions, acts or strategies) is the number and type of alternatives though may be dependent on the previous decisions made and on what has happened subsequently to those decisions under the control of the decision maker e.g. conditioning a market survey to know the likely demand of an item.

States of Nature are the future conditions (also called consequences, events or scenarios) not under the control of the decision maker e.g. state of economy (inflation), a weather condition, a political development, act of God, etc.

The States of Nature are mutually exclusive and collectively exhaustive with respect to any decision problem.

Payoff: Payoff is a numerical value (outcome) resulting from each possible combination of alternatives and states of nature. The values of payoff are always conditional values because of unknown states of nature. Payoff is measured within a specified period (e.g. yearly) and this period is referred to as **the decision horizon**. The payoffs considered in most decisions are monetary which are measured in terms of money market share, or other measures.

2.2 Person's Product Moment Correlation Analysis according to Nwabuokei (1986)

The Product Moment Coefficient of Correlation

If the relationship lies between X and Y, a linear, a precise quantitative measure of the degree of correlation between the two variables is the Pearson's Product Moment Coefficient of Correlation. Designated by the letter, r, the Pearson's Product Moment Coefficient of correlation is calculated by the formula

$$\frac{\sum_i^n (x-\bar{x})(y-\bar{y})}{nS_xS_y}$$

$$\frac{\sum_i^n (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 \sum (y-y)^2}}$$

Where S_x = standard deviation of the x values
S_y = standard deviation of the y values.

Another form of the formula for finding the product moment coefficient of correlation is given as

$$\frac{\frac{1}{n}\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\left[\frac{1}{n}\sum (x-\bar{x})^2\right]\left[\sum \frac{1}{n}(y-\bar{y})^2\right]}}$$

For easier arithmetic computation of r, formula above is usually written in the form;

$$r = \frac{\sum xy - n\,\bar{x}\bar{y}}{\sqrt{[\sum x^2 - n\bar{x}^2][\sum y^2 - n\bar{y}^2]}}$$

$$r = \frac{n\sum xy - \sum x.\sum y}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}}$$

The values of the **product moment coefficient of correlation** vary between the limits ±1. A positive sign indicates positive correlation, while a negative sign shows negative correlation. When r = – 1 the correlation is perfect and negative. When the r approaches +1, we have a strong, positive correlation. When r approaches – 1, we have a strong, negative correlation. When r approaches zero, the correlation is weak. A value of r = 0, indicates the absence of correlation.

(d) The Coefficient of Determination and the Coefficient of Non-Determination

i. The Coefficient of Determination

The linear correlation between X and Y may be considered as a measure of the proportion of the variation in Y explained by the regression equation. The total variation in Y can be represented as the sum of explained variation and the sum of unexplained variation as follows:

Total variation = Explained variation + Unexplained variation

$$i.e. \sum (y_i - \bar{y})^2 = \sum (y_x - \bar{y})^2 + \sum (y_i - y_x)^2 \qquad (9)$$

$$The\ ratio\ \frac{\sum (y_i - y_x)^2}{\sum (y_i - y_x)^2}\ is\ the\ proportion\ of the\ total\ variation\ that\ may\ be\ unexplained$$

by the regression equation.

The coefficient of determination which is defined as

$$r^2 = 1 - \frac{\sum (y_i - y_x)^2}{\sum (y_i - y_x)^2} \qquad (5)$$

$$= 1 - \frac{Unexplainedvariation}{Totalvariation} \qquad (6) \qquad (10)$$

$$= \frac{Explainedvariation}{Totalvariation}$$

represents the proportion of the total variation in Y that has been explained by the regression equation.

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ii. Computational Formula for r^2,

The computational formula for finding the coefficient of determination is given as

$$r^2 = \frac{a\sum y + b\sum xy - n\,\bar{y}^2}{\sum y^2 - n\bar{y}^2} \quad (11)$$

$$\text{The coefficient of correlation becomes } r = \sqrt{r^2} \quad (12)$$

iii. The Coefficient of Non determination

The value $(1 - r^2)$ is called the coefficient of non determination. It measures the proportion of the variation of the Y values that has not been explained by the regression equation; that is the variation in Y due to factors other than X. The square root of the coefficient of non-determination.

$$T = \sqrt{1 - r^2} \quad (13)$$

is called the **coefficient of alienation.** This coefficient measures the extent of departure from perfect correlation.

2.3 Coefficient of Determination and Regression Equation

The Coefficient of Determination is calculated using the values of independent (x) and dependent (y) variables in a linear relationship. The slope or gradient of the regression (b) and the intercept on the dependent variable axis (a) are calculated to determine the regression equation as stated below.

$$\bar{X} = \frac{\sum X}{n} = \bar{Y} = \frac{\sum Y}{n} \quad (14)$$

where $\bar{X}$ = the sample mean of the independent variable.

$\bar{Y}$ = the sample mean of the dependent variable.

n = the number of samples or variables.

The slope or the gradient of the linear regression equation is determined by

$$b = \frac{\sum XY - n\bar{X}\bar{Y}}{\sum X^2 - n\,\bar{X}^2};\ a = \bar{Y} - b\bar{X} \quad (15)$$

The regression equation = y = a + bx (16)

The sample coefficient of determination (R or r^2)

$$r^2 = \frac{a\sum Y - b\sum XY - n\bar{Y}^2}{\sum Y^2 - n\,\bar{Y}^2} \quad (17)$$

The sample coefficient of determination (r^2) is the proportion of variation in the dependent variable (Y) that has been accounted for or explained by the variation in independent variable (X). it is a very useful assessment of how closely the regression equation fits the data. This shows how the total variation in the dependent variable (Y) is explained by the relationship between the independent variable (X) and the dependent variable (Y) in a regression equation to ascertain whether additional variable need to be added or it is a linear relationship.

The coefficient of correlation (r) is the square root of the coefficient of determination i.e. $r = \sqrt{r^2}\ or\ \sqrt{R}$. It measures the probability that there is a genuine relationship between the variables and that it has not risen by chance

The value of the coefficient of correlation (r) ranges from (–0) to (+1). This value of zero shows no correlation. As the value of –1 shows a perfect strong negative correlation while the value of +1 shows a perfect strong positive correlation.

III. METHODOLOGY

The methodology involves the use of correlation and regression analysis with posterior probability as the dependent variable (y) against the prior probability as the independent variable (x) to confirm their relationship at first, second and third iteration bayesian decision model at the river basin.

IV. ANALYSIS AND DISCUSSION OF RESULTS

4.1 Pearson's Product Moment Correlation Coefficient (PPMCC) between Prior and Posterior Probability of 1st Iteration values and Testing the Hypothesis

The Prior and Posterior Probabilities were correlated to determine the coefficient of correlation on Table 1.

Table 1: PPMCC of Prior and Posterior Probabilities Outcomes at 1st Iteration

S/N	**Prior (X)**	**Posterior (Y)**	**XY**	**X^2**	**Y^2**
(i)	0.02	0.006	0.00012	0.0004	0.000036
(ii)	0.07	0.046	0.00322	0.0049	0.002116
(iii)	0.03	0.012	0.00036	0.0009	0.000144

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(iv)	0.04	0.013	0.00052	0.0016	0.000169
(v)	0.09	0.061	0.00549	0.0081	0.003721
(vi)	0.10	0.080	0.008	0.01	0.0064
(vii)	0.09	0.061	0.00549	0.0081	0.003721
(viii)	0.07	0.050	0.0035	0.0049	0.0025
(ix)	0.08	0.051	0.00408	0.0064	0.002601
(x)	0.41	0.620	0.2542	0.1681	0.3844
Total	**1.00**	**1.00**	**0.28498**	**0.2134**	**0.405808**

$$r = \frac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}$$

$$r = \frac{10 \times 0.28498 - (1.00)(1.00)}{\sqrt{[10 \times 0.2134 - (1.0)^2][10 \times 0.405808 - (1.0)^2]}}$$

$$= \frac{1.8498}{\sqrt{[1.134 \times 3.05808]}} = \frac{1.8498}{\sqrt{3.4679}} = \frac{1.8498}{1.8622} = 0.9933$$

Discussion of Results in Table 1

(i) The coefficient of correlation r = 0.9933 which shows that there is a strong positive correlation in the prior and posterior probabilities used in the Bayesian decision analysis of the Multi - purpose/Multi-objective Anambra/Imo River Basin Development Projects.

(ii) Using another measure, the Coefficient of Determination (R) which is the square of coefficient of correlation we have, R (or r^2) = 0.9933^2 = 0.9867 shows that 98.67% of the total variation in Posterior probabilities is explained by the variation in the Prior probabilities for the 1st iteration while the coefficient of non-determination ($1-r^2$) = 1 – 0.9867 = 0.0133 or 1.33 percent of the variation in Posterior probabilities is attributable to other factors not explained by the regression function. **This result will reject the null hypothesis and accept the alternative hypothesis that there is a relationship between the prior and posterior probabilities on first iteration used in the Bayesian Decision Analysis.**

(iii) The degree of freedom (d_f) was determined as $d_f = P_1 + P_2 - 2 = 10 + 10 - 2 = 18$. Referring to the table of critical values of r at 0.05 and 0.01 level of significance, we have $r_{0.05}$ at d_f (18) = 0.4435 and $r_{0.01}$ at d_f (18) = 0.5614. The critical values of r at 0.05 and 0.01 level of significance with d_f = 18 are 0.4435 and 0.5614 respectively. Since the computed coefficient of correlation (r = 0.9933) is greater than the critical values of $r_{0.05}$ = 0.4435 and $r_{0.01}$= 0.504, we reject the null hypothesis and accept the alternative hypothesis.

The regression function was as shown in the graph in Figure 1

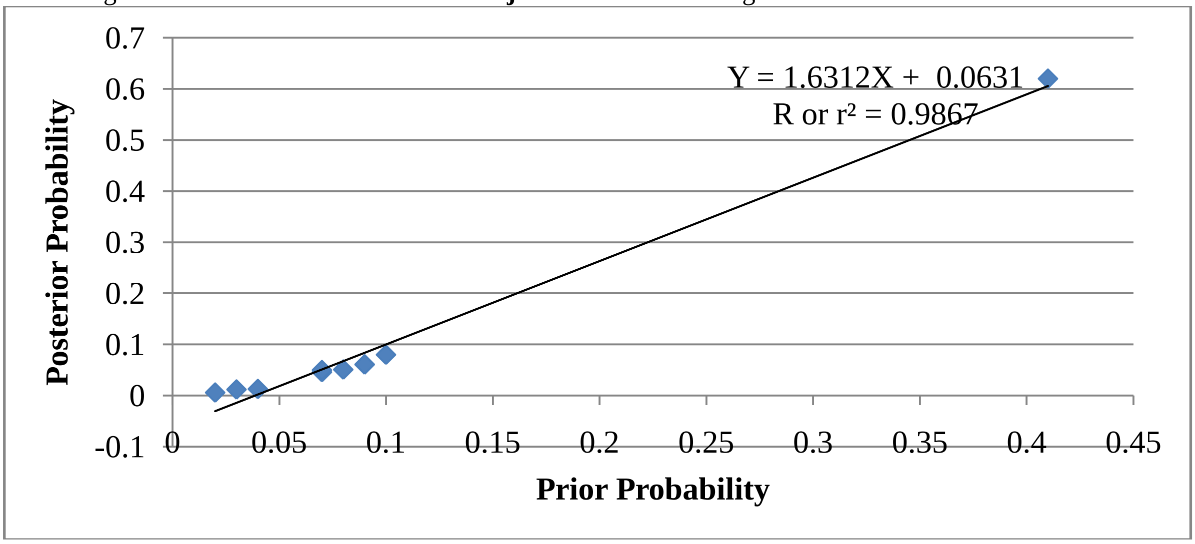

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Figure 1: The Graph of Posterior probability versus Prior probability of the regression function on first iteration

Discussion of result in Figure 1:

(i). The graph of the posterior probability outcomes versus prior probability on first iteration shows the linear function of best fit at Y = 1.631X + 0.0631.

(ii) The value of R or r^2 from the graph is 0.9867 which corresponds exactly with the value of r^2 (0.9867) obtained from the calculated value of coefficient of determination, then r = 0.9933..

(iii). The implication is that 98.67 % of the total variation in posterior probabilities outcomes is explained by the variation in prior probability outcomes and the values has not risen by chance.

(iv) The coefficient of non-determination $(1 – r^2)$ = 1 – 0.9867 = 0.0133. The implication of this is that only 1.33% of the variation in posterior probabilities outcomes is attributable to other functions not explained by the regression function. The values of r above 90 % confirm a strong perfect positive correlation.

(v) The coefficient of alienation $(\mathrm{T}) = \sqrt{1-\mathrm{r}^2} = \sqrt{0.0133}$ = 0.1153 shows the extent of departure from perfect correlation. This confirms the result from the first test that a strong perfect positive relationship exists between the posterior and prior probabilities.

(vi) As in the test of correlation coefficient, for the first iteration, the computed correlation is greater than the critical values of r at 0.05 level of significance = 0.4438 and r at 0.01 level of significance = 0.5614 with d_f= 18. This also confirmed the result from the first test that a strong perfect relationship exists between the posterior and prior probabilities.

4.2 Pearson's Product Moment Correlation Coefficient between Prior and Posterior Probability of 2nd iteration values

The Prior and Posterior Probabilities were correlated to determine the coefficient in Table 2.

Table 2: PPMCC of Prior and Posterior Probabilities Outcomes at 2nd Iteration

S/N	Prior (X)	Posterior (Y)	XY	X^2	Y^2
(i)	0.006	0.0017	0.000010	0.000036	0.00003
(ii)	0.046	0.024	0.001104	0.002116	0.000576
(iii)	0.012	0.0040	0.000048	0.000144	0.000016
(iv)	0.013	0.0043	0.000056	0.000169	0.000018
(v)	0.061	0.0345	0.002105	0.003721	0.000119
(vi)	0.080	0.0532	0.004256	0.0064	0.002830
(vii)	0.061	0.0345	0.002105	0.003721	0.000119
(viii)	0.050	0.0276	0.00138	0.0025	0.000762
(ix)	0.051	0.0281	0.001433	0.002601	0.000790
(x)	0.620	0.7881	0.488622	0.3844	0.621102
Total	**1.00**	**1.00**	**0.501119**	**0.405808**	**0.626335**

$$\mathrm{r} = \frac{\mathrm{n}\sum \mathrm{XY} - \sum \mathrm{X} \sum \mathrm{Y}}{\sqrt{[\mathrm{n}\sum \mathrm{X}^2 - (\sum \mathrm{X})^2][\mathrm{n}\sum \mathrm{Y}^2 - (\sum \mathrm{Y})^2]}}$$

$$\mathrm{r} = \frac{10 \times 0.501119 - (1.00)(1.00)}{\sqrt{[10 \times 4.05808 - (1.0)^2][10 \times 0.626335 - (1.0)^2]}}$$

$$= \frac{4.01119}{\sqrt{[3.05808 \times 5.26335]}} = \frac{4.01119}{\sqrt{16.09575}} = \frac{4.01119}{4.01195} = 0.9998$$

r = 0.9998, R or $(r^2) = 0.9998^2$ = 0.9996 or 99.96 %

Discussion of Results in Table 2

(i). The coefficient of correlation, r = 0.9998 which shows that there is a strong positive correlation in the prior and posterior probabilities used in the Bayesian decision analysis of the Multi -purpose/Multi-objective Anambra/Imo River Basin Development Projects on second iteration.This is an improvement on the first iteration.

(ii). Using another measure, the coefficient of determination (R) which is the square of coefficient of correlation R (or r^2) = 0.9996 = 0.99.96 shows that 99.96 % of the total variation in Posterior probabilities is explained by the variation in the Prior probabilities for the second iteration.

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(iii). The coefficient of non-determination ($1-r^2$) = 1 – 0.9996 = 0.0004 or 0.04 percent shows that 0.04% of the variation in Posterior probabilities is attributable to other factors not explained by the regression function.

The coefficient of alienation $I = \sqrt{1 - r^2}$

$$= \sqrt{0.0004} = 0.02$$

(iv) This describes that the extent of departure from perfect correlation is only two (2) percent.

(v) **This result will reject the null hypothesis and accept the alternative hypothesis that there is a genuine relationship between the prior and posterior probabilities on the second iteration used in the Bayesian Decision model.**

(vi) The result of the test of correlation coefficient for first iteration shows that the computed coefficient of correlation is greater than the critical values of r at 0.05 level of significance = 0.4435 and r at 0.01 level of significance = 0.5614 with d_f = 18. This confirms the result which rejects the null hypothesis and accepts the alternative hypothesis.

The graph of posterior and prior probabilities on second iteration is shown in Figure 2.

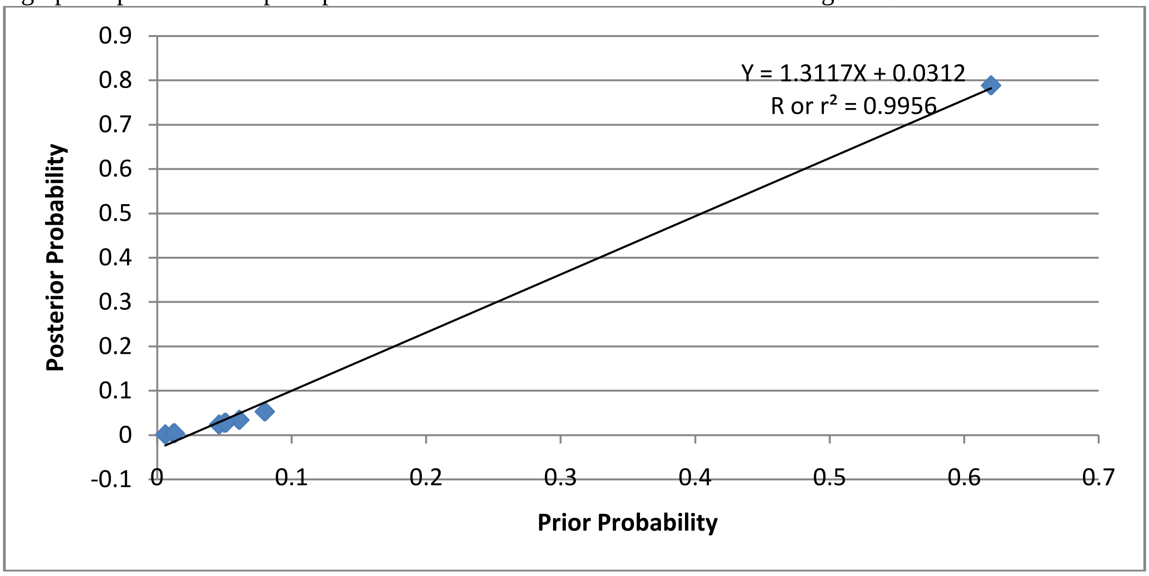

Figure 2: The graph of posterior and prior probabilities on second iteration and the regression function equation

Discussion of the Results in Figure 2:

(i). The graph of linear functions of posterior versus prior probability outcomes on second iteration shows the best lie of fit at Y = 1.3117 X + 0.0312

(ii) The estimation of the graphical function for R or r^2 of 0.9956 is approximately equal to the calculated value of 0.9996 of coefficient of determination, then $r = \sqrt{0.9956} = 0.9978$.

(iii) The implication from the graphical result shows that 99.56 % of the total variation in posterior probability outcomes is explained by the variation of prior probability outcomes and the values has not risen by chance.

(iv) The coefficient of non-determination (0.0044) and the coefficient of alienation $(T) = \sqrt{1 - r^2} = \sqrt{0.0044}$ = 0.0663.

(v) The result is consistent with result on calculated values and confirms the result of the hypothesis test on that there is a perfect relationship between the posterior and prior probability outcomes on the second iteration.

4.3 Pearson's Product Moment Correlation Coefficient between Prior and Posterior Probability of 3rd iteration values/Hypothesis Testing.

The Prior and Posterior probabilities outcomes were correlated to determine the coefficient of correlation between the variables as shown on Table 3.

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Table 3: PPMCC of Prior and Posterior Probabilities Outcomes at 3rd Iteration

S/N	Prior (X)	Posterior (Y)	XY	X^2	Y^2
(i)	0.0017	0.0003	0.0000051	0.000003	0.00000009
(ii)	0.024	0.0122	0.0002928	0.000576	0.00014884
(iii)	0.0040	0.0012	0.0000048	0.000016	0.00000144
(iv)	0.0043	0.0009	0.00000387	0.000018	0.00000081
(v)	0.0345	0.0175	0.00060375	0.000119	0.00030625
(vi)	0.0532	0.0315	0.0016918	0.002831	0.00101124
(vii)	0.0345	0.0175	0.00060375	0.000119	0.00030625
(viii)	0.0276	0.0149	0.00041124	0.000762	0.00022201
(ix)	0.0281	0.0134	0.0003765	0.000790	0.00017956
(x)	0.7881	0.8906	0.70188186	0.621102	0.79316836
Total	**1.00**	**1.00**	**0.7058**	**0.626335**	**0.79534485**

$$r = \frac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}$$

$$r = \frac{10 \times 0.7058 - (1.00)(1.00)}{\sqrt{[10 \times 0.626335 - (1.0)^2][10 \times 0.79534485 - (1.0)^2]}}$$

$$r = \frac{6.0557688}{\sqrt{[5.26335 \times 6.9534485]}} = \frac{6.0557688}{\sqrt{36.598433}} = \frac{6.0557688}{6.0497} = \frac{6.056}{6.050} = 1.001009$$

r = 1.00, R or (r^2) = 1.000 or 100.00 %

Discussion of Results in Table 3

(i) This shows that the coefficient of correlation r = 1.00 at 3rd iteration which indicates a strong perfect positive correlation in the prior and posterior probabilities used in the Bayesian decision model analysis of the Multi - purpose/Multi-objective Anambra/Imo River Basin Development Projects.

(ii) Using another measure, the coefficient of determination (R or r^2) which is the square of coefficient of correlation R (or r^2) = 1.00. This shows that 100 % of the total variation in Posterior probabilities is explained by the variation in the Prior probabilities for the third iteration and has not risen by chance.

(iii) The coefficient of non-determination ($1-r^2$) = 1 – 1 = 0. This shows that no proportion of the variation in Posterior probabilities is attributable to other factors not explained by the regression function.

(iv) The coefficient of alienation, $T = \sqrt{1 - r^2} =$ 0 which shows that there is no departure of the variables from perfect correlation.

(v) From the result obtained in first and second iteration, the critical value (r) of correlation coefficient for d_f = 18 at 0.05 level of significance and at 0.01 level of significance are r = 0.4438 and r = 0.5614 respectively. This shows that the computed r value is greater than the critical r values thus rejecting the null hypothesis as already stated.

(vi) Referring to the above result, the **null hypothesis** is rejected while the **alternative hypothesis** is accepted that there is a strong perfect relationship between the prior and posterior probabilities on the third iteration used in the Bayesian Decision Analysis is accepted.

The graph of posterior versus prior probability outcome on third iteration is shown in Figure 3.

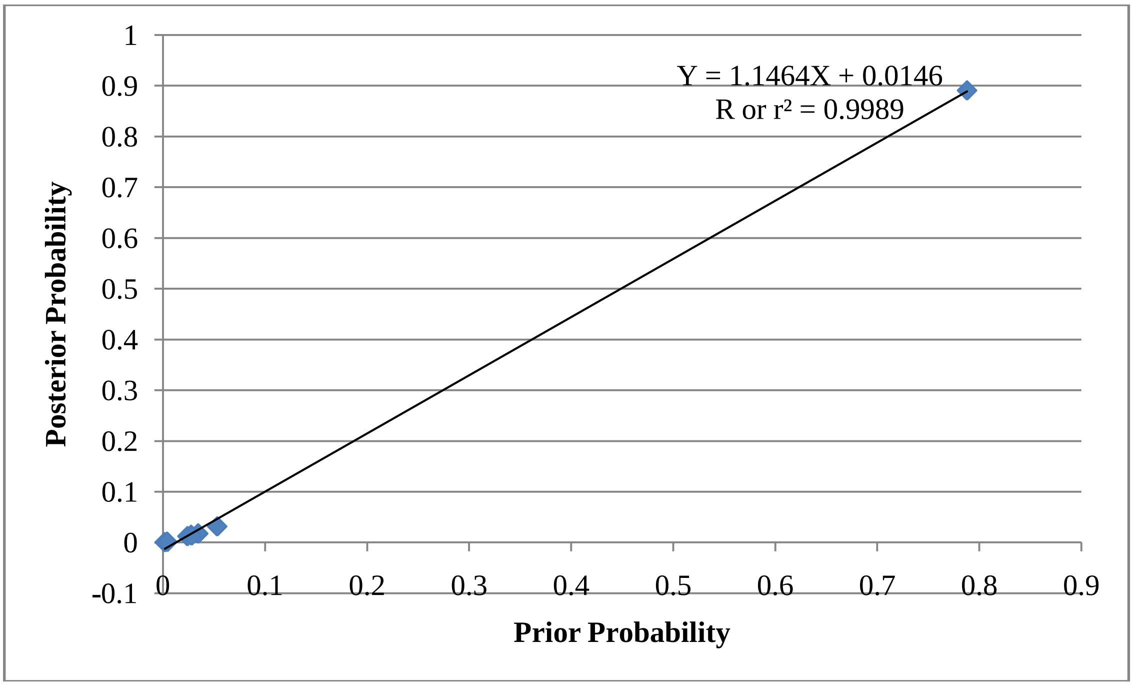

Figure 3: The graph of Posterior versusPrior probabilities on third iteration and regression function equation

Discussion of Results in Figure 3

i. The graph of the regression function of posterior probability outcomes versus prior probability outcomes on third iteration is linear relationship which shows the line of best fit at Y = 1.1464 X + 0.0146.

ii. The estimation of the graphical function for R or r^2 = 0.9989 and r = $\sqrt{R} = \sqrt{0.9989}$ = 0.9994 is very consistent with the value obtained from the calculated value of R or r^2 = 1.000 and r = 1.0000.

iii. The graphical results show that 99.89 % is the total variation in posterior probabilities is explained by the variation in prior probabilities on third iteration which the calculated value expressed as 1.00 or 100%.

iv. The coefficient of non-determination is zero, the coefficient of alienation is zero from the calculated values while from the graphical values $1 - r^2 = 1 - 0.9989 = 0.0011$ which is the coefficient of non-determination and, the coefficient of alienation (T) = $\sqrt{1 - r^2}$ = $\sqrt{0.0011}$ = 0.0332

v. The coefficient of alienation shows that there is 3.32 % extent of departure from perfect correlation.

vi. The result of these tests from the graphical analysis is consistent with the results obtained from the calculated values.

vii. The graphical estimation of r = $\sqrt{0.9989}$ = 0.9994, and the computed r value of 1.000 are greater than the critical value of correlation coefficient as stated before which affirms the result of hypothesis tested for $r_{0.05}$ = 0.4438 or $r_{0.01}$ = 0.5614.

viii. This also confirms the rejection of the null hypothesis and acceptance of the alternative hypothesis that there is a strong perfect genuine relationship between the posterior and prior probability outcomes on third iteration.

4.4 Testing the Hypothesis

The Degree of Freedom (D_f) = $P_1 + P_2 - 2$

Where; P_1 = number of values in the independent variable column.

P_2 = number of values in the dependent variable column.

The Degree of Freedom (D_f) = $P_1 + P_2 - 2 = 10 + 10 - 2 = 18$.

The Critical Values of Coefficient of Correlation (r) using Critical Values of Pearson's Correlation Coefficient with D_f = 18 are:

At 0.05 level of significance, r = 0.4438.

At 0.01 level of significance, r = 0.5614.

The Coefficient of Correlation (r) = 0.9933 on first (1st) iteration; r = 0.9998 on second (2nd) iteration; r = 1.00 on the third (3rd) iteration.

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The results show that the calculated value of Coefficient of Correlation (r) = 0.9933, 0.998 and 1.00, at first, second and third iterations respectively are greater than the critical values of Coefficient of Correlation (r) at degree of freedom (D_f) = 18 which is 0.4438 at 0.05 level of significance and 0.5614 at 0.01 level of significance.

The results of the analysis shows a strong positive linear correlation which confirms the results of other tests that there is a genuine linear relationship between the posterior and prior probabilities used in the analysis of the Bayesian Decision Model theory analysis. Therefore we reject the Null hypothesis and accept the Alternative hypothesis.

4.5 Discussion of Experimentation on Optimization of River Basin Resources Utilization and Climate Variability Analysis Using the Bayesian Model

(i) The Bayesian Decision Model optimization is best for situation of uncertainty.ie. state of nature which are the future conditions (also called consequences, events or scenarios) associated with climate change or climate variability.

(ii) The Bayesian theory describes the magnitude of difference between alternative actions and provides a variety of estimates for consideration which the result of policy iteration algorithm on third iteration has shown.

(iii) The full capacity utilization of river basin assets of Irrigation Agriculture, Hydro-electric power generation, Water supply, Navigation, Drainage/Dredging, Flood Control, Recreation/Tourism, Erosion Control, Plantation/Forestry, Reservoir/Gullies are the veritable tools to combat climate change impacts on the river basin.

(iv) The Bayesian Decision Theory presents the prioritization of the development projects according to the degree of returns from the expected monetary values with the amount of money released to the river basin.

V. CONCLUSION AND RECOMMENDATIONS

The Pearson's Product Moment Correlation Coefficient was calculated for the prior and posterior probabilities for the first, second and third iterations. The coefficients of correlation (r) were 0.9933 for the first, 0.9998 for the second and 1.00 for the third iterations respectively. This shows a strong perfect correlation. The coefficient of determination (R or r^2) shows that the total variation in posterior probabilities is explained by the total variation in the prior probabilities with the value of 98.67 percent for first iteration, 99.96 percent for the second iteration and 100 percent for the third iteration. This means that no additional variable is required and the values have not risen by chance. The result obtained from correlation resulted to the rejection of Null hypothesis and acceptance of the alternative hypothesis that there is a genuine relationship between the prior and posterior probabilities used in the Bayesian Decision model analysis.

There should be measures to encourage the use of green and clean energy while implementing the purpose/objectives in a multi-purpose/multi-objective Anambra-Imo River basin to reduce the impact of soil erosion, flood disaster, failure of reservoirs and dams, improve hydro-electric power generation, improve water supply, and check insecurity etc. that ravage our living environment.

REFERENCES

[1]. Adrian, E. R. (1994). University of Washington (1994), Technical Report no. 254 Department of Statistics, University of Washington Technical Report no. 571.

[2]. Al Raadif, A. (1999). Integrated Water Resources Management (IWRM): An approach to the future crises. Desalination, **124**(1-3): 145-153.

[3]. Amusu, I. (1980). Fisheries development and management in the river basins of Nigeria. FAO Document Repositor,**1**: 20-25.

[4]. Anyadike, A.N.C. (2009). Statistical Methods for Social and Environmental Sciences. Ibadan, Nigerias Spectrum Books Limited.

[5]. Anyata, B.U. (2009). System Analysis and Numerical Methods in Civil Engineering, University of Benin City, Nigeria

[6]. Arora, K. R. (2002). Irrigation, Water PowerandWaterResourcesEngineering. 4th ed. Delhi: Standard Publishers Distribution.

[7]. Barrow, C. J. (1998). Dei^loggientPlanjj|ng and! Management. WorldDevelopment,**26**(1): l/il-186

[8]. Bukar A. C., Adamu, I. A., Gani, B. and Rasheed, O. O. (2018). Stream Ordering as a Tool For Effective River Basin Development: Iz.ampies From Romadugu

International Journal of Advances in Engineering and Management (IJAEM)
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Yobe River Basin, Nigeria. Journal of Resources Development and Management, **44**

[9]. Chang, L.F., Lin, C.H. and Su, M.D. (2008). Application of Geographic Weighted Regression to Establish Flood-Damage Functions Reflecting Spatial Variation. Water SA, **34**(2): 209-215.

[10]. Eme, L.C. (2013). Interactive Optimization for Model and Prototype: A Case Study of the Twelve Nigerian River Basin Engineering Development Scheme. American Academic & Scholarly Research Journal, **7**(4): 55 – 61.

[11]. Eme, L.C. (2010). Simulation Optimization for Model and Prototype using Non Parametric Method: A Case Study of Anambra/Imo River Basin Engineering Development Scheme. American Academic & Scholarly Research Journal, **7**(4): 63 – 61.

[12]. Ezenweani, R. S. (2012). A Comprehensive Collation of River Basins in Nigeria: Benefits and River Basin Development Planning and Management. International Journal of Scientific & Engineering Research Volume A Issue 1.

[13]. Klare, M.T. (2001). The New Geography of Conflict. Foreign Affairs, **80**(4): 49–61,

[14]. Klare, M.T. (2001). The New Geography of Conflict. Foreign Affairs, **80**(4): 49–61,

[15]. Kohini, D., Tek, M., Geoff, C. and l.axmi, P. D. (2013). Indigenous Knowledge for Climate Change Induced Flood Adaptation in Nepal, The International Journal for Climate Change,**5** (1)

[16]. Kohini, D., Tek, M., Geoff, C. and l.axmi, P. D. (2013). Indigenous Knowledge for Climate Change Induced Flood Adaptation in Nepal, The International Journal for Climate Change,**5** (1)

[17]. Molle, F. (2006). Development Trajectories of River Basins: A Conceptual Framework". Water International, **29** (1): 70–80.

[18]. Nwabuokei, P.O. (1986). Fundamentals of Statistics. Koruna books, Chuka Printing Company ltd. ISBN 978 – 2258-06-7. pp 241 – 338.

[19]. Sharma, J.K. (2008). Operations Research Theory Applications, Third Edition Rajiv Beri for Macmillan India ltd, 2/10 Ansari Road, Daryajanj New Delhi 110002.