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The Buckling Behaviour of Clamped Simple Clamped Isoptropic Plate Elemnt under 3rd Order Functionals

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ABSTRACT
The research centers on the buckling effect of rectangular Clamped-Simple-Clamped- Fixed plate Isotropic plate. This was done using 3rd energy Functional. The ClSmClFx plate was considered as the direct independent plate. That means the material properties like the flexural rigidity, poison ratio and young elastic modulus of elasticity are the same round about the shape of the object. The shape functions were first derived and then the various integral values of the differentiated shape functions, of the various boundary conditions were all gotten. Based on the derived results, the stiffness coefficients of the various boundary arrangements were also formulated. Upon further minimizations , the Third order strain energy equation was derived and further expansion Third order strain energy equation gave rise to the Third Order Overall Potential Energy Functional. The Third Order Overall Potential Energy Functional, with respect to the amplitude was further integrated and this gave a result known as the Lead equation. Further minimization of the Lead equation gave rise to the Vital buckling load equations. Next to this was the formulation of the non-dimensional buckling load parameters which, m/n ranging from 1.0 to 2.0, and considering it at the interval of 0.1. The relationship of the non-buckling load parameters against the various aspect ratios was shown on the graph.

Symbols and Meanings
P_E Overall Potential Energy Functional
S_E Strain Energy
E_x External Energy
N_e Normal Stress
N_a Normal Strain
xl external Load
sk Stiffness coefficients,
ClSmClFx Clamped Simple Clamped Fixed

I. INTRODUCTION

A plate element can be considered as a structural element having straight or curves boundaries, and also possessing three dimensions known as the primary, secondary and tertiary dimension. The tertiary dimension also known as the plate thickness are usually very small compared to other dimensions. The isotropic rectangular ClSmClFx plate have all their material properties in all directions as the same and so they classified as direction independent element. Stability analysis sometimes is referred to as the plate buckling has been a subject of study in solid structural mechanics for a long time now. Although the buckling analysis of rectangular plates has received the attention of many researchers for several centuries Prior to this time,. other researchers have gotten solution using both the Second and Fourth the Order energy functional for Buckling of plate. None of the scholars have any work on buckling of plate using Third order energy functional and so the resolution of the buckling tendency of CLAMPED SIMPLE CLAMPED FIXED isotropic plate using third order energy functional is the gap the work tends to fill. The plates arrangement can be as shown

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Stage 1

1.1 The Buckling Load Equation Load Coefficient

The first stage of the work deals on the formulation process of the needed buckling load coefficients. With the stress and strain as the fundamental parameters, the constitutive relations were formulated. Overall potential energy, P_E is the summation of Strain energy S_E, and External Work, E_x given as: $P_E = S_E + E_x$ i

To derive the S_E strain energy the product of normal stress and normal strain in x direction is considered as $N_{ex} N_{ax} = \frac{Ez^2}{1-\mu^2}\left(\left[\frac{\partial^2 f}{\partial x^2}\right]\left[\frac{\partial^2 f}{\partial x^2}\right] + \right.$ μ∂2f∂x∂y2 ii

while their product in y direction is considered as

$$N_{ey} N_{ay} = \frac{Ez^2}{1-\mu^2}\left(\left[\frac{\partial^2 f}{\partial y^2}\right]\left[\frac{\partial^2 f}{\partial y^2}\right] + \mu\left[\frac{\partial^2 f}{\partial x\,\partial y}\right]^2\right) \quad \text{iii}$$

And finally the product of the in-plane shear stress and in-plane shear strain is given as: $\rho_{xy}\,\delta_{xy} = 2\,\frac{Ez^2(1-\mu)}{(1-\mu^2)}\left[\frac{\partial^2 f}{\partial x\,\partial y}\right]^2$ iv

adding all together gives

$N_{ex} N_{ax} + N_{ey} N_{ay} + \rho_{xy}\,\delta_{xy} = \frac{Ez^2}{1-\mu^2}\left(\left[\frac{\partial^2 f}{\partial x^2}\right]\left[\frac{\partial^2 f}{\partial x^2}\right] + \right.$ 2∂2f∂x∂y2+∂2f∂y2∂2f∂y2 v

But $S_E = \frac{1}{2}\iint_{xy} \overline{S_E}\, dxdy$ where

$\overline{S_E} = \frac{Ez^2}{1-\mu^2}\int\left(\left[\frac{\partial^2 f}{\partial x^2}\right]\left[\frac{\partial^2 f}{\partial x^2}\right] + 2\left[\frac{\partial^2 f}{\partial x\,\partial y}\right]^2 + \right.$ ∂2f∂x2∂2f∂x2 vi

Upon minimisation of the expressions above, the third order strain energy equation is gin as $S_E = \frac{G_v}{2}\int_0^n\int_0^m\left(\frac{\partial^3 f}{\partial x^3}\cdot\frac{\partial f}{\partial x} + 2\frac{\partial^3 f}{\partial x\,\partial y^2}\cdot\frac{\partial f}{\partial x} + \frac{\partial^3 f}{\partial y^3}\cdot\frac{\partial f}{\partial y}\right) dxdy$ vii

with the external load as

$$xl = -\frac{Bx}{2}\int_0^n\int_0^m\left(\frac{\partial f}{\partial x}\right)^2 dxdy \quad \text{viii}$$

The third order total potential energy functional is expressed mathematically as

$$P_E = \frac{G_v}{2}\int\int\left(\frac{\partial^3 f}{\partial x^3}\cdot\frac{\partial f}{\partial x} + 2\frac{\partial^3 f}{\partial x^2\,\partial y}\cdot\frac{\partial f}{\partial y} + \frac{\partial^3 f}{\partial y^3}\cdot\frac{\partial f}{\partial y}\right) dxdy - \frac{Bx}{2}\int\int\frac{\partial^2 f}{\partial x^2} dxdy \quad \text{ix}$$

Rearranging the total potential energy equation in terms of non dimensional parameters, the buckling load equation is gotten as

$$BL_x = \frac{\frac{G_v}{a^2}\int_0^1\int_0^1\cdot\left(\left[\frac{\partial^3 f}{\partial J^3}\right]\cdot\frac{\partial f}{\partial J} + 2\frac{1}{p^2}\left[\frac{\partial^3 f}{\partial J\,\partial I^2}\right]\cdot\frac{\partial f}{\partial J} + \frac{1}{p^4}\left[\frac{\partial^3 f}{\partial I^3}\right]\cdot\frac{\partial f}{\partial I}\right) dJdI}{\int_0^1\int_0^1\left(\frac{\partial f}{\partial J}\right)\cdot\left(\frac{\partial f}{\partial J}\right) dJdI} \quad \text{x}$$

Stage 2

1.2 Derivation of Shape Function

On getting the critical buckling load, the various shape functions were derived by considering three major support conditions, namely Fixed support which was denoted as Fx, Simple support which is denoted as Sm and Clamped support which is denoted as Cl. For Simple support condition, the deflection equation F and the 2nd order derivative of the deflection equation F^2, were equated to zero and simultaneous equations were formed by considering J = 0 at the left hand support for X axis and I = 1 at the right. Also considering the top as J = 1 and I = 1 at the bottom support for the Y axis. For the Clamped support condition, the deflection equation, F and 1st order derivative of the deflection equation, F^1, were equated to zero and simultaneous equations were formed by considering J = 0 at the top support and I = 0 at the bottom support for the Y axis, while at the Right hand support, J = 1 while I = 1 at the left support for X axis. These equations were solved simultaneously to obtain the various values of the primary and secondary dimensions (n_1, m_1, n_2, m_2 n_3, m_3, n_4 andm_4) for the CiSiCFi plate element. Where J and I are non-dimensional axis parallel to X and Y axis respectively as earlier explained.

1.3 The Shape Function For Clamped-Simple-Clamped-Fixed Plate Element

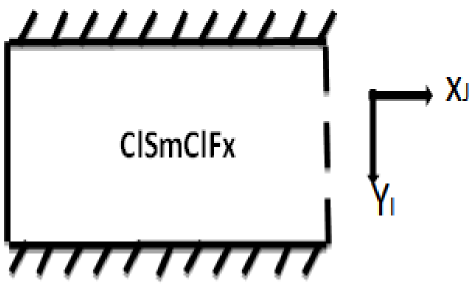


Fig 1a Isotropic Rectangular ClSmClFx Plate

The case of horizontal Direction (X- X axis)

Fig 1b Simple-Fixed Support on x-x axis

Considering the X- X axis

But $F_x = n_o + n_1J + n_2 J^2 + n_3J^3 + n_4J^4 + n_5J^5$ xi

$F_x^1 = n_1 + 2n_2J + 3n_3J^2 + 4n_4J^3 + 5n_5J^4$ xii

$F^2 = 2n_2 + 6n_3J + 12n_4J^2 + 20n_5J^3$ xiii

$F^3 = 6n_3 + 24n_4J + 60n_5J^2$ ivx

Introducing the boundary conditions, reduces the Equations xi-ivx as

At the left support, J = 0

When $F_x = 0$

$F_x = 0 = n_o + 0 + 0 + 0 + 0$ vx

$n_o = 0$

Also when $f_x^{ii} = 0$ xx

$F^{ii} = 0 = 2n_2 + 0 + 0 + 0$ xxi

$2n_2 = 0$ xxii

$n_2 = 0$ xxii

At the right support, J =1

$F_x^1 = n_1 + 0 + 3n_3 + 4n_4 + 5n_5 = -\frac{2n_5}{3}$ xxiii

Further simplifying Equation 15 gives

$n_1 = -\frac{2n_5}{3} - 3n_3 - 4n_4 - 5n_5$ xxiv

Also for the second derivative of the Deflection,

$F^2 = 0 = 0 + 6n_3 + 12n_4 + 20n_5$ xxv

Making n_3 the subject gives

$n_3 = \frac{-12n_4 - 20n_5}{6}$ xxvi

$n_3 = \frac{-10n_5}{3} - 2n_4$ xxvii

For the third derivative of the Deflection,

$F^3 = 0 = 6n_3 + 24n_4 + 60n_5$ xxviii

$F^3 = 0 = 6n_3 + 24n_4 + 60n_5$ xxix

$n_3 = \frac{-60n_5 - 24n_4}{6}$ xxx

Comparing Equation xxvii and xxx gives

$\frac{-10n_5}{3} - 2n_4 = \frac{-60n_5 - 24n_4}{6}$ 2i

Bringing the like terms together and further simplifying gives

$n_4 = \frac{-10n_5}{3}$ 2ii

But substituting Equation 2ii into Equation 2i gives

$n_3 = \frac{-60n_5 - 24(\frac{-10n_5}{3})}{6}$ 2iii

Further simplification gives

$n_3 = \frac{10n_5}{3}$ 2iv

Putting Equations 2ii and 2iv into Equation xxiv gives

$n_1 = -\frac{2n_5}{3} - 3(\frac{10n_5}{3}) - 4(\frac{-10n_5}{3}) - 5n_5$ 2v

$n_1 = -\frac{7n_5}{3}$ 2vi

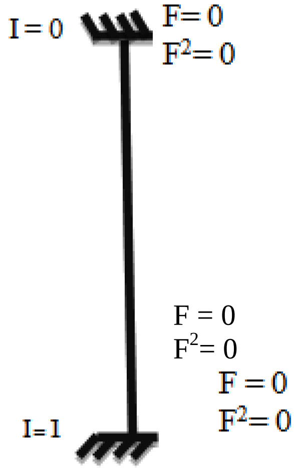
Recall that $F_x = n_o + n_1J + n_2 J^2 + n_3J^3 + n_4J^4 + n_5J^5$ 2vii

Putting the derived values into Equation 2vii gives

$F_x = n_5 \left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right)$

2viii

The case of Vertical Direction (Y- Y axis)



$F_y = m_o + m_1I + m_2 I^2 + m_3 I^3 + m_4 I^4$

2ix

The first derivative on Y axis gives

$F_y^1 = m_1 + 2m_2 I + 3m_3I^2 + 4m_4 I^3$

2x

Considering the boundary conditions on the clamped ends gives

At I = 0,

$F_y = 0 = m_o + 0 + 0 + 0 + 0$ 3i

Leaving $m_o = 0$ 3ii

Also

$F_y^1 = m_1 + 0 + 0 + 0 + 0$ 3iii

$m_1 = 0$ 3iv

At I = 1,

$F_y = 0 = 0 + 0 + m_2 + m_3 + m_4$ 3v

$m_2 + m_3 = -m_4$ 3vi

$F_y^1 = 0 = 0 + 2m_2 + 3m_3 + 4m_4$ 3vii

$m_2 = -m_3 - m_4$ 4i

Putting Equation 3vii into the first derivatives gives

$F_y^1 = 0 = 0 + 2(-m_3 - m_4) + 3m_3 + 4m_4$ 4ii

Opening the bracket gives

$m_3 + 2m_4 = 0$ 4iii

That means $m_3 = -2m_4$ 4iv

Putting it back into Equation 4i gives

$m_2 = -(-2m_4) - m_4$ 4v

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$m_2 = +m_4$ 4vi

Substituting the derived values into Equation 2ix gives

$F_y = m_4I^2 - 2m_4I^3 + m_4I^4$ 4vii

$F_y = m_4(I^2 - 2I^3 + I^4)$ 4viii

That means $F = F_x * F_y = m_4(I^2 - 2I^3 + I^4) * n_5(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 4ix

$f = f_x * f_y = m_4 n_5 (I^2 - 2I^3 + I^4)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 4x

The shape function is give as $(I^2 - 2I^3 + I^4)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 6i

Equation 6i is further differentiated at different stages , from where the stiffness coefficients were derived. These includes

$\frac{\partial f}{\partial J} = (I^2 - 2I^3 + I^4)(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4)$ 6ii

$\frac{\partial^2 f}{\partial J \partial I} = (-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4)(2I - 6I^2 + 4I^3)$ 6iii

$\frac{\partial f}{\partial J \partial I^2} = (-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4)(2 - 12I + 12I^2)$ 6iv

$\frac{\partial^2 f}{\partial J^2} = (I^2 - 2I^3 + I^4)(-\frac{7}{3} + 60J - 10 * 4J^2 + 20J^3)$ 6v

$\frac{\partial^3 f}{\partial J^3} = (I^2 - 2I^3 + I^4)(60 - 80J + 60J^2)$ 6vi

also

$\frac{\partial f}{\partial I} = (2I - 6I^2 + 4I^3)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$

6vii

$\frac{\partial^2 f}{\partial I^2} = (2 - 12I + 12I^2)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 6viii

$\frac{\partial^3 f}{\partial I^3} = (-12 + 24I)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 6ix

Integrating the product Equation 6ii and 6vi give the first stiffness coefficient. That is

$sk_1 = \int_0^1 \int_0^1 \frac{\partial^3 f}{\partial J^3} * \frac{\partial f}{\partial J} dIdJ$ 6x

$sk_1 = \int_0^1 \int_0^1 \left[(I^2 - 2I^3 + I^4)(60 - 80J + 60J^2) * (I^2 - 2I^3 + I^4)(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4)\right] dIdJ$

6xi

bringing the like terms together gives

$= \int_0^1 \int_0^1 \left[(I^2 - 2I^3 + I^4)(I^2 - 2I^3 + I^4) *\right.$

(60¯80/+60/2)(−73+30/2¯40/33+5/4)dIdJ

6xii

multiplying them gives

$= \int_0^1 \int_0^1 \left[(I^2(I^2 - 2I^3 + I^4) - 2I^3 (I^2 - 2I^3 +\right.$

/4+ /4/2−2/3+ /4*

60−73+30/2¯40/33+5/4¯80/−73+30/2¯40/33+5/4

+60/2−73+30/2¯40/33+5/4

dIdJ 7i

further minimization yields

$sk_1 = 0.789 * 0.8296$

$= 0.65455$

also integrating the product Equation 6iv and 6i give the second stiffness coefficient.

That is

$sk_2 = \int_0^1 \int_0^1 \frac{\partial^3 f}{\partial J \partial I^2} * \frac{\partial f}{\partial J} dIdJ$ 7ii

$sk_2 = \int_0^1 \int_0^1 \left[\left(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J^4\right)(2 - 12I +\right.$

12/2*(/2 −2/3+ /4)(−73+30/2¯40/33+5/4)dIdJ

7iii

Bring the like terms together gives

$\int_0^1 \int_0^1 \left[(2 - 12I + 12I^2)(I^2 - 2I^3 + I^4) *\right.$

−73+30/2¯40/33+5/4(−73+30/2¯40/33+5/4)dIdJ

7iv

Multiplying the like terms gives

$\int_0^1 \int_0^1 \left[(2 (I^2 - 2I^3 + I^4) - 12I (I^2 - 2I^3 +\right.$

/4+ 12/2(/2 −2/3+

/4)*−73(−73+30/2¯40/33+5/4)+30/2(−73+30/2¯40/33+5/4)¯40/33(−73+30/2¯40/33+5/4(−73+30/2¯40/33+5/4))+5/4dIdJ 7v

$sk_2 = 0.01891 * 2.11599$

$= 0.0400137$

Furthermore integrating the product Equation 6ix and 6vii give the third stiffness coefficient. That is

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$sk_3 = \int_0^1 \int_0^1 \frac{\partial^3 fk}{\partial I^3} * \frac{\partial fk}{\partial I} dIdJ$ 7vi

$sk_3 = \int_0^1 \int_0^1 \Big[(-12 + 24I)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J5) * (2I - 6I2 + 4I3) \quad (-7J3 + 10J3^3 - 10J4^3 + J5) dIdJ$ 7vii

$sk_3 = 0.0001579 * 3.77781 = 0.00059651$

and finally integrating the product Equation 6ii by 6ii give the sixth stiffness coefficient. That is

$sk_6 = \int_0^1 \int_0^1 (\frac{\partial fk}{\partial J} * \frac{\partial fk}{\partial J}) dIdJ$ 7viii

$sk_6 = \int_0^1 \int_0^1 \Big[(I^2 - 2I^3 + I4)(-\frac{7}{3} + 30J^2 - \frac{40J^3}{3} + 5J4) * (I2 - 2I3 + I4)(-7^3 + 30J2 - 40J3^3 + 5J4) dIdJ$ 7ix

Collecting the like terms together gives

$= \int_0^1 \int_0^1 \Big[(I^2 - 2I^3 + I4) * (I^2 - 2I^3 + I4)(-7^3 + 30J2 - 40J3^3 + 5J4)(-7^3 + 30J2 - 40J3^3 + 5J4) dIdJ$ 7x

Opening the brackets gives

$= \int_0^1 \int_0^1 \Big[(I^2 (I^2 - 2I^3 + I4) - 2I^3 (I^2 - 2I^3 + I4) + I4(I2 - 2I3 + I4)) * (-7^3(-7^3 + 30J2 - 40J3^3 + 5J4) + 30J2(-7^3 + 30J2 - 40J3^3 + 5J4) - 40J3^3(-7^3 + 30J2 - 40J3^3 + 5J4) + 5J4(-7^3 + 30J2 - 40J3^3 + 5J4)) dIdJ$ 8i

$sk_6 = 0.018111 * 0.8478 = 0.0153545$

Reducing Equation 2iii in terms of the stiffness coefficients gives

$BL_x = \frac{D(sk_1 + 2\frac{1}{p^2}sk_2 + \frac{1}{p^4}sk_3)}{sk_6 m^2}$ 8ii

Substituting the real values in to Equation 8ii gives

$BL_x = \frac{D(0.65455 + 2\frac{1}{p^2}0.0400137 + \frac{1}{p^4}0.00059651)}{0.0153545\, m^2}$ 8iii

The Equation 8iii represents the critical buckling load coefficient. The result is expressed in terms of the aspect rations. At different aspect ratios, the coefficient was determined and tabulated as detailed below.

II. RESULTS AND DISCUSSION.

From then derived values, the stiffness coefficients, which is the integral functions of the differential values of the shape functions and the coefficients of the smallest load needed to cause buckling were derived. The critical buckling load coefficients were derived and recorded at different aspect ratios. The first table represents the values of the stiffness coefficients while the other contains the critical buckling coefficients for the aspect ratio of m/n, both for the previous and present study. The values of the m/n ranges from highest value to smallest values with arithmetic increase of i/10. From the values generated in the tables, it was observed that as the aspect ratio increases from smallest to the biggest value, the critical buckling load decreases as shown both in the present and previous research.

Stiffness Coefficients from Previous Work are stated below

sk_1 : 0.67096 ,
sk_2 : 0.04043,
sk_3: 0.006047,
sk_6: 0.0159444

also for the case of Present Work as detailed

sk_1 : 0.65455,
sk_2: 0.0400137,
sk_3: 0.0400137,
sk_6: 0.0153545,

Table 1.3 and 1.4 contain the actual values of the critical buckling coefficients after substituting the aspect ratios at an arithmetic increase of 0.1. Starting from the highest value to the lowest value.

Table 1.3 Critical buckling load values for ClSmClFx Plate from Previous/Present.

m/n		2	1.9	1.8	1.7	1.6
B		43.9346	44.0759	44.2415	44.4373	44.6711
BL_x	Previous	43.3728	43.5151	43.6826	43.8814	44.1201
	Present	43.9346	44.0759	44.2415	44.4373	44.6711

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Table 1.3 cont'd.

m/n		1.5	1.4	1.3	1.2	1.1	1
B		44.9533	45.2985	45.7268	46.2674	46.9632	47.88
$BL_{x}\frac{G}{n^2}$	Previous	44.4101	44.7674	45.2148	45.7859	46.5315	47.5319
	Present	44.9533	45.2985	45.7268	46.2674	46.9632	47.88

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