

# Optimizing Drilling Using Step-wise Linear Regression Rate of Penetration Model

John Lander Ichenwo and Barinem Peter

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## ABSTRACT

In nature, it is an open secret that commercial quantities of petroleum reserves are produced and buried under the earth. Since the discovery of the several varied uses of petroleum and its derived product, the need to retrieve economical quantities of petroleum have ever been on the rise.

However the fact that petroleum is produced and being stored beneath the surface has made it practically impossible to retrieve or obtain economically viable quantities of petroleum without making a channel through which the underground fluid can be harnessed to the surface of the earth where they are needed to serve various useful purposes.+

Man in his attempt to obtain this economically viable product had come up with several means to bring this economically significant fluid to the surface, one very key stage in the recovery of petroleum is the construction of a conduit through which the fluid can find free passage from the bottom hole to the surface. This is widely known as "Drilling".

Drilling cuts across all procedures designed to safely make a channel from the surface to the subsurface where the presence of economical quantities of petroleum has been confirmed. The modern day drilling utilizes complex equipment collectively known as the drilling rig to make a hole to the subsurface.

Usually very few operators own their rigs, implying, they rent drilling rigs throughout the duration of the drilling along with other tools needed to give insight on the progress of the drilling activity. This ranks drilling a cost intensive activity, accounting for up to 45% of the total cost of petroleum exploration and production and if not checked can exceed.

This has reinstated the need for significant improvements in the efficiency of drilling as to minimize the drilling costs whilst ensuring the safety of the drilling crew. A field of study comprising series of procedures and routines have been dedicated to the end of drilling efficiency

improvements consequently overall time and cost reduction.

This field of study is termed "Drilling Optimization". Drilling optimization is achieved by various techniques however this project focuses on drilling optimization by Rate of Penetration optimizations which seeks to achieve the accurate estimation of the optimal rate of penetration and afterwards the optimization of the controlling variables influencing the rate of penetration to achieve the optimum rate of penetration.

In achieving accurate estimation of the rate of penetration, there is need to formulate a ROP model.

This paper is written to complement the works of previous researchers in the areas of drilling optimizations.

In this paper, a stepwise linear regression model is developed to aid accurate estimation of the ROP, the procedures for this model development is outlined in details in this project.

**Keywords:** rate of penetration; ROP optimization; rotary speed; weight on bit; nonlinear regression.

## I. INTRODUCTION

Drilling a complex cost intensive activity is such that cannot be eliminated in the sourcing and production of the economical fluid, petroleum, therefore the need for various techniques to reduce the cost of drilling has been set in place. One such technique is the improvements of the efficiency of the drilling process termed drilling optimization.

One of the fields in drilling optimization is ROP optimization which is the focus of this project. In ROP optimizations, there's a need to obtain an optimal value of the controlling variables that will in turn produce an optimal rate of penetration leading to reduction in loss time occurrence consequently cost savings.

ROP optimizations requires a relationship to accurately estimate or forecast the ROP be derived and afterwards the significant improvement of the rate of penetration by selecting the optimal controlling variables settings that will produce the

best ROP in line with standard safe practices in a series of procedures regarded as optimization.

This paper seeks to produce a model to aid accurate estimation of the ROP using a stepwise linear regression model. In establishing a valid mathematical relationship to predict the ROP. There's a need to determine the variables that influences the Rate Of Penetration.

Based on the work of researchers in previous literature, it is established that the ROP is affected by certain parameters, Bourgoyne and Young in their ROP Model, published in 1974 established that the ROP was affected by eight parameters, the weight exerted on the bit, rotation speed, formation hardness, pressure differential, mud characteristics, drillability of the formation, hydraulics, bit characteristics.

Maurer in his theoretical equation for roller cutter bits, established that the rate of penetration is influenced by the bit weight, rotary speed, strength of rock and bit size.

Galle and woods, in their early researches using graphical and analytical techniques established the rate of penetration was affected by the weight on bit, formation drillability and the rotation speed.

Therefore this paper adopts to model the ROP as functions of two basic parameters namely the rotation speed and the effective weight on bit. The reason for the choice of these two is that they can be controlled and directly controlled by the drilling team hence they are termed operating parameters unlike parameters like the formation drillability and pore pressure gradients which cannot be controlled.

Furthermore, based on findings from previous literature, the relationship between the ROP and the rotation speed is a nonlinear relationship exhibiting an extremum and an inflexion point, similarly the plot of the ROP with the effective weight on bit per inch of the drilling bit diameter is shown to also exhibit similar trend, rising steeply and peaking at some maximum and then an inflexion establishing a nonlinear relationship.

This project recognizes the nonlinear relationship existing between the ROP and the WOB and RPM, however this project uses a linear model adapted for fitting and estimating nonlinear

trends. The linear stepwise regression model, SLRM which fits nonlinear trend using somewhat similar to the method of exhaustion by breaking down the curve into finite sections called steps and fitting each steps with a straight line giving the SLRM a high degree of accuracy in estimating nonlinear trends.

This project aims to establish the accuracy of the SLRM in ROP estimation and prediction. The procedures and results of the SLRM in ROP estimation is given below.

## II. MATERIALS AND METHODS

Linear step wise Regression Model basically is a linear regression performed at discrete number of data intervals called steps. The steps is chosen based on the nature and properties of the data to be modeled. This is achieve through data visualization or through detailed observation of the data trend.

From our data trend, a step size of 3 data point was selected

Generally, the step wise linear regression model should be of the format below:

Assuming no footage gained at a zero revolution per minute rotation speed and a zero pound force weight exerted on the bit:

Where ROP = Drilling rate of penetration [ft/hr]  
WOB = Effective pound force weight exerted per inch of the bit [lb-f/in]  
RPM = Rotation rate of the drill string [Rev/ min]  
and .

To obtain the values of the unknown coefficient, real time drilling data was obtained from a well, Name of well withheld but will be referred to as well G, the data is cleansed of unwanted parameters leaving only the operating parameters, the effective weight exerted per inch of the drilling bit and the rotation speed of the drill string as well as the Rate of penetration, each data was obtained in interval of 100 fts starting from the 5000ft till the 12000ft, being the longest drilled hole section.

Linear stepwise regression model requires that the data to be modeled be broken into steps of finite number of data points and then regression is carried out each small step looping through the whole data in steps.

Selection of step-size can be done in two ways and requires a knowledge of basic statistical

trends, it can be achieved by visualizing the data and graphically establishing relationships by observing the turning points in the data, on the other hand, the step size can be effectively selected by viewing the trend in the values of the dependent variable, observing points where the value decreases or increases and at what point there is an effective increase or a decrease.

For this model, an effective step size of 3 data points was chosen, carrying out the regression over the whole data, the procedures is displayed for the first and last steps.

The regression requires the value of the constant to be known and for two unknown constants, a minimum of two linear equation is required. This is generated using the normal equation for the linear least squares method. The equation are written in their matrix form for easy computations. The general format for the equations in matrix form is given as:

Where  $b$  = vector of the unknowns  
 $X$  = matrix of the parameters  
 $C$  = vector of the dependent variable constants

Therefore the following matrix are obtained:

To obtain, a numerical solution to these unknown constants, we need to substitute the numerical values for this sums into the matrix, this is achieved using real time data obtained from an oil well, which for confidential reasons we shall call well G.

Well G data comprises data for drilling parameters inclusive of the weight on bit, rotation speed, viscosity, tooth wear, equivalent circulating density, pore gradients and other drilling parameters.

First we extract only the required data for our model, being the effective weight on bit, the rate of penetration and the rotation speed, afterwards our data was cleansed of irregularities and smoothened to better visualize the data trends.

This data was used to prepare the table of values using a statistical tool developed to aid stepwise linear regression computation. Screenshot of the table of values of sums displayed below.

ROP [ft/hr]	ROP.WOB	WOB [lbf/in]	RPM	RPM.WOB	WOB^2
34.9	757.33	21.7	201	4361.7	470.89
40.9	1096.12	26.8	202	5413.6	718.24
45.1	1316.92	29.2	132	3854.4	852.64
59.9	2114.47	35.3	130	4589	1246.09
54.6	1938.3	35.5	105	3727.5	1260.25
61.3	2059.68	33.6	173	5812.8	1128.96
56.5	2124.4	37.6	129	4850.4	1413.76
82.3	3003.95	36.5	118	4307	1332.25
33.5	1236.15	36.9	103	3800.7	1361.61
31.03	1185.346	38.2	103	3934.6	1459.24
80.4	3183.84	39.6	140	5544	1568.16
80.2	3031.56	37.8	135	5103	1428.84
88.2	3413.34	38.7	153	5921.1	1497.69
72.9	2828.52	38.8	166	6440.8	1505.44
82.6	3295.74	39.9	150	5985	1592.01
44.3	1740.99	39.3	175	6877.5	1544.49
42.9	1767.48	41.2	145	5974	1697.44
40.3	1785.29	44.3	146	6467.8	1962.49
<b>152.5</b>	<b>3988.81</b>	<b>103.6</b>	<b>737</b>	<b>18861.5</b>	<b>2712.58</b>

Figure 1. Table of values for data showing sum for the steps

Substituting, the numerical values for each of the sums as obtained from the table of values, the following matrices are obtained:

The solutions to the constants are obtained using Gaussian elimination technique as shown below:

The aim of the Gaussian elimination obtains solution to linear equation in matrix format

using a two phase series of procedures. The first phase aims to obtain a triangular matrix using series of elimination techniques known as “forward elimination” and then obtain the values for the unknown using another series of techniques that works in the opposite direction from the starting point termed “backwards substitution”. We proceed with the forward elimination as shown below:

The forward elimination seeks to get rid of unwanted variables which are variable in the triangular positions, first we establish a pivot row, which will be the reference row for the elimination operation. The selected pivot is row 1. Reducing the first element in row 1 to unity for easy computation by carrying out the following operations.

Eliminate the first element in the second row by carrying out the following operations

Observing the matrix of parameters above, you notice the matrix is an upper triangular matrix,

therefore the forward elimination stage is complete. Proceeding to the backward substitution. We started out from row 1, with row 1 as the pivot, therefore backward substitution begins from row 2.

Therefore,

And from Row 1,

Therefore;

Substituting the values for and in equation 1, we obtain the rate of penetration model for the first step.

This procedure is repeated for subsequent steps till the whole data is covered.

### III. RESULTS AND DISCUSSION

The accuracy of the ROP estimations from the SLRM was validated by comparing the SLRM predictions with the ROP obtained from field data. The results of the comparative analysis is outlined using a tabular and graphical outputs shown below.

Table 1. SLRM ROP estimations against field data ROP

SLRM ROP (ft/hr)	Field Data ROP (ft/hr)	Standard Error %
31.6	31.9	1.5
34.7	34.9	1.9
41.4	40.9	1.6

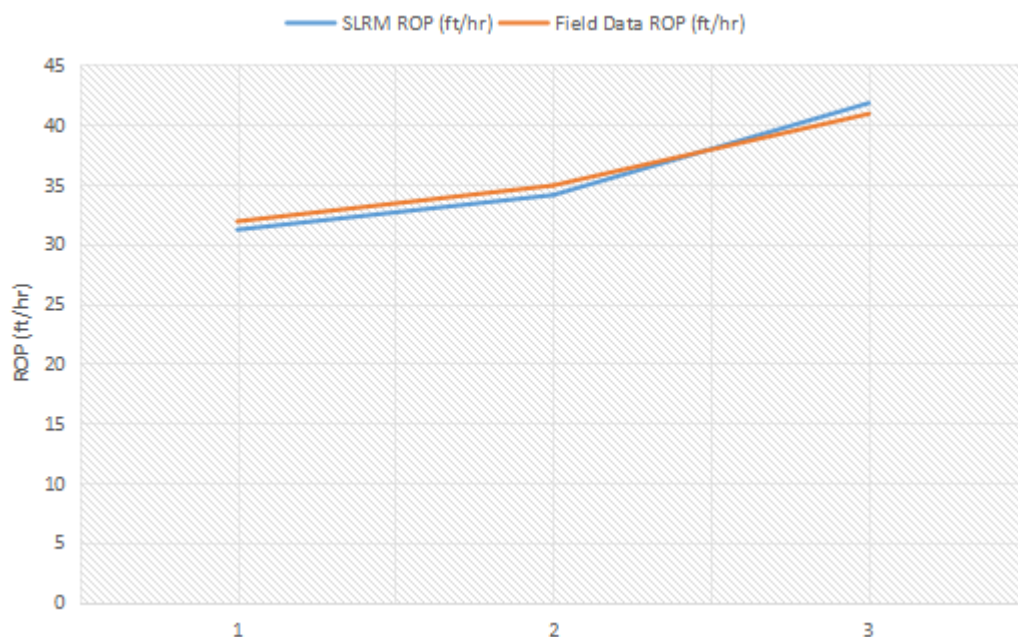


Figure 2. SLRM ROP performance against Field data ROP.

The graph above displays the performance of the SLRM ROP prediction against the actual ROP for the first step, hence three data points.

As indicated by the legend, the blue trend line indicates the readings of the stepwise linear regression model whilst the orange trend line denotes the ROP as obtained from the field data. The graph reveals an excellent estimation of the ROP by the SLRM notable by the proximity of the blue and orange trend lines.

Furthermore, the SLRM was compared with other ROP model, the widely used in the petroleum industry being the BYM, therefore the SLRM was paired against the BYM relative to the actual data, the result is displayed in a graphical output below

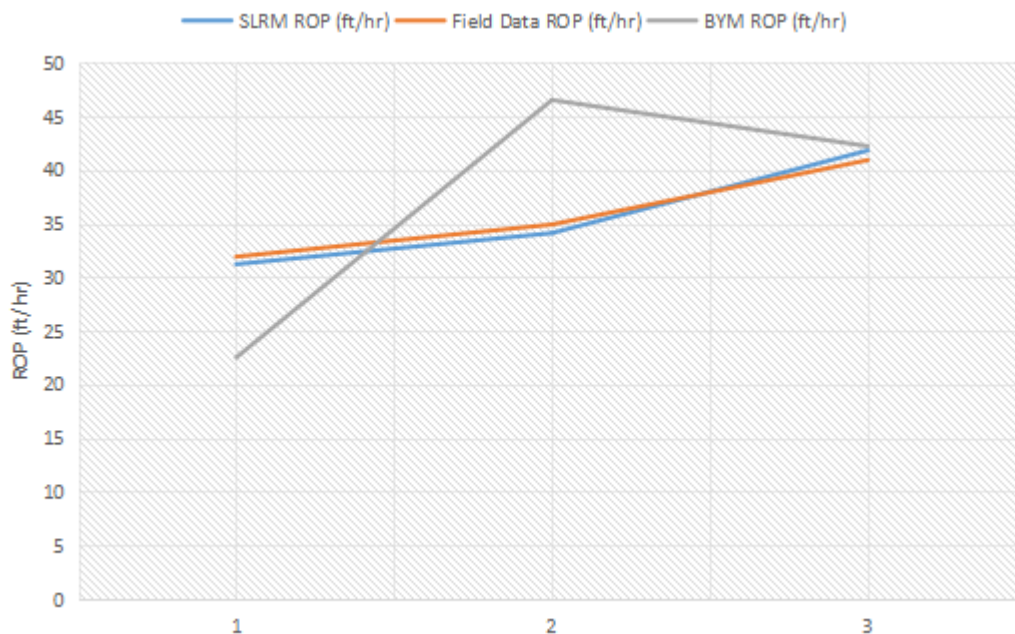


Figure 3. SLRM and BYM ROP predictions with reference to ROP from field data

As seen from the graph above, the estimations of the SLRM ROP is indicated by the blue trend line, with that of BYM represented by the gray coloured trend line, with the orange trend line indicating the ROP as obtained from the field data.

Accuracy can be established graphically by observing the deviations or distance of each trend line ROP predictions from the ROP indicated in the field data.

Observing the deviations, the BYM has the greatest deviation representative of the fact that it has the least accuracy in prediction relative to the actual ROP.

This sets the SLRM as a good choice for use in ROP estimation and optimization.

#### IV. CONCLUSION

With the increase in the number of oil wells drilled daily across the globe, there is need

for research and formulation of optimization strategies to achieve overall drilling efficiency, minimal loss time and consequently drilling cost reductions. This is possible through deliberate research efforts targeted to understand drilling parameters along with it influencing factors.

This work established the rate of penetration (ROP) can be estimated using offset field data using stepwise linear regression models requiring drilling parameters that are readily available and easily controllable.

This new model has proven a better prediction tool when paired with other model like the BYM with a correlation coefficient of 97.2% and a standard error of no more than 2% in it predictions.

Furthermore, the results of this model can be employed by drillers in making informed decision on optimal settings of effective Weight on



bit (WOB) and Rotation speed (RS) to yield optimal ROP during drilling.

In addition, the works and procedures employed herein is useful for adoption for drilling optimizations in any geography, however adequate corrections will need be factored in, one of which is the use of drilling data from the geography in the modelling procedures.

Overall the SLRM has proven to be efficient and suitable for curve fitting and estimation, therefore its useful in nonlinear trend fitting and estimation, which includes but not limited to trigonometric functions, transcendental functions and other higher order functions, however accurate knowledge of data trend is needed to improve the estimation accuracy of the SLRM.

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