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The Effect of Suction/Injection on MHD Oscillatory Flow of Jeffrey Fluid with Heat Source/Sink through a Porous Medium

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ABSTRACT
The effect of suction/injection on unsteady MHD Oscillatory flow of Jeffrey fluid with heat source/sink through porous medium under slip condition was investigated. The dimensionless governing equations are solved using perturbation technique. The analytical expressions for the velocity, temperature, skin friction and Nusselt number of the fluid have been obtained. The effects of flow parameters of Jeffrey fluid, Suction/injection, Grashof number, Hartmann number, slip parameter, porosity parameter, radiation parameter and frequency of the oscillation are studies. The result obtained for suction/injection parameter was computed for velocity and skin friction, which shows that increasing Grashof number and slip parameter has enhancing effect on velocity. It was also observed that skin friction increases with increasing injection parameter while decreases with increasing suction parameter.

Keywords: Jeffrey fluid**,** Suction/injection**,** Heat transfers, Oscillatory flow, Heat source/sink, Porous Medium, Slip condition, MHD

I. INTRODUCTION

The effect of suction/injection on MHD Oscillatory flow of Jeffrey fluid with heat source/sink through porous medium are encountered in a wide range of engineering and industrial applications such as molten iron flow, recovery extraction of crude oil, geothermal systems.

The phenomenon of slip-flow regime has attracted the attention of a large number of scholars due to its wide ranging application. The problem of the slip flow regime is very important in this era of modern science, technology and vast ranging industrialization. In practical applications, the particle adjacent to a solid surface no longer takes the velocity of the surface. The particle at the surface has a finite tangential velocity, it slips along the surface. Many researchers carried out their research work in this area. Some of these include, Makinde and Mhone (2005) who examined heat transfer to MHD oscillatory flow in a channel filled with porous medium. Mehmood and Ali (2007) extended the work of Makinde and Mhone (2005) by considering the fluid slip at the lower wall. Kumar et al. (2010) extended the work of Mehmood and Ali (2007) by employing perturbation technique to the problem. The effect of viscous dissipative heat on three dimensional oscillatory flows with periodic suction velocity has been studied by Sahim (2010). While Jha and Ajibade(2010) reported some interesting results on the free convective oscillatory flows induced by time dependent boundary conditions. Hamza et al. (2011) investigate the transient heat transfer to MHD oscillatory flow through porous medium under slip condition and oscillating temperature.

The effect of magnetic field on free convective flow of a viscous incompressible fluid past an infinite moving porous hot vertical plate in the presence of porous medium and radiation was analyzed by Singh et al. (2012). Uwanta and Hamza (2012) investigate unsteady heat transfer flow of a viscous, incompressible, electrically conductive fluid through porous medium with periodic suction and temperature oscillation. Kavita et al. (2012) investigate the influence of heat transfer on MHD oscillatory flow of Jeffrey fluid in a channel. In their work they observed that, the axial velocity increases with increasing Jeffrey

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fluid. Also, the maximum velocity occurs at the centerline of the channel while the minimum at the channel walls. Moreover, the velocity is more of Jeffrey fluid than that of Newtonian fluid. Additionally, Aruna Kumari et al. (2012) studied the effect of heat transfer on MHD oscillatory flow of Jeffrey fluid in a channel with slip effect at a lower wall where the expressions for the velocity and temperature are obtained analytically. Asadullah et al. (2013) consider the MHD flow of a Jeffrey fluid in converging and diverging channels. The flows between non parallel walls have a very significant role in physical and biological sciences. Idowa et al. (2013) studied the effect of heat and mass transfer on unsteady MHD Oscillatory flow of Jeffrey fluid in a horizontal channel with Chemical Reaction. Adesanya and Makinde (2014) studied MHD Oscillatory slip flow and heat transfer in a channel filled with porous medium. Al-Khafajy (2016) investigates the effect of heat transfer on MHD Oscillatory flow of Jeffrey fluid with variable viscosity through porous medium. Ahmad and Ishak (2017) studied steady two-dimensional mixed convection boundary layer flow and heat transfer of a Jeffrey fluid over a stretched sheet immersed in a porous medium in the presence of a transverse magnetic field.

Yale et al. (2019) studied transient heat transfer to MHD oscillatory flow of Jeffry fluid through a porous medium under slip condition and oscillating temperature. The authors have not considered the problem under effect of suction/injection on Oscillatory flow of Jeffry fluid with heat source/sink through a porous medium in their work. Therefore in the present study, the effect of suction/injection on Oscillatory flow of Jeffrey fluid, slip conditions, heat source/sink, magnetic field and radiative heat transfer on unsteady flow of conducting optically thin fluid through a porous medium have been analyzed.

II. MATHEMATICAL FORMULATION

Consider the flow of a conducting optically thin fluid in a channel filled with saturated porous medium under the influence of an externally applied homogeneous magnetic field and radiative heat transfer. It is assumed that the fluid has a small electrical conductivity and the electromagnetic force produced is very small. Take a Cartesian coordinate system (x, y), where x lies along the center of the channel, y is the distance measured in the normal section.

Within the frame of the above the assumptions of a Boussinesq incompressible fluid model of the equations governing the motion are given as:

$$\frac{\partial u'}{\partial t'} - v'\frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \frac{v}{(1+\beta_1)}\frac{\partial^2 u'}{\partial y'^2} - \frac{v}{k'}u' - \frac{\sigma_e B_0^2 u'}{\rho} + g\beta(T' - T'_0) \quad (1)$$

$$\frac{\partial T'}{\partial t'} - v'\frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q'}{\partial y'} \pm \frac{Q'}{\rho c_p}(T' - T'_0) \quad (2)$$

With boundary conditions

$$\left.\begin{array}{l} u' - \gamma^*\frac{\partial u'}{\partial y'} = 0,\ T' = T'_0, \quad \text{on } y' = 0 \\ u' = 0, T' = T'_0 + (T'_\omega - T'_0)\cos\omega' t' \quad \text{on } y' = a \end{array}\right\} \quad (3)$$

Where u' is the axial velocity, t' is time, ω' is frequency of the oscillation, T' the fluid temperature, p is the pressure gravitational force, c_p is the specific heat at constant pressure, k is the thermal conductivity, q is the radiative heat flux, β is the coefficient of volume expansion, k' is the porous medium permeability coefficient, B_0 is the electromagnetic induction, σ_e is the conductivity of the fluid, ρ is the denity of the fluid, v is the kinematics viscosity coefficient. It is assumed that walls temperature T'_0, T'_ω are high enough to induce radiative heat transfer, and γ^* is the dimensionless slip parameter. Following Makindi and Mhone (2005), it is assumed that the fluid is optically thin with a relatively low density and the radiative heat flux is given by :

$$\frac{\partial q'}{\partial y'} = 4\alpha^2(T'_0 - T') \quad (4)$$

Where α is the mean radiation absorption coefficient.

The following dimensionless variables and parameters are introduced:

$$\left.\begin{array}{l} x = \frac{x'}{a} \Rightarrow x' = xa,\ y = \frac{y'}{a} \Rightarrow y' = ya, V_0 = \frac{v'}{U} \Rightarrow v' = V_0 U,\ u = \frac{u'}{U} \Rightarrow u' = uU, \\ w = \frac{w'a}{U} \Rightarrow w' = \frac{wU}{a}, t = \frac{t'U}{a} \Rightarrow t' = \frac{ta}{U}, \theta = \frac{T' - T'_0}{T'_\omega - T'_0} \Rightarrow T' = T'_0 + \theta(T'_\omega - T'_0), \\ p = \frac{ap'}{\rho v U} \Rightarrow p' = \frac{p\rho v U}{a},\ \gamma = \frac{\gamma^*}{a} \Rightarrow \gamma^* = \gamma a,\ S^S = \frac{1}{Da},\ Da = \frac{K'}{a^2} \Rightarrow k' = a^2 Da \\ N^2 = \frac{4\alpha^2 a^2}{k},\ H^2 = \frac{a^2\sigma_e B_0^2}{\rho v},\ Gr = \frac{g\beta(T'_\omega - T'_0)a^2}{vU},\ Pe = \frac{Ua\rho c_p}{k},\ Re = \frac{Ua}{v}, Q = \frac{Q'a^2}{k} \end{array}\right\} \quad (5)$$

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Where U is the flow mean velocity, the dimensionless governing equations together with appropriate boundary conditions can be written as:

$$Re[\frac{\partial u}{\partial t} - V_0 \frac{\partial u}{\partial y}] = -\frac{\partial P}{\partial x} + \frac{1}{(1+\beta_1)}\frac{\partial^2 u}{\partial y^2} - (H^2 + S^2)u$$
(6)

$$Pe\left[\frac{\partial \theta}{\partial t} - V_0 \frac{\partial \theta}{\partial y}\right] = \frac{\partial^2 \theta}{\partial y^2} + (N^2 \pm \theta)\theta$$
(7)

Subject to the boundary condition

$$\left.\begin{array}{l} u - \gamma \frac{\partial u}{\partial y} = 0, \theta = 0 \text{ at } y = 0 \\ u = 0, \theta = \cos \omega t \text{ at } y = 1 \end{array}\right\}$$
(8)

Where β_1 is the Jeffrey fluid, V_0 is suction/injection parameter, Q Heat source/sink parameter, Gr is the thermal Grashof number, Ha is the Hartmann number, N is the radiation parameter, Pe is the Peclet number, Re is the Reynolds number, Da is the Darcy number, Pr is the Prandtl number, γ is the slip parameter and s is the porous medium shape factor.

III. METHOD OF SOLUTION

In order to solve equations (6), (7) and (8) for purely oscillatory flow, let the pressure gradient, fluid velocity and temperature be:

$$u(y,t) = u_0(y)e^{i\omega t} + u_1(y)e^{-i\omega t}$$
(9)

$$\theta(y,t) = \theta_0(y)e^{i\omega t} + \theta_1(y)e^{-i\omega t}$$
(10)

Assume that $-\frac{\partial P}{\partial x} = \lambda[e^{i\omega t} + e^{-i\omega t}]$

Where λ <0 for favorable pressure, ω is the frequency of the oscillation. Substituting the above expressions: (9) and (10) into (6), (7) and (8), we obtained

$$\theta_0'' + PeV_0\theta_0' + N_1^2\theta_0 = 0$$
(11)

$$\theta_1'' + PeV_0\theta_1' + N_2^2\theta_1 = 0$$
(12)

$$u_0'' + \alpha ReV_0 u_0' - M_1^2 u_0 = -\alpha\lambda\text{-}\alpha Gr\theta_0$$
(13)

$$u_1'' + \alpha ReV_0 u_1' - M_2^2 u_1 = -\alpha\lambda\text{-}\alpha Gr\theta_1$$
(14)

With the following boundary conditions

$$\left.\begin{array}{l} u_0 - \gamma u_0' = 0, u_1 - \gamma u_1' = 0, \theta_0 = 0, \theta_1 = 0, at\ y = 0 \\ u_0 = 0,\ u_1 = 0,\ \theta_0 = \frac{1}{2},\ \theta_1 = \frac{1}{2} \quad at\ y = 0 \end{array}\right\}$$
(15)

Where $\eta^2 = N^2 \pm Q, \alpha = 1 + \beta_1$

$N_1^2 = (\eta^2 - Pei\omega), N_2^2 = (\eta^2 + Pei\omega)$

$M_1^2 {=} \alpha\,(H^2 + S^2 + Rei\omega)$ And $M_2^2 {=} \alpha\,(H^2 + S^2 - Rei\omega$

Equations (11) to (15) are solved and the solution for fluid temperature and velocity are given as follows;

$$\theta(y,t) = \left[\frac{1}{2(e^{-r_2} - e^{r_1})}e^{-r_2 y} - \frac{1}{2(e^{-r_2} - e^{r_1})}e^{r_1 y}\right]e^{i\omega t} + \left[\frac{1}{2(e^{-r_4} - e^{r_3})}e^{-r_4 y} - \frac{1}{2(e^{-r_4} - e^{r_3})}e^{r_3 y}\right]e^{-i\omega t}$$

$$\text{Or}\,\theta(y,t) = [Be^{-r_2 y} + Ae^{r_1 y}]e^{i\omega t} + [De^{-r_4 y} + Ce^{r_3 y}]e^{-i\omega t}$$
(16)

$$u(y,t) = [Ee^{r_5 y} + Fe^{-r_6 y} + G + H_1 e^{r_1 y} + H2e{-}r2yei\omega t{+}[Jer7y{+}Ke{-}r8y{+}X{+}H3er3y{+} H_4 e^{-r_4 y}]e^{-i\omega t} \quad (17)$$

Skin friction or shear stress is given by;

$$\tau_0 = \frac{d\theta}{dy}/_{y=0} = [Ar_1 - Br_2]e^{i\omega t} + [Cr_3 - Dr_4]\,e^{-i\omega t} \quad (18)$$

$$\tau_1 = \frac{d\theta}{dy}/_{y=1} = [Ar_1 e^{r_1} - Br_2 e^{-r_2}]e^{i\omega t} + [Cr_3 e^{r_3} - Dr_4 e^{-r_4}]e^{-i\omega t} \quad (19)$$

Nusselt number or rate of heat transfer is given by;

$$Nu_0 = \frac{du}{dy}/_{y=0} = [Er_5 - Fr_6 + H_1 r_1 - H2r2ei\omega t{+}[Jr7{-}Kr8{+}H3r3{-}H4r4]e{-}i\omega t \quad (20)$$

$$Nu_1 = \frac{du}{dy}/_{y=1} = [Er_5 e^{r_5} - Fr_6 e^{-r_6} + H_1 r_1 e^{r_1} - H2r2e{-}r2ei\omega t{+}[Jr7er7{-}Kr8e{-}r8{+}H3r3er3{-}H4r4e{-}r4]e{-}i\omega t$$
(21)

IV. GRAPHICAL RESULTS AND DISCUSSION

To analyze the effect of suction/injection on MHD Oscillatory flow of Jeffrey fluid with heat source/sink through porous medium, the velocity u, and skin friction ($\tau_0$andτ_1) profiles are depicted graphically against y for different values of different parameters; Jeffrey fluidβ_1, suction/injection parameter V_0, Heat source/sink parameter Q, thermal Grashof number Gr, Hartmann number H, radiation parameter N, Peclet number Pe, Reynolds number Re, Darcy number Da, Prandtl number Pr, slip parameter γ and porous medium shape factor s. We made use of the following parameter values except otherwise indicated, $\beta_1 = 0.01$, Re = 1, S = 1, Ha = 1, Gr = 10, Pe = 1, N = 1, λ=−1, γ = 0.5 andω = 1.

The velocity profiles have been studied and presented in Figures 1 to 5. The velocity profiles for different values of Grashof number Gr (Gr = 1, 5, 10) is shown in figure 1. It is observed that the velocity increase with increasing Grashof number. The Velocity profiles for different values of slip parameter (γ=0.1, 0.5, 1 and γ=-o.1, -o.5, -1) is shown in figure 2. It is observed that the

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velocity increase with increasing slip parameter. The velocity profiles for different values of the Hartmann number (Ha = 1, 2, 3), heat source parameter (Q=1, 5, 10) and Jeffrey fluid parameter (β_1=0.1, 0.2, 0.3) are shown in Figures 3, 4 and 5 respectively. It observed that the velocity decreases with increasing Hartmann number, heat source and Jeffrey parameter.

The variation of the skin friction on the porous plate with material parameters are shown in figure 6 to 9. The skin friction profiles for different values of slip parameter (γ=0.1, 0.2, 0.3), Jeffrey fluid parameter (β_1=0.1, 0.2, 0.3) and heat source/sink parameter (Q=1, 2, 3) are shown in figure 6, 7 and 8 respectively. It is observed that skin friction decreases with increasing of slip parameter, Jeffrey fluid parameter and heat source parameter. The skin friction profiles for different values of suction/injection parameter (V_0=0.1, 0.5, 1) is shown in figure 9. It is observed that skin friction increases with increasing injection parameter while decreases with increasing suction parameter.

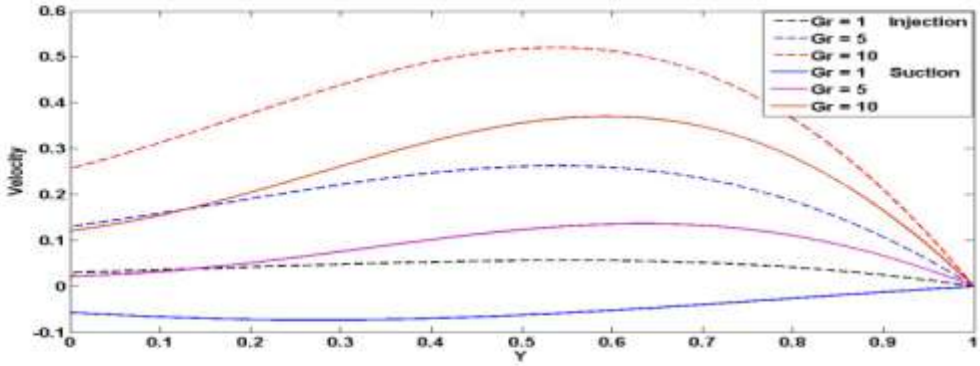

Figure 1: Velocity profiles for different values of Grashof number Gr

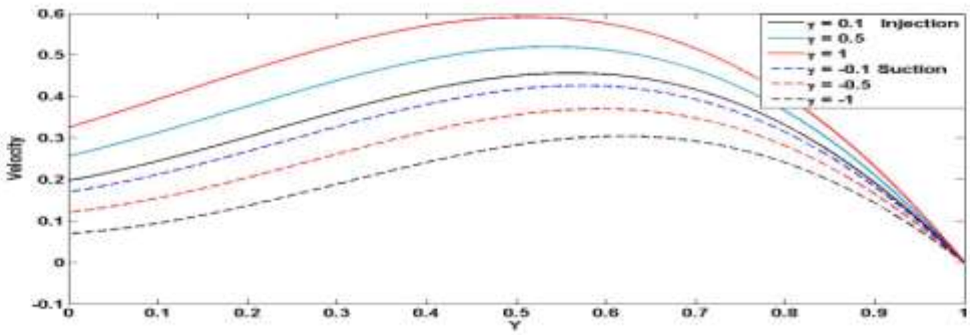

Figure 2: Velocity profiles for different values of slip parameter

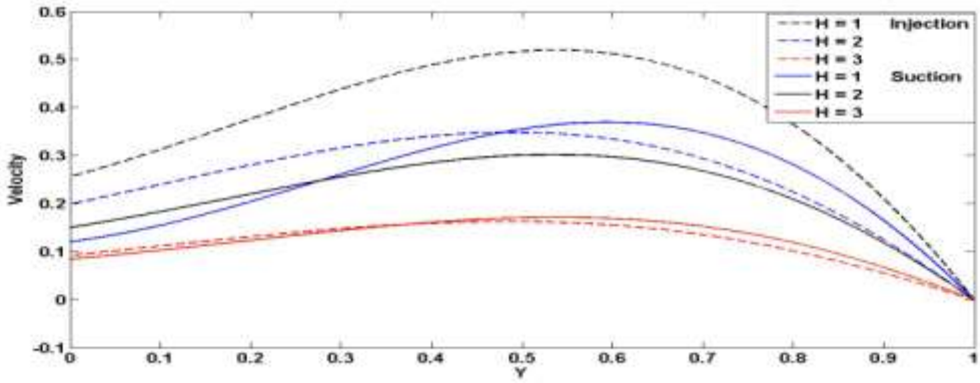

Figure 3: Velocity profiles for different values of Hartmann number

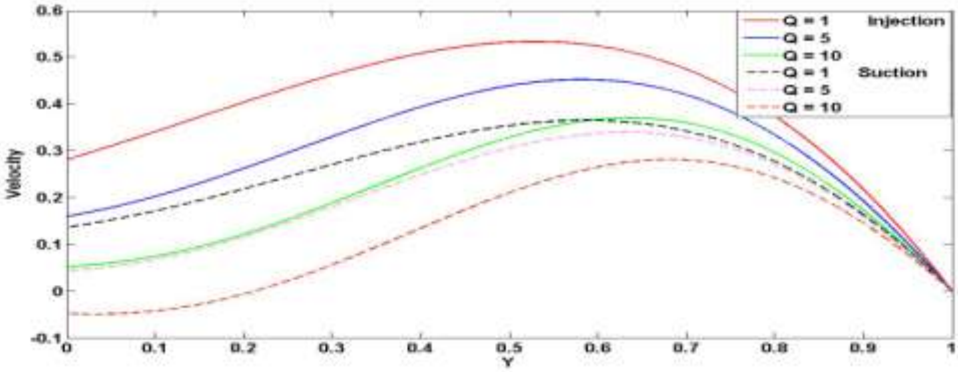

Figure 4: Velocity profiles for different values of heat source/sink parameter

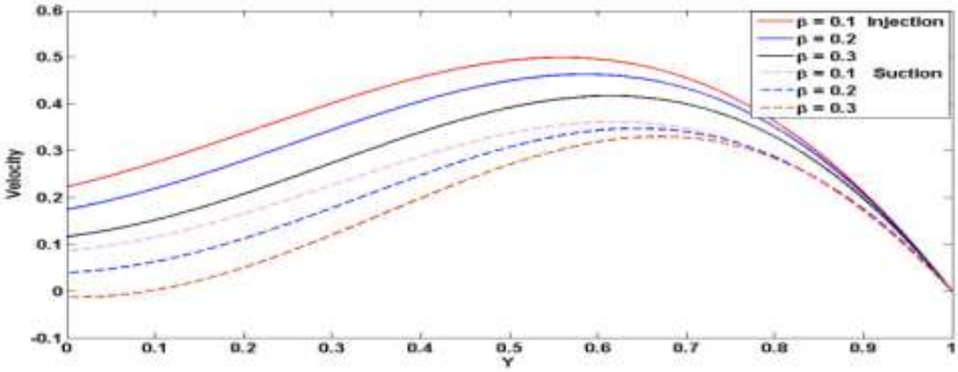

Figure 5: Velocity profiles for different values of Jeffrey fluid parameter

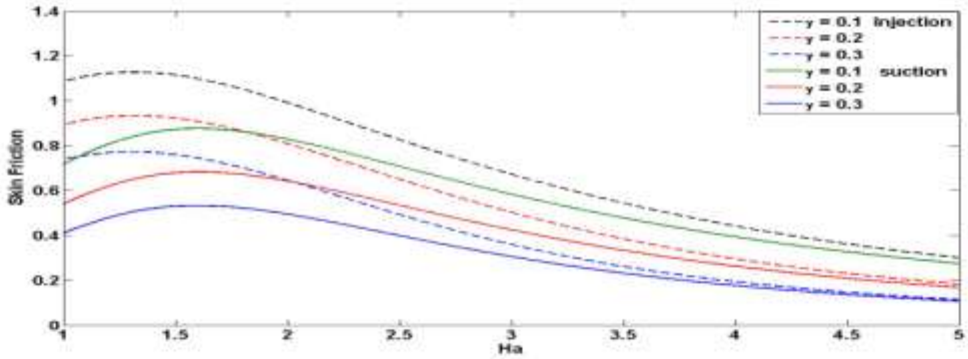

Figure 6: skin friction profiles for different values slip parameter

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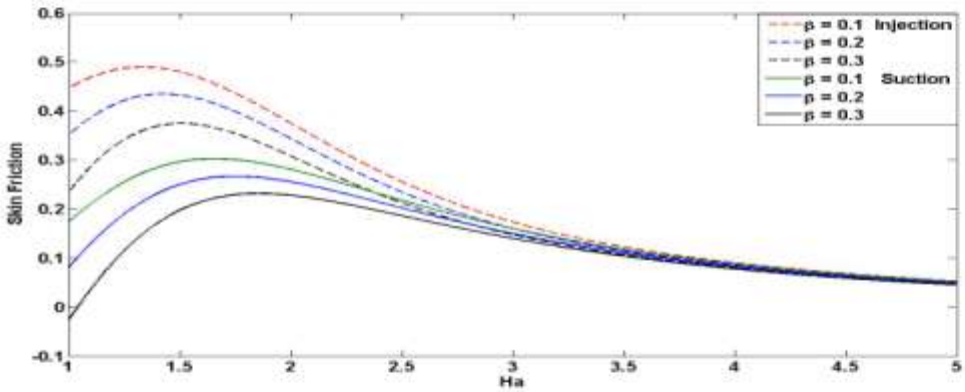

Figure 7: Skin friction profiles for different values of Jeffrey fluid parameter

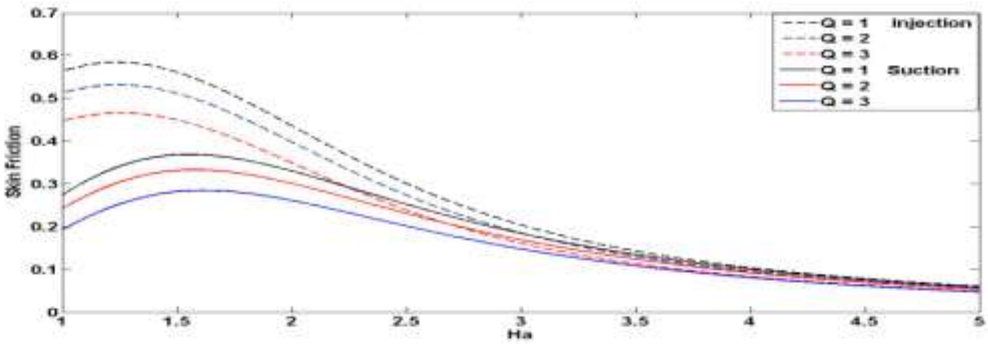

Figure 8: Skin friction profiles for different values of heat source/sink parameter

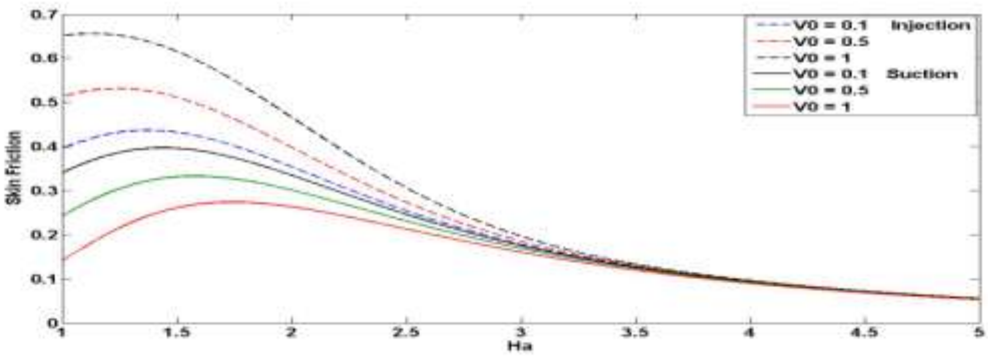

Figure 9: Skin friction profiles for different values of suction/injection parameter

V. CONCLUSIONS

This paper analyzes the effect of suction/injection on MHD Oscillatory flow of Jeffrey fluid with heat source/sink through a porous medium. The velocity and skin friction profiles are obtained analytically. The effect of different parameters namely, the Grashof number, Hartmann number, Jeffrey fluid parameter, Porosity parameter, Slip parameter, heat source/sink parameter and suction/injection parameter are studied. The conclusions of the study are as follows;

(i) It is observed that increasing Grashof number and slip parameter has enhancing effect on velocity.
(ii) It observed that the velocity decreases with increasing Hartmann number, heat source and Jeffrey fluid parameter.
(iii) It is observed intensification of slip parameter, Jeffrey fluid parameter and heat source

parameter has diminishing effect on skin friction.

(iv) It is observed that skin friction increases with increasing injection parameter while decreases with increasing suction parameter.

REFERENCES

[1]. Adesanya, S.O and Makinde, O.D. (2014). MHD Oscillatory slip flow and heat transfer in channel filled with porous media. U.P.B. Science Bull. Series A, Vol. 76, Iss. 1.

[2]. Al-khafajy, D.G.S (2016). Effects of heat transfer on MHD oscillatory flow of Jeffrey fluid with variable viscosity through porous medium. Advances in Applied Science Research, 7(3): PP. 179-186.

[3]. Ahmad, K and Ishak, A. (2017). Magnetohydrodynamic (MHD) Jeffrey fluid over a stretching vertical surface in a porous medium. Propulsion and Power Research. 6(4): PP 269–276.

[4]. Aruna Kumari B., Ramakrishna Prasad K. and Kavita K. (2012). Slip Effects on MHD Oscillatory Flow of Jeffrey Fluid in a Channel with Heat Transfer. International Journal of Mathematics Arch. 3 (8), PP 2903 – 2911.

[5]. Asadullah M., Umar K., Nareed A., Raheela M. and Mohyud – Din S.T. (2013). MHD Flow of a Jeffrey Fluid in Converging and Diverging Channels. International Journal of Mathematics science. 6 (2): PP 92 – 106.

[6]. Hamza, M. M, Isah, B. Y and Usman, H. (2011). Unsteady heat transfer to MHD oscillatory flow through porous medium under slip condition. International Journal of computer Application, 33(4), PP. 0975-8887.

[7]. Iduwu, A.S, Joseph, K.M and Daniel, S. (2013). Effect of Heat and Mass Transfer on Unsteady MHD Oscillatory Flow of Jeffrey Fluid in a Horizontal Channel with Chemical Reaction. Journal of Mathematics (IOSR-JM), 8(5) PP. 74-87.

[8]. Jha,B.K and Ajibade, A.O. (2010). Free convective flow between vertical porous plates with periodic heat input, Z. Angew. Math. Mech. Journal of Applied Mathematics and physics, Pp.1-9.

[9]. Kavita, K, Ramakrishna, P.K, and Kumari, A. (2012). Influence of heat transfer on MHD Oscillatory flow of Jeffrey fluid in a channel. Advances in Applied Science Research, 3 (4): PP 2312-2325.

[10]. Kumar, A., Varshney, C. L and Sajjan,L. (2010). Perturbation Techniques to Unsteady M H D Periodic Flow of Viscous Fluid through a Planner Channel. Journal of Eng. Technol. Res., 2(4): PP 73-81.

[11]. Makinde, O.D and Mhone, P. Y. (2005). Heat Transfer to MHD Oscillatory Flow in a Channel Filled with Porous Medium. Rom Journal of Physics, 50(9-10), Pp. 931 – 938.

[12]. Mehmood, A and Ali, A. (2007). The Effect of Slip Condition on Unsteady MHD Oscillatory Flow of a Viscous Fluid in a Planner Channel. Rom. Journal of Physics, 52(1-2), Pp. 85-91.

[13]. Yale, I.D, Aisha, A. H, Mustapha, I. and Muhammad, M. H. (2019). Unsteady Heat Transfer to MHD Oscillatory flow of Jeffrey fluid in channel filled with porous material. IJSRP V 9 p9178

[14]. Sahim, A. (2010). The effect of viscous dissipative heat on three dimensional oscillatory flows with periodic suction velocity. Indian J. Sci. Technol., 3(3): PP 276-283.

[15]. Singh, P. and Gupta, C.B. (2012). MHD Free Convective of Flow of Viscous Fluid through a Porous Medium Bounded by an Oscillating Porous Plate in Slip Flow Regime with Mass Transfer. Indian J. Theor. Physics 53 (2): Pp. 111 – 120.

[16]. Uwanta, I.J and Hamza, M.M. (2012). Transient Heat Transfer Flow in a channel with porous medium and periodic suction. Research Journal of Mathematics and statistics, 4(4): PP 89-93.