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The Effect of Linear, Parabolic and Inverted Parabolic Temparature Gradients on Rayleigh Benard Non Darcy Two - Component Convection in a Two Layered System with Dufour Effect

[1*]Shivaraja. J. M, [2]Shreelatha. C. S, [3]Menaka. B
[1,2&3]*Assistant Profesors,Department of Mathematics,Bengaluru North University,Tamaka, Kolar-563101, Karnataka*

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ABSTRACT: The mathematical formulation of the problem Rayleigh Benard Non Darcy Two-Component (RBNDTC) Convection in a two layered system has been investigated for linear, parabolic and inverted parabolic temperature gradients with Dufour effect. Both lower and upper boundaries is rigid . At the interface, the normal velocity, mass, normal stress, mass flux, heat, shear stress, heat flux is assumed to be continuous. The non-dimensionalised and normalized ODE's are solved by making use of Regular perturbation method. The influence of various physical parameters on the Rayleigh number versus depth ratio is discussed and results are depicted graphically.

Key words: Dufour effect, Two-Component Convection, Rayleigh Number, Composite Layer Temperature Gradients.

I. INTRODUCTION

The layers of fluid which is subjected to temperature gradient then we can have observed that convection clearly when the gradient crosses critical value. This type of convections can be occurred due to the influence of induced density gradient, induced evaporation and induced surface tension gradients. In a Composite layered system the exchange of light fluid particles to its top and heavy fluid particles to its bottom this process will happens continuously because of presence of gravitational forces. The double diffusive/Two component convection in a composite layers has become very crucial in now days because of its wide range of applications in geophysics especially in the field of saline geothermal, a comprehensive literature review concerning Soret and Dufour effects are mentioned below. Partha et al [15] investigated the influence of non darcy, Dufour and Soret effects in a porous medium then Narayana et al [14] has extended current work by considering horizontal flat plate which immersed in porous medium. The transformed system of ODEs contains the physical parameters such as diffusivity ratio, Dufour parameter,, buoyancy ratio parameter and Soret parameter. The changes in coefficients of heat mass transfer along with two more parameters such has Soret and Dufour is portrayed through graphically. Krishnamurthy et al [10] investigated the phenomena, where cross diffusion coefficients (Dufour and Soret) becomes very much significant in the non darcy medium which contains vertical surface to analyse two component free convection, the Keller-Box technique is used to solve the equations which concerns to boundary layers are obtained from non linear PDEs. A detailed discussions are carried out on some important physical parameters such as Soret number, Lewis number, Dufour number, Groshof number, Sherwood number and Nesslet number depicted graphically. In Darcy porous medium Narayana et al [14], Postelnicu [16] and Partha et al [15] reported that a huge changes in the numerous values of both Soret and Dufour parameters because there is a sign changes in the nondimensional quantities such as mass and heat transfer parametrs. They are also observed that there is a heat reversal for some combinations of Dufour and Soret parametrs in the region of boundary layers. In a porous medium Moorthy et al

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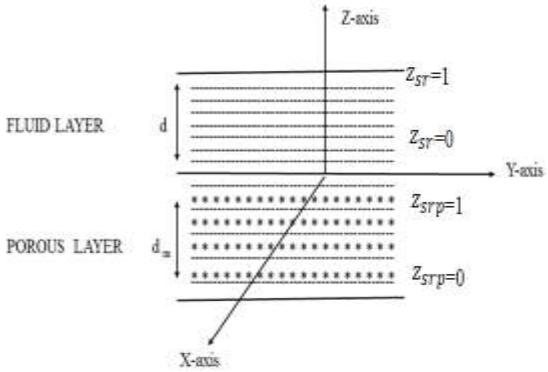
[12-13] discussed Dufour and Soret effects upon natural convection by considering vertical surface and variable viscosity after that the current work is extended for horizontal surface. Ahlers et al [4] gives review on Rayleigh Benard convection in gas mixtures with horizontal fluid layer which acts as stationary. Chilla et al [7] investigates some important new perspectives concerns to turbulent RBC, extensively Ahlers et al [3] investigates logarithmic temperature profiles for the same then Brown [4] has continued the current work by including non Boussinesq approximation.

II. MATHEMATICAL FORMULATION

Composite system/Two-layered which has rigid-rigid boundaries with different temperature and concentration. The region-1 and region-2 are occupied by the fluid and porous layers of thickness d and d_m respectively. Coordinate system is taken at the interface of fluid and porous layer moreover $\mathcal{Z}$ −.axis is upward direction.



Boussinesq approximation is taking an account because of density variation, governing equations continuity, momentum, temperature, concentration and state equations are taken under the above mentioned conditions.

For Region – 1(fluid layer)

$$\nabla.\overrightarrow{q_{sr}} = 0 \quad (1)$$

$$\rho_{0f}\left[\frac{\partial\overrightarrow{q_{sr}}}{\partial t} + (\overrightarrow{q_{sr}}.\nabla)\overrightarrow{q_{sr}}\right] = -\nabla P_{sr} + \mu_{sr}\nabla^2\overrightarrow{q_{sr}} - \rho_{sr}\, g_{sr}\,\hat{k} \quad (2)$$

$$\frac{\partial T_{sr}}{\partial t} + (\overrightarrow{q_{sr}}.\nabla)T_{sr} = \kappa_f\nabla^2 T_{sr} + \kappa_{Tf}\,\nabla^2 C_{sr} \quad (3)$$

$$\frac{\partial C_{sr}}{\partial t} + (\overrightarrow{q_{sr}}.\nabla)C_{sr} = \kappa_{srf}\,\nabla^2 C_{sr} \quad (4)$$

$$\rho_{sr} = \rho_{0f}[\,1 + \alpha_{sr}(C_{sr} - C_{0f}) - \alpha_{2f}(T_{sr} - T_{0f})] \quad (5)$$

For Region – 2 (porous layer)

$$\nabla_{srp}.\overrightarrow{q_{srp}} = 0 \quad (6)$$

$$\left[\frac{\rho_{0f}}{\phi_m}\right]\frac{\partial\overrightarrow{q_{srp}}}{\partial t_{srp}} = -\nabla_{srp}\,p_{srp} - \frac{\mu_f}{K_{srp}}\overrightarrow{q_{srp}} + \mu_{srp}\,\nabla^2_{srp}\,\overrightarrow{q_{srp}} - \rho_{srp}\,g_{srp}\,\hat{k} \quad (7)$$

$$A\left[\frac{\partial T_{srp}}{\partial t_{srp}}\right] + \left(\overrightarrow{q_{srp}}\nabla_{srp}\right)T_{srp} = \kappa_p\nabla^2_{srp}\,T_{srp} + \kappa_{Tp}\,\nabla^2_{srp}\,C_{srp} \quad (8)$$

$$\phi\left[\frac{\partial C_{srp}}{\partial t_{srp}}\right] + \left(\overrightarrow{q_{srp}}\nabla_{srp}\right)C_{srp} = \kappa_{srp}\,\nabla^2_{srp}\,C_{srp} \quad (9)$$

$$\rho_{srp} = \rho_{0f}\left[\,1 - \alpha_{srp}\left(T_{srp} - T_{0f}\right) + \alpha_{2p}\left(C_{srp} - C_{0f}\right)\right] \quad (10)$$

Where,
$\overrightarrow{q_{sr}} = (u, v, w)$, $t, \mu_{sr}, P_{sr}, \rho_{sr}, g_{sr}, T_{sr}, \kappa_f, \kappa_p, \kappa_{Tf}, C_{sr}, \alpha_{sr}, \alpha_{2f}, \phi_m, K_{srp}, \mu_{srp}, A = \frac{(\rho_{of}\, C_p)_m}{(\rho_{rf}\, C_p)_f}, C_P, \rho_{0f}$ are the fluid layer velocity vector, time, viscosity of fluid, pressure, density of fluid, gravity, temperature, thermal diffusivity, solutal diffusivity, Dufour coefficient, concentration, thermal expansion coefficient, solutal analog of α_{2f}, porosity of the porous medium, permeability of the porous medium, porous medium effective viscosity, heat capacities ratio, specific heat and referring fluid respectively. In the porous medium the quantities are referred by the subscripts p.

Basic/Primary state conditions are implem ented to the governing equations which are in the form of partial differential equations (PDE's) to discover the answers of basic state. The set-up is taken into consideration of undergoing super-imposition with infinitely small perturbations. The equations are then linearized using the units of perturbations after that resulting equations are

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going to be non- dimensionalised and normalized which turns PDE's are reduced to system of ODE's.

For region – 1(Fluid layer)

$$[D_{sr}{}^2 - a_{sr}{}^2](D_{sr}{}^2 - a_{sr}{}^2)W_{sr} = R_{sr}\, a_{sr}{}^2 \theta_{sr} - R_{srf}\, a_{sr}{}^2\, S_{sr} \quad (11)$$

$$(D_{sr}{}^2 - a_{sr}{}^2)\Theta + W_{sr} + D_{uf}(D_{sr}{}^2 - a_{sr}{}^2)\, S_{sr} = 0 \quad (12)$$

$$[\tau(D_{sr}{}^2 - a_{sr}{}^2)]\, S_{sr} + W_{sr} = 0 \quad (13)$$

For region −2(porous layer)

$$[\widehat{\mu}\beta^2(D_{srp}^2 - a_{srp}^2) - 1](D_{srp}^2 - a_{srp}^2)W_{srp} = R_{rp}\, a_{srp}{}^2 \theta_{srp} - R_{srp}\, a_{srp}{}^2 S_{srp} \quad (14)$$

$$(D_{srp}^2 - a_{srp}^2)\Theta_{srp} + W_{srp} + D_{up}(D_{srp}^2 - a_{srp}^2)S_{srp} = 0 \quad (15)$$

$$[\tau_{pm}(D_{srp}^2 - a_{srp}^2)]S_{srp} + W_{srp} = 0 \quad (16)$$

III. BOUNDARY CONDITIONS

$$W_{sr}(1) = 0, D_{sr}W_{sr}(1) = 0, D_{sr}\Theta_{sr}(1) = 0, D_{sr}\, S_{rf}(1) = 0$$

$$\left.\begin{array}{c} \hat{T}W_{sr}(0) = W_{srp}(1), \hat{T}\hat{d}D_{sr}W_{srp}(0) = D_{sr}W_{srp}(1), \Theta_{sr}(0) = \hat{T}\Theta_{srp}(1) \\ D_{sr}\Theta_{sr}(0) = D_{srp}\Theta_{srp}(1), S_{sr}(0) = \hat{S}S_{srp}(1), D_{sr}S_{sr}(0) = D_{srp}S_{srp}(1) \\ \hat{T}\hat{d}^3\beta^2\begin{bmatrix} D_{sr}{}^3W_{sr}(0) \\ -3a^2D_{sr}W_{sr}(0)\end{bmatrix} = \begin{bmatrix} -D_{srp}W_{srp}(1) \\ +\hat{\mu}\beta^2[D_{srp}{}^3W_{srp}(1) - 3a_{srp}{}^2D_{srp}W_{srp}(1)]\end{bmatrix} \\ \hat{T}\hat{d}^2[(D_{sr}{}^2 + a^2{}_{sr})W_{sr}(0)] = \hat{\mu}(D_{srp}^2 + a_{srp}^2)W_{srp}(1) \end{array}\right\}$$

$$W_{srp}(0) = 0, D_{srp}W_{srp}(0) = 0, D_{srp}\Theta_{srp}(0) = 0, D_{srp}S_{srp}(0) = 0$$

IV. REGULAR PERTURBATION METHOD

For the consistent mass and heat fluxes, expanding the physical parameters in terms of horizontal wave number "a_{sr}" and "a_{srp}"

$$\begin{bmatrix} W_{sr} \\ \Theta_{sr} \\ S_{sr}\end{bmatrix} = \sum_{j=0}^{\infty} a_{sr}{}^{2j}\begin{bmatrix} W_{rfj} \\ \Theta_{rfj} \\ S_{rfj}\end{bmatrix} \;\&\; \begin{bmatrix} W_{srp} \\ \Theta_{srp} \\ S_{srp}\end{bmatrix} = \sum_{j=0}^{\infty} a_{srp}{}^{2j}\begin{bmatrix} W_{srpj} \\ \Theta_{srpj} \\ S_{srpj}\end{bmatrix}$$

To find the Solution of zero order equations using the arbitrary factors as mention below

$$W_{sr\,0}(z_{sr}) = 0, \quad W_{srp\,0}(z_{srp}) = 0, \quad S_{sr\,0}(z_{sr}) = \hat{S}, \quad \Theta_{sr\,0}(z_{sr}) = \hat{T}, \quad \Theta_{srp\,0}(z_{srp}) = 1, \quad S_{srp\,0}(z_{srp}) = 1,$$

First order equations are mention below.

For Fluid layer ($\mathbf{z_{sr}} \in [\mathbf{0}, \mathbf{1}]$)

$$D_{sr}{}^4W_{sr\,1} - R_{sr}\,\hat{T} + R_{srf}\,\hat{S} = 0 \quad (17)$$

$$D_{sr}{}^2\Theta_{sr\,1} + W_{sr\,1} + D_u(D_{sr}{}^2S_{sr\,1} - \hat{S}) - \hat{T} = 0 \quad (18)$$

$$\tau D_{sr}{}^2S_{sr\,1} + W_{sr\,1} - \tau\,\hat{S} = 0 \quad (19)$$

For porous layer ($\mathbf{z_{srp}} \in [\mathbf{0}, \mathbf{1}]$)

$$\hat{\mu}\beta^2 D_{srp}{}^4W_{srp\,1} - D_{srp}{}^2W_{srp\,1} - R_{rp} + R_{srp} = 0 \quad (20)$$

$$D_{srp}{}^2\Theta_{srp\,1} + W_{srp\,1} + D_{up}D_{srp}{}^2S_{srp\,1} - D_{up} - 1 = 0 \quad (21)$$

$$\tau_m D_{srp}{}^2S_{srp\,1} + W_{srp\,1} - \tau_m = 0 \quad (22)$$

Boundary conditions related to 1^{st} order equations are given below.

$$W_{sr\,1}(1) = 0, D_{sr\,1}W_{sr\,1}(1) = 0, D_{sr\,1}\Theta_{sr\,1}(1) = 0, D_{sr\,1}\, S_{sr\,1}(1) = 0$$

$$\hat{T}W_{sr\,1}(1) = \hat{d}^2W_{srp\,1}(0), \hat{T}D_{sr\,1}W_{sr\,1}(1) = \hat{d}D_{sr\,1}W_{srp\,1}(0), \Theta_{sr\,1}(1) = \hat{d}^2\hat{T}\Theta_{srp\,1}(0)$$

$$D_{sr\,1}\Theta_{sr\,1}(1) = \hat{d}^2D_{srp\,1}\Theta_{srp\,1}(0), \; S_{sr\,1}(1) = \hat{S}\hat{d}^2S_{srp\,1}(0), D_{sr\,1}S_{sr\,1}(1) = D_{srp\,1}S_{srp\,1}(0)$$

$$\hat{T}\hat{d}\beta^2D_{sr\,1}{}^3W_{sr\,1}(1) = -D_{srp\,1}W_{srp\,1}(0) + \hat{\mu}\beta^2[D_{srp\,1}{}^3W_{srp\,1}(0)]$$

$$\hat{T}D_{sr\,1}{}^2W_{sr\,1}(1) = \hat{\mu}D_{sr\,1}{}^2W_{sr\,1}(0)$$

$$W_{srp\,1}(0) = 0, \quad D_{srp\,1}W\, r_{srp\,1}(0) = 0, \quad D_{srp\,1}\Theta_{srp\,1}(0) = 0, D_{srp\,1}S_{srp\,1}(0) = 0$$

The equations (17) and (20) are solved using the relevant boundary conditions then we get velocity distributions as below,

$$W_{sr\,1}(z_{sr}) = \left[\frac{\hat{T}R_{sr} - \hat{S}R_{Srf}}{24}\right](C_1 + C_2z_{sr} + C_3z_{sr}{}^2 + C_4z_{sr}{}^3 + z_{sr}{}^4) \quad (23)$$

$$W_{srp\,1}(z_{srp}) = \frac{(R_{srp} - R_{rp})z_{srp}{}^2}{2} + \frac{(R_{srp} - R_{rp})}{P^2} + \mathcal{M} \qquad (24)$$

$$\mathcal{M} = C_5 + C_6 z_{srp} + C_7 e^{pz_{srp}} + +C_8 e^{-pz_{srp}}$$

V. COMPATIBILITY CONDITION

The differential equations corresponding to concentration and temperature, along with corresponding boundary conditions gives the compatibility condition as below.

$$\left\{ \begin{array}{l} \int_0^1 W_{sr1}(z_{sr})\, dz_{sr} - \frac{D_u}{\tau}\int_0^1 W_{sr1}(z_{sr})\, f(z_{sr}) dz_{sr} \\ + \widehat{d^2}\int_0^1 W_{srp1}(z_{srp})\, dz_{srp} - \frac{D_{up}}{\tau_m}\widehat{d^2}\int_0^1 W_{srp1}(z_{srp})\, g(z_{srp}) dz_{srp} \end{array} \right\} = \widehat{d^2} + \widehat{T} \qquad (25)$$

In the above expression, $f(z_{sr})$ and $g(z_{srp})$ are taken according to basic temperature gradients.

5.1 Linear temperature Profile(LTP)

For this temperature profile $f(z_{sr}) = 1$ & $g(z_{srp}) = 1$ (**26**) and Rayleigh number ($\mathcal{R}$)is obtained by using **(25)** and **(26)** then.

$$\mathcal{R}_{LTP} = \frac{\widehat{T} + \widehat{d}^2 + Rs\widehat{S}(\mathcal{M}_{01} - \lambda_1 \mathcal{M}_{02})}{\widehat{T}(\mathcal{M}_{01} - \lambda_1 \mathcal{M}_{02})}$$

Where,

$$\beta_1 = \sqrt{D_a}\,; \beta_2 = [\widehat{d}\beta^2]^{-1}; \alpha = \sqrt{\left[\frac{1}{\mu\,\beta^2}\right]}; \alpha_1 = e^{\alpha}; \alpha_2 = e^{-\alpha}\,; \Psi_3 = 12\,\alpha_2(2\widehat{d} + 2\mu - \beta_2)$$

$$\Psi_2 = 24\,\widehat{d}\,\alpha\,\alpha_2(1-\alpha_2) + 24\,\mu\alpha^2\alpha_2^2 + 12\,\alpha\alpha_2^2\beta_2(1-\alpha-\alpha_2)$$

$$\Psi_1 = 24\,\widehat{d}\,\alpha(1-\alpha_2) + 24\,\mu\alpha^2 + 12\,\alpha\,\beta_2(\alpha_2+\alpha-1)$$

$$\Psi_4 = 12\,\widehat{d}^{\,2}\alpha_2 + 24\,\widehat{d}\,\alpha_2 + 12\,\mu\alpha_2 - 4\beta_2\alpha_2; \Delta = \tau_m\widehat{d}^{\,2} - D_{up}\,\widehat{d}^{\,2}; \delta = \frac{\tau - D_u}{\tau\,\widehat{T}};$$

$$\Psi_5 = [24\,\widehat{d}^{\,2}(\alpha+\alpha_2-1)\alpha_2 + 24\,\widehat{d}\alpha\alpha_1(1-\alpha_1) + +12\mu\alpha^2\alpha_2^2 + 4\beta_2\alpha\alpha_2^2(1-\alpha-\alpha_1)]$$

$$\Psi_6 = 24\,\widehat{d}^{\,2}(1-\alpha\alpha_2-\alpha_2) + 24\,\widehat{d}\,\alpha(1-\alpha_2) + 12\mu\alpha^2 + 4\alpha\beta_2(\alpha_2+\alpha-1)$$

$$\omega_{20} = \widehat{d}\,(\alpha+\alpha_2-1);\ \omega_{19} = \frac{\alpha\,(1-\alpha_2)}{2};\ \omega_{18} = \alpha^2\alpha_2; \omega_{17} = \alpha\alpha_2^2(1-\alpha-\alpha_1);$$

$$\omega_{16} = \frac{\Delta}{\alpha\tau_m};\ \omega_{15} = \frac{\alpha\Delta}{2\,\tau_m};\ \omega_{14} = \frac{\Delta}{\tau_m};\ \omega_{13} = \alpha\alpha_1(1-\alpha\alpha_2-\alpha_2);\ \omega_{12} = \frac{\alpha(1-\alpha_2)}{2\alpha_2}$$

$$\omega_{11} = \delta\,\widehat{d}\,; \omega_{10} = \alpha^2\alpha_1\,; \omega_9 = \frac{\delta\,\mu}{6}; \omega_8 = \alpha(\alpha_2+\alpha-1)\alpha_2; \omega_7 = \frac{\delta\,\beta_1}{24}$$

$$\Psi_7 = \omega_7 + \omega_9 + \frac{\omega_{11}}{2} + \frac{\omega_{11}\widehat{d}}{2} + \frac{1}{6}; \lambda_1 = \frac{\beta^2\widehat{d}^{\,3}}{\widehat{\kappa}}$$

$$\Psi_8 = \omega_7\omega_8 + \omega_9\omega_{10} + \omega_{11}\omega_{12} + \omega_{11}\omega_{13} - \omega_{14} - \omega_{15} + \omega_{16}(\alpha_2 - 1);$$

$$\Psi_9 == \omega_7\omega_{17} + \omega_9\omega_{18} + \omega_{11}\omega_{19} + \omega_{11}\omega_{20} - \omega_{14} + \omega_{15} + \omega_{16}\alpha_2 - \omega_{16};$$

$$\omega_6 = \Psi_8(\Psi_6\Psi_3 - \Psi_4\Psi_1);\ \omega_5 = \Psi_9(\Psi_4\Psi_2 - \Psi_5\Psi_4); \omega_4 = \Psi_8\ \ \alpha_2\widehat{T}(4\Psi_6 - \Psi_1)$$

$$\omega_3 = \Psi_9\ \ \alpha_2\widehat{T}(\Psi_2 - 4\Psi_5); \omega_2 = \Psi_5\Psi_1 - \Psi_6\Psi_2$$

$$\mathcal{M}_{01} = \frac{\omega_3}{\omega_2} + \frac{\omega_4}{\omega_2} + \frac{\tau - D_u}{120}; \mathcal{M}_{02} = \Psi_7 + \frac{\omega_5}{\omega_2} + \frac{\omega_6}{\omega_2}$$

5.2 Parabolic Temperature Profile(PTP)

For this temperature profile $f(z_{sr}) = 2z_{sr}$ & $g(z_{srp}) = 2z_{srp}$ (**27**) and Rayleigh ($\mathcal{R}$) number is obtained by using **(25)** and **(27)** then.

$$\mathcal{R}_{PTP} == \frac{\widehat{\beta^2}d^3}{\widehat{k}};\ \mathcal{M}_1 = \frac{\hbar_3}{\hbar_2} + \frac{\hbar_4}{\hbar_2} + \frac{3\tau\tau_m - 3D_R\tau_m}{360}; \mathcal{M}_2 = \mathring{A}_3 + \frac{\hbar_5}{\hbar_2} + \frac{\hbar_6}{\hbar_2};$$

$$\hbar_7 = \frac{g(\mathbb{C} + g - 1)}{\mathbb{C}};\ \ \mathring{A}_3 = \hbar_{10} - \hbar_8 + \hbar_{12} + \hbar_{22} + \frac{\tau\widehat{d}^2\,\tau_m}{6} + \frac{\tau\widehat{d}^2 D_{MR}}{4};$$

$$\mathring{A}_1 = \hbar_7\hbar_8 + \hbar_9\hbar_{10} + \hbar_{11}\hbar_{12} + \hbar_{13} + \hbar_{14} - 2\hbar_{15}\tau\widehat{d}^2 D_{MR};$$

$$\mathring{A}_2 = \hbar_8\hbar_{16} + \hbar_{10}\hbar_{17} + \hbar_{12}\hbar_{18} + \hbar_{19} + \hbar_{20} - 2\hbar_{15}\tau\widehat{d}^2 D_{MR};$$

$$\hbar_8 = \left(\frac{\tau\tau_m}{4} - \frac{2D_R\tau_m}{5}\right)\left(6\tau\widehat{\beta^2 d}\right)^{-1}; \hbar_9 = g^2\mathbb{C}; \hbar_{10} = \left(\frac{\tau\tau_m}{3} - \frac{D_R\tau_m}{2}\right)\left(\frac{\mu}{2\widehat{T}}\right);$$

$$\hbar_{11} = g(1 - \mathbb{C})l; \hbar_{12} = \left(\frac{\tau\tau_m}{2} - \frac{2D_R\tau_m}{3}\right)\left(\frac{\widehat{d}}{\widehat{T}}\right);\ \hbar_{18} = g(1 - \mathbb{C});$$

$$\hbar_{13} = \frac{\widehat{d}^2(1 - g\mathbb{C} - \mathbb{C})(\tau\tau_m - D_R\tau_m)}{\widehat{T}\,\mathbb{C}}; \hbar_{14} = \tau\tau_m\widehat{d}^2\left(-1 - \frac{g}{2} + \frac{l-1}{g}\right);$$

$$\hbar_{15} = \frac{-3-2\mathcal{g}}{6} + \frac{\mathcal{g}\,\mathrm{l} - \mathrm{l} + 1}{\mathcal{g}^2};\ \hbar_{16} = \mathcal{g}\,\mathbb{C}(1-\mathcal{g}-\mathrm{l});\ \hbar_{17} = \mathcal{g}^2\,\mathbb{C};$$

$$\hbar_{18} = \mathcal{g}(1-\mathbb{C});\ \hbar_{19} = \frac{\hat{\mathrm{d}}^2(-1+\mathcal{g}+\mathbb{C})(\tau\tau_m - \mathrm{D_R}\tau_m)}{\hat{\mathrm{T}}};\ \hbar_{22} = \frac{\hat{\mathrm{d}}^2(\tau\tau_m - \mathrm{D_R}\tau_m)}{\hat{\mathrm{T}}};$$

$$\hbar_{20} = \tau\tau_m\hat{\mathrm{d}}^2\left(-1+\frac{\mathcal{g}}{2}-\frac{\mathbb{C}-1}{\mathcal{g}}\right);\ \hbar_{21} = \frac{2\,\mathcal{g}-3}{6} + \frac{1-\mathbb{C}-\mathcal{g}\,\mathbb{C}}{\mathcal{g}^2};$$

5.3 Inverted Temperature Profile(ITP)

For this temperature profile $f(z_{sr}) = 2(1 -$ zsr&fzsrp=21−zsrp**(28)** and Rayleigh number $(\mathcal{R})$is obtained by using **(25)** and **(28)** then.

$$\mathcal{R}_{ITP} = \left\{ \left[\frac{\hat{\mathrm{d}}^2}{\left[\left[\frac{T_L-T_0}{T_0-T_U}\right](\mathcal{M}_1 - \hbar_1\mathcal{M}_2)\right]}\right] + \left[\frac{\mathrm{R_{srp}}\left[\frac{C_L-C_0}{C_0-C_U}\right](\mathcal{M}_1-\hbar_1\mathcal{M}_2)}{\left[\frac{T_L-T_0}{T_0-T_U}\right](\mathcal{M}_1-\hbar_1\mathcal{M}_2)}\right] + \left[\frac{\mathbf{1}}{(\mathcal{M}_1-\hbar_1\mathcal{M}_2)}\right] \right\}$$

Where,

$$\hbar_1 = \frac{\widehat{\beta^2}\mathrm{d}^3}{\hat{\mathrm{k}}};\ \mathcal{M}_1 = \frac{\hbar_3}{\hbar_2} + \frac{\hbar_4}{\hbar_2} + \frac{6-2\mathrm{D_R}}{720};\ \mathcal{M}_2 = \mathrm{Å}_3 + \frac{\hbar_5}{\hbar_2} + \frac{\hbar_6}{\hbar_2};$$

$$\hbar_7 = \frac{10-4\mathrm{D_R}}{240\,\hat{\mathrm{d}}\hat{\mathrm{T}}\hat{\beta}^2};\ \mathrm{Å}_3 = \hbar_9 - \hbar_7 + \hbar_{11} + \frac{\hbar_{13}}{2} + \frac{\hat{\mathrm{d}}^2}{6} - \frac{\hat{\mathrm{d}}^2\mathrm{D_{MR}}}{12\,\tau_m};$$

$$\mathrm{Å}_1 = \hbar_7\hbar_8 + \hbar_9\hbar_{10} + \hbar_{11}\hbar_{12} + \hbar_{13}\hbar_{14} + \hat{\mathrm{d}}^2(\hbar_{15} + \hbar_{16});$$

$$\mathrm{Å}_2 = \hbar_8\hbar_{16} + \hbar_{10}\hbar_{17} + \hbar_{12}\hbar_{18} + \hbar_{19} + \hbar_{20} - 2\hbar_{15}\tau\hat{\mathrm{d}}^2\mathrm{D_{MR}};$$

$$\hbar_8 = \mathcal{g}(\mathbb{C}+\mathcal{g}-1)\mathrm{l};\ \hbar_9 = \frac{\mu(2-\mathrm{D_R})}{\hat{\mathrm{T}}\,12};\ \hbar_{10} = \frac{\hbar_{16}}{\mathbb{C}};\ \hbar_{12} = \mathcal{g}(-\mathbb{C}+1)\mathrm{l};$$

$$\hbar_{11} = \frac{\hat{\mathrm{d}}(3-2\mathrm{D_R})}{6\,\hat{\mathrm{T}}} + \frac{\hat{\mathrm{d}}^2(1-\mathrm{D_R})}{2\hat{\mathrm{T}}};\ \hbar_{13} = \frac{\hat{\mathrm{d}}^2(1-\mathrm{D_R})}{2\hat{\mathrm{T}}};\ \hbar_{14} = \frac{1-\mathbb{C}\mathcal{g}-\mathbb{C}}{\mathbb{C}};$$

$$\hbar_{15} = -1 - \frac{\mathcal{g}}{2} + \frac{\mathrm{l}-1}{\mathcal{g}} + \frac{\mathrm{D_{MR}}}{\tau_m};\ \hbar_{16} = \frac{\mathrm{D_{MR}}\,\mathcal{g}}{\tau_m\,3} - \frac{\mathrm{D_{MR}}\,(\mathrm{l}-\mathcal{g}-1)2}{\tau_m\mathcal{g}^2};\ \hbar_{18} = \mathcal{g}^2\,\mathbb{C};$$

$$\hbar_{17} = \mathcal{g}\,\mathbb{C}(1-\mathcal{g}-\mathrm{l});\ \hbar_{19} = \mathcal{g}(1-\mathbb{C});\ \hbar_{20} = -1+\mathcal{g}+\mathbb{C};$$

$$\hbar_{21} = -1 + \frac{\mathcal{g}}{2} - \frac{\mathbb{C}-1}{\mathcal{g}} + \frac{\mathrm{D_{MR}}}{\tau_m};\ \hbar_{16} = \frac{-\mathrm{D_{MR}}\,\mathcal{g}}{\tau_m\,3} - \frac{\mathrm{D_{MR}}\,(\mathcal{g}+\mathbb{C}-1)2}{\tau_m\mathcal{g}^2}$$

VI. GRAPHICAL INTERPRETATIONS

The physical configuration of the problem Rayleigh Benard Non Darcy Two – Component (RBNDTC) Convection in a two layered system has been investigated for uniform temperature Gradient/profile (linear, parabolic and inverted parabolic) with diffusion thermal (Dufour effect). To find the eigenvalue for dimensionless factors such as viscosity (μ), Darcy number (D_a) , Solutal Rayleigh number (R_s) , Dufour number for fluid layer (D_{uf}), Dufour number for porous layer (D_{up}), variations in the Rayleigh numbers R_{LTP}, R_{PTP} and R_{ITP} against the depth ratio $(\hat{d})$ for all three uniform temperature gradients are plotted. Now set $D_{up} = 1, D_{uf} = 1, R_s = 500, D_a = 0.0001, \tau_m = 0.25, \tau = 0.25, \mu = 1, S = 1, T = 1$.

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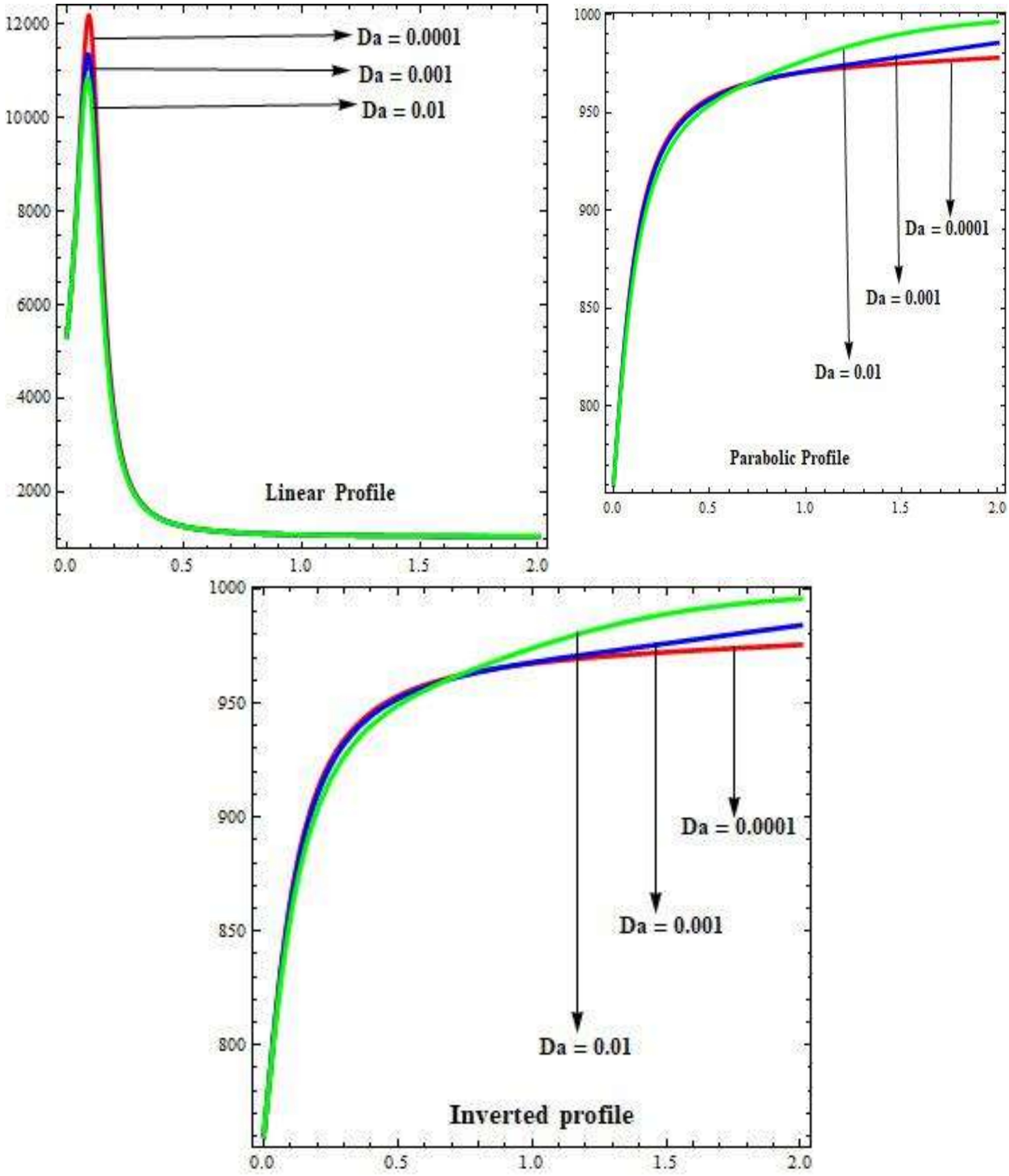
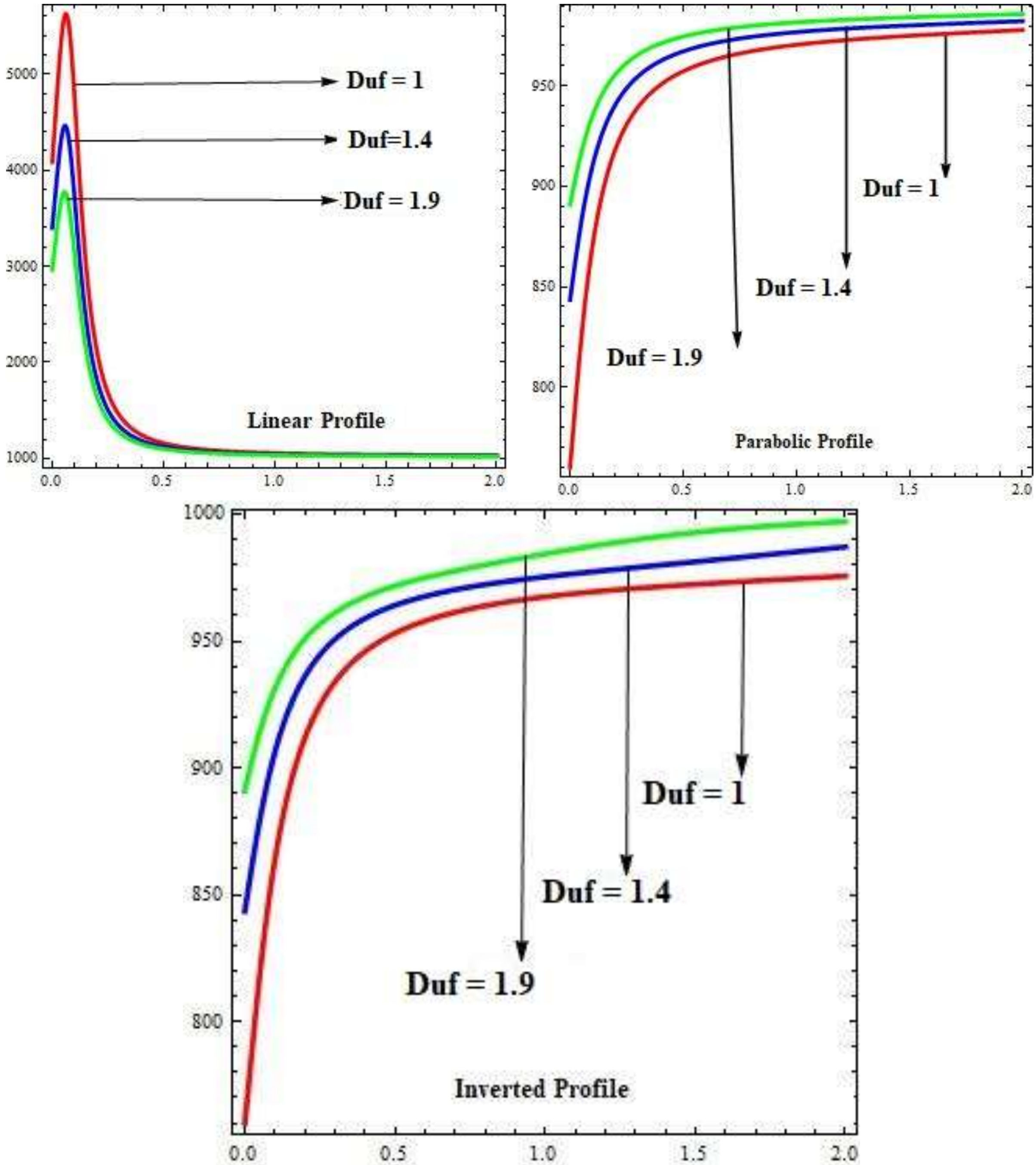
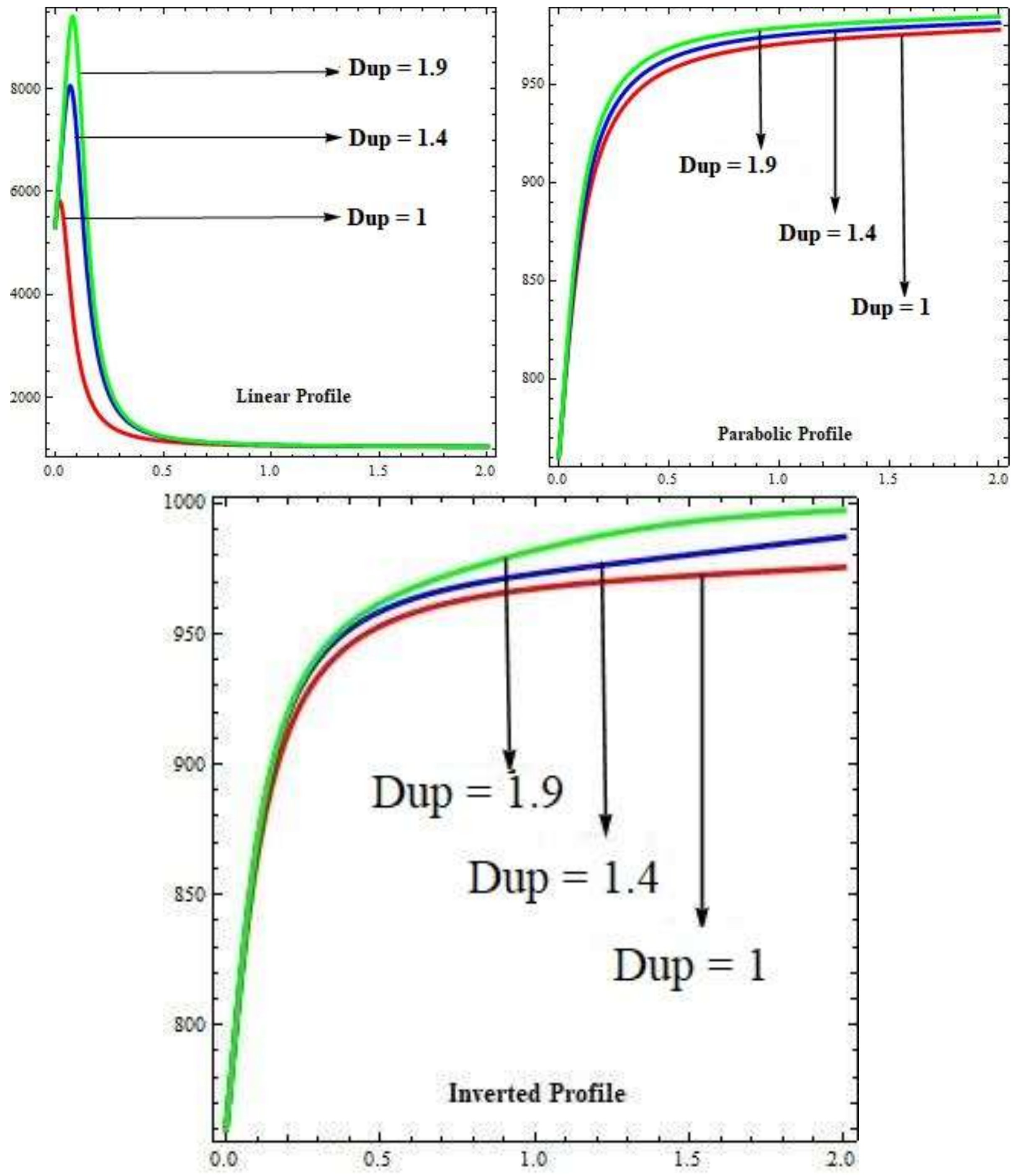


The Figure 1 portrays influence of (D_a) on Rayleigh number R



The Figure 2 portrays influence of (D_{uf}) on Rayleigh number R



The Figure 3 portrays influence of (D_{up}) on Rayleigh number R

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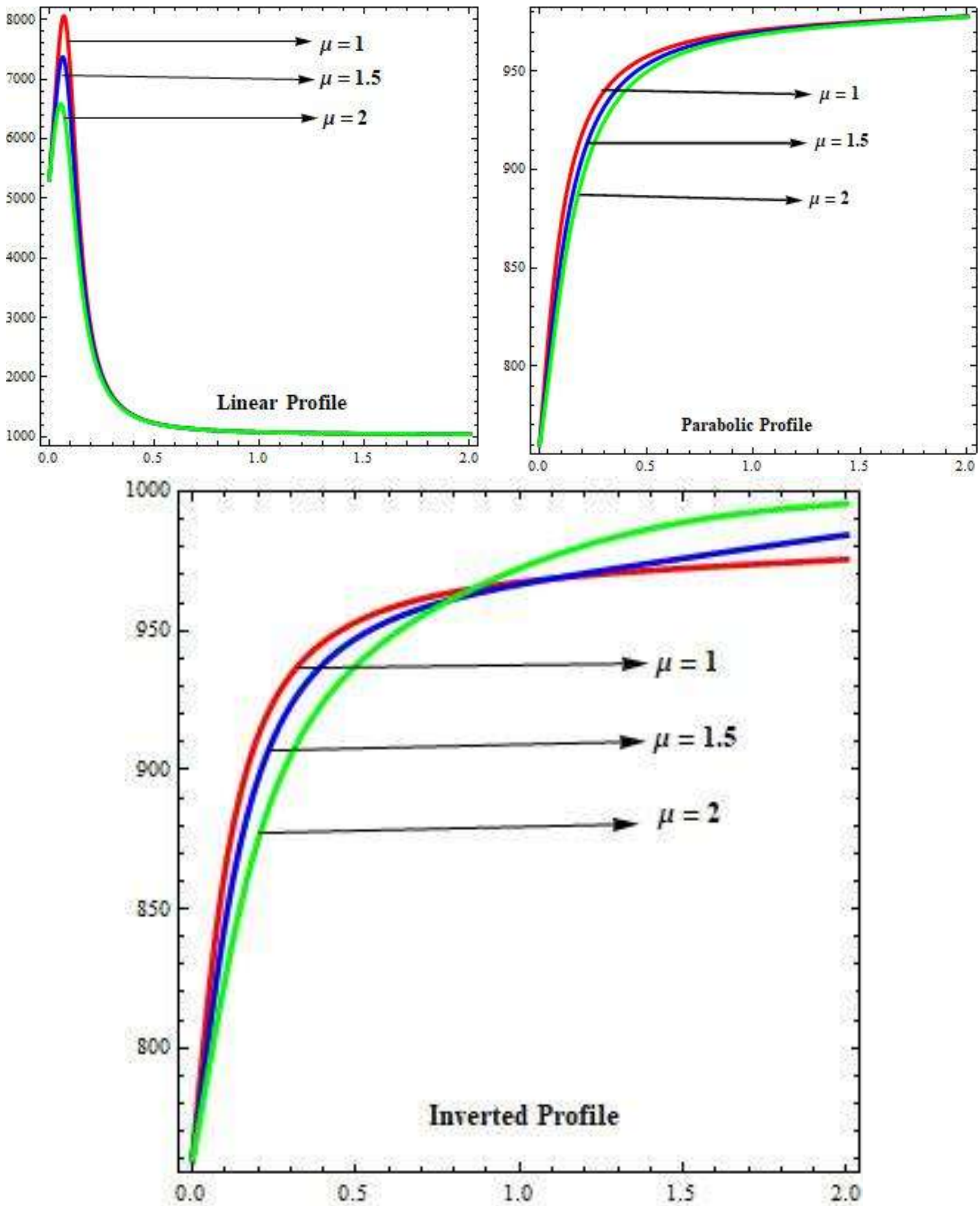
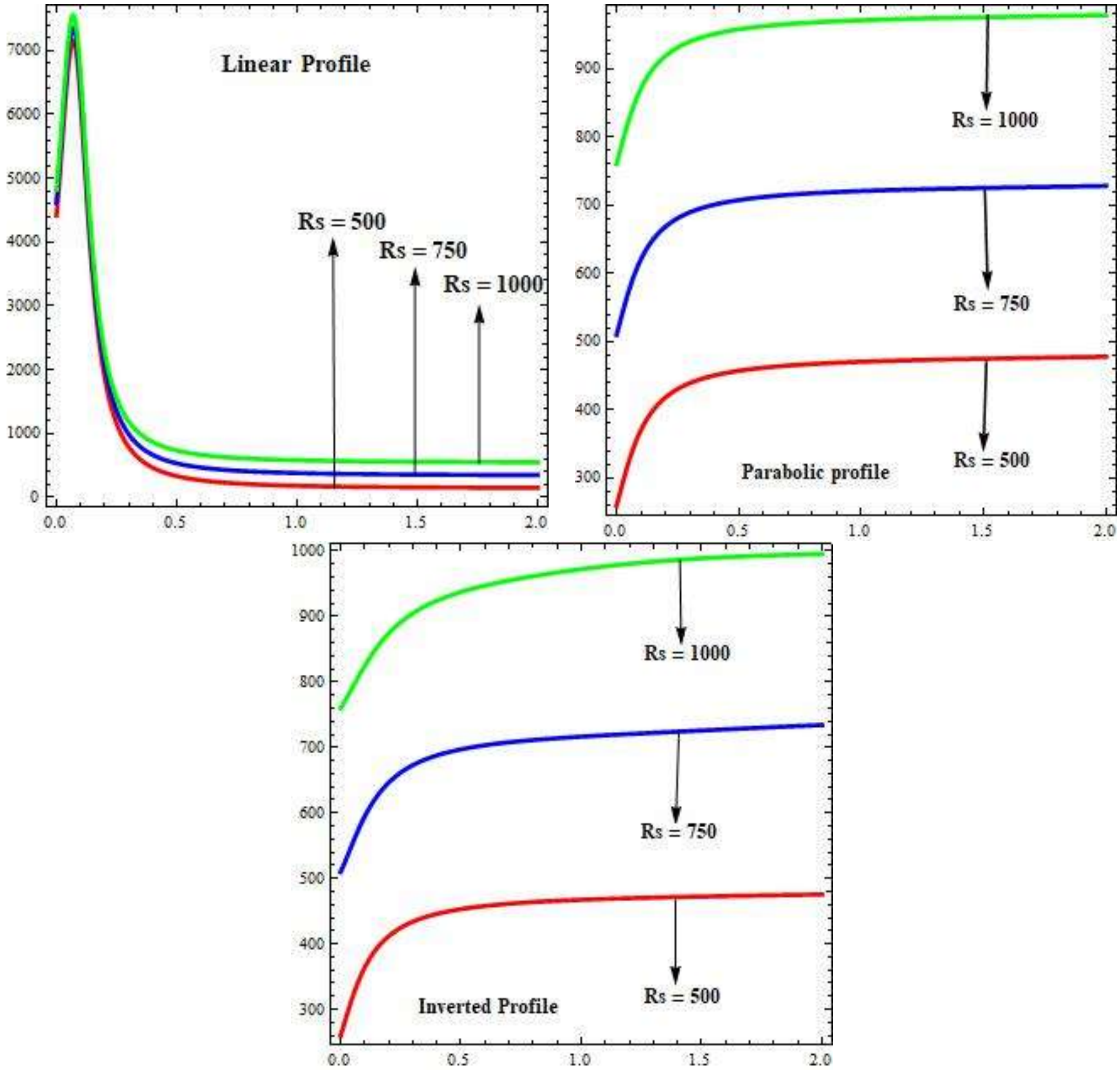


The Figure 4 portrays influence of (μ) on Rayleigh number R



The Figure 5 portrays influence of (R_s) on Rayleigh number R

Figure 1portrays the difference in critical Rayleigh numbers R_{LTP} , R_{PTP} and R_{ITP} against the depth ratio $\hat{d}$for various values of $D_a = 0.0001, 0.001$ and $0.01$for LTP, PTP and ITP. The Graphs shows that for linear temperature profile D_a has a destabilizing the composite system and for parabolic and inverted parabolic temperature profile D_a has a stabilizing the composite System influence on (RBNDTC) because of R_{LTP} de-escalating as the value of D_a increase, R_{PTP} and R_{ITP}escalates as the value of D_a increasing so that the composite system is unstable for parabolic and inverted parabolic temperature profiles and stable for linear temperature profile.

Figure 2portrays the difference in critical Rayleigh numbers R_{LTP} , $R_{PTP}$and R_{ITP}against the depth ratio $\hat{d}$for various values of $D_{uf} = 1, 1.4$ and 1.9 for LTP, PTP and ITP. The graphs shows that the Dufour parameter for porous layer D_{up} has a stabilizing influence on (RBNDTC) because of R_{LTP} , $R_{PTP}$and R_{ITP}escalate as the values of D_{up} increases. Resistance of flow is increases for improved values of D_{up} so the system is stable for all the three temperature profiles. And also Dufour parameter for fluid layer D_{uf} compare with D_{up} it plays a same role as in case of PTP and ITP moreover it plays quite opposite role for LTP has shown in Figure 3. Figure 4 portrays the difference in critical Rayleigh numbers R_{LTP} , $R_{PTP}$and R_{ITP}against the depth ratio $\hat{d}$ for various values of $\mu = 1, 1.5$ and 2 for LTP, PTP and ITP. The graphs shows that μ has destabilizing inuence on (RBNDTC) convection because of R_{LTP} , R_{PTP} and R_{ITP} de-escalate as the improved values of μ, so the system is unstable for the three temperature profiles. Figure 5portrays the difference in critical Rayleigh numbers R_{LTP} , R_{PTP} and R_{ITP} against the depth ratio $\hat{d}$ for various values of $R_s = 500, 750$ and 1000 for LTP, PTP and ITP. The graphs shows that R_s has stabilizing influence on (RBNDTC) convection because of R_{LTP} , R_{PTP} and R_{ITP} escalate as the improved values of R_s, so the system is stable for all the three temperature profiles. Moreover also conclude that the deviation of curves in parabolic and inverted parabolic temperature profiles are more compared to linear temperature profile.

VII. CONCLUSION

For the effect of LTP, PTP and ITP on the onset of (RBNTDTC) Convection in a Two layered/Composite system along with diffusion thermal/Dufour effect has been explored. The expressions for the LTP, PTP and ITP Rayleigh numbers are found as functions of various dimensionless quantities and their influence on stability of composite system is depicted graph-ically. And the following findings are made from the study. (RBNTDTC) Convection in a Composite layer system with Dufour effect is solved in closed form using Regular Perturbation method and the following deductions are made from the study.

i. The composite system is destabilized by the parameters Darcy number (D_a) and Dufour number related to fluid layer $(\mathrm{D_{uf}})$ because of linear temperature profile (LTP) moreover composite system is stabilizes for parabolic temperature profile (PTP) and inverted parabolic temperature profile (ITP).
ii. The physical parameter viscosity ratio (μ) destabilizes the composite system for all the three temperature profiles (LTP, PTP) and (ITP).
iii. The physical parameters Dufour number concerns to porous layer $(\mathrm{D_{up}})$ and Rayleigh number concerns to solute $(\mathrm{R_s})$ are stabilizes the composite system for all the three temperature profiles (LTP, PTP) and (ITP).

REFERENCES

[1]. Appidi, Lakshmi, Bala Siddulu Malga, Sweta Matta, and P. Pramod Kumar. "Heat and mass transfer for Soret and Dufour's consequences on unsteady MHD free convection flow over a porous media with heat absorption." Heat Transfer 50, no. 8 (2021): 8492-8505. https://doi.org/10.1002/htj.22286
[2]. Ahlers, Guenter, Eberhard Bodenschatz, and Xiaozhou He. "Logarithmic temperature profiles of turbulent Rayleigh–Bénard convection in the classical and ultimate state for a Prandtl number of 0.8." Journal of fluid mechanics 758 (2014): 436-467. https://doi.org/10.1017/jfm.2014.543
[3]. Ahlers, Guenter, Eberhard Bodenschatz, Denis Funfschilling, Siegfried Grossmann, Xiaozhou He, Detlef Lohse, Richard JAM Stevens, and Roberto Verzicco. "Logarithmic temperature profiles in turbulent Rayleigh-Bénard convection." Physical review letters 109, no. 11 (2012): 114501. https://doi.org/10.1103/physrevlett.109.11450
[4]. Brown, Eric, and Guenter Ahlers. "Temperature gradients, and search for non-Boussinesq effects, in the interior of turbulent Rayleigh-Bénard convection." EPL Europhysics Letters 80, no. 1 (2007): 14001. https://doi.org/10.1209/0295-5075/80/14001
[5]. Bodenschatz, Eberhard, Werner Pesch, and Guenter Ahlers. "Recent developments in Rayleigh-Bénard convection." Annual review of fluid mechanics 32, no. 1 (2000): 709-778.
[6]. Bergé, P., and M. Dubois. "Rayleigh-bénard convection." Contemporary Physics 25, no. 6 (1984): 535-582.
[7]. Chillà, Francesca, and Joerg Schumacher. "New perspectives in turbulent Rayleigh-Bénard convection." The European Physical Journal E 35, no. 7 (2012): 1-25. https://doi.org/10.1140/epje/i2012-12058-1
[8]. Kairi, R. R., and P. V. S. N. Murthy. "Effect of double dispersion on mixed convection heat and mass transfer in a non-Newtonian fluid-saturated non-Darcy porous medium." Journal of Porous Media 13, no. 8 (2010). https://doi.org/10.1615/JPorMedia.v13.i8.60
[9]. Kumar, B. V., and SVSSNVG Krishna Murthy. "Soret and Dufour effects on double-diffusive free convection from a corrugated vertical surface in a non-Darcy

porous medium." Transport in porous media 85, no. 1 (2010): 117-130. https://doi.org/10.1115/1.4003813

[10]. Krishna Murthy, S. V. S. S. N. V. G., B. V. Rathish Kumar, Peeyush Chandra, Vivek Sangwan, and Mohit Nigam. "A study of double diffusive free convection from a corrugated vertical surface in a Darcy porous medium under Soret and Dufour effects." Journal of heat transfer 133, no. 9 (2011). https://doi.org/10.1115/1.4003813

[11]. Lakshmi Narayana, P. A., and P. V. S. N. Murthy. "Soret and Dufour effects on free convection heat and mass transfer from a horizontal flat plate in a Darcy porous medium." Journal of heat transfer (2008): 104504. https://doi.org/10.1115/1.2789716

[12]. Moorthy, M. B. K., and Karuppiah Senthilvadivu. "Soret and Dufour effects on natural convection flow past a vertical surface in a porous medium with variable viscosity." Journal of Applied Mathematics 2012 (2012). https://doi.org/10.1155/2012/634806

[13]. Moorthy, M. B. K., T. Kannan, and K. Senthilvadivu. "Soret and Dufour effects on natural convection heat and mass transfer flow past a horizontal surface in a porous medium with variable viscosity." WSEAS Transactions on Heat and Mass Transfer 8, no. 4 (2013): 74-83.

[14]. Narayana, PA Lakshmi, and P. Sibanda. "Soret and Dufour effects on free convection along a vertical wavy surface in a fluid saturated Darcy porous medium." International Journal of Heat and Mass Transfer 53, no. 15-16 (2010): 3030-3034.https://doi.org/10.1016/j.ijheatmasstransfer.2010.03.025

[15]. Partha, M. K., P. V. S. N. Murthy, and G. P. Raja Sekhar. "Soret and Dufour effects in a non-Darcy porous medium." Journal heat transfer 605-610 (2006). https://doi.org/10.1115/1.2188512

[16]. Postelnicu, Adrian. "Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects." International Journal of heat and mass transfer 47, no. 6-7 (2004): 1467-1472. https://doi.org/10.1016/j.ijheatmasstransfer.2003.09.017

[17]. Reddy, D. Srinivas, and K. Govardhan. "Effect of viscous dissipation, Soret and Dufour effect on free convection heat and mass transfer from vertical surface in a porous medium." Procedia Materials Science 10 (2015): 563-571.https://doi.org/10.1016/j.mspro.2015.06.007

[18]. Sarma, G. Sreedhar, and K. Govardhan. "Thermo-diffusion and Diffusionthermo effects on free convective heat and mass transfer from vertical surface in a porous medium with viscous dissipation in the presence of Thermal radiation." Archives of Current Research International 3, no. 1 (2016): 1-11.

[19]. Srinivasacharya, D., and O. Surender. "Non-Darcy mixed convection in a doubly stratified porous medium with Soret-Dufour effects." International Journal of Engineering Mathematics (2014). https://doi.org/10.1155/2014/126218

[20]. Sarma, G. Sreedhar, Ra K. Prasad, and K. Govardhan. "The combined effect of chemical reaction, thermal radiation on steady free convection and mass transfer flow in a porous medium considering Soret and Dufour effects." IOSR Journal of Mathematics 8, no. 2 (2013): 67-87.

[21]. Tsai, R., and J. S. Huang. "Numerical study of Soret and Dufour effects on heat and mass transfer from natural convection flow over a vertical porous medium with variable wall heat fluxes." Computational Materials Science 47, no. 1 (2009): 23-30. https://doi.org/10.1016/j.commatsci.2009.06.009