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# Waste to Wealth in Enugu Urban Solid Waste generation using Jacobi's Iteration Optimization Model

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**ABSTRACT:** The planning and management of Municipal of Solid Waste (MSW) in Enugu Urban using Jacobi's iteration optimization model is a study intended to solve the problem of indiscriminate disposal of solid waste in Enugu metropolis. The intention is to maximize the economic benefit of proper solid waste handling and provide a better clean environment. The aim is to develop optimization solution strategies for solid waste planning and management in Enugu urban. The existing solid waste planning practices were reviewed with a view to determine solid waste volume, characteristics and Jacobi's Iteration Optimization Model was used to model the cost effective solid waste management in order to convert waste to wealth. The methodology used include characterization of solid waste deposited at selected dumpsites, and estimation of total volume of waste generated per capita per day. This solid waste volume was projected for fifty (50) years based on 2006 census figure of Enugu urban. Theresult of Jacobi's Iteration Optimization Model converged at $X_1 = 2162$, $X_2 = 2222$, $X_3 = 3880$ and $X_4 = 3343$ for e-waste, plastics, ceramics and metals respectively. The objective function was maximized at ₦21.07million per ton per day or ₦ 147.51million per ton per week or ₦ 7.67 billion per ton per annum. The work concluded that using Jacobis iteration optimization model, substantial wealth can be created from solid waste. This will make solid waste a big market for Enugu Waste Management Authority (ESWAMA), the inhabitants and the communities which will lead to better managed clean environment. It was recommended that processing of waste products and its use for various agricultural, industrial and commercial purposes will create wealth in Enugu. The contribution to knowledge shows that with respect to the use of an optimal policy which converges at the expected points is maximized.

**Keywords:** solid waste planning and management, Jacobi's iteration, optimization model, waste to wealth.

## I. INTRODUCTION

The planning and management of solid waste in Enugu the capital city of Enugu state has been an issue of concern to successive government in Enugu both at state and local government areas. Various strategies to tackling the problem enunciated by successive governments are inconsistent with the global standard.Solid waste problem in Enugu contributes to the contamination of the streams, river, land and the atmosphere. Waste disposal operations are becoming increasingly sophisticated with specialist companies and facilities, leaving the developing countries to rise up to the challenges.

There have been an increase in population of Enugu Urban from 3,170 people in 1921, 12, 959 in 1931, 138, 874 in 1963, 385,735 in 1983 to 722,664 in 2006 (NPC Report, 2006) and it is projected to be 900,319 in 2020. Enugu state recorded a total of 3, 267, 837 people in 2006 with a population density of 268 persons per square kilometers while the average national density is about 96 persons per sq.km. The urban population concentration is high with densities ranging between 300-600 persons per sq.km (NPC, 2006).

Tchobanoglous (2009) opined that; Human activities generate waste materials that are often discarded because they are considered useless. These wastes are normally solid, and the word "waste" suggests that the material is useless and unwanted.

It is essential to be aware that many of these waste materials can be reused, and thus they can become a source for industrial production or energy generation if managed properly. Waste management has become one of the most significant problems of our time because any modern society of life produce enormous amounts of waste and most people want to preserve their lifestyle, while also protecting the environment and public health. Industry, private citizens and state legislature are searching for means to reduce it or dispose of it safely and economically. In recent years, various states legislatures have passed more laws dealing with solid waste management than with any other related issues of waste. It is essential to examine background materials on issues and challenges involved in the management of municipal solid waste (MSW) and determine the information on specific technologies and management options.

Tchobanoglous and Kreith (2002) emphasized that historically, waste management has been an engineering function. It is related to the evolution of a technological society, which along with the benefits of mass production has also created problems that require the disposal of solid wastes. The flow of materials in a technological society and the resulting waste generation – are illustrated schematically in Figure 1. Wastes are generated during the mining and production of raw materials, such as the tailings from a mine or the discarded husks from a cornfield. After the materials have been mined, harvested, or otherwise procured, more wastes are generated during subsequent steps of the processes that generate goods for consumption by society from these raw materials. It is apparent from control the solid waste disposal problem is that it is generated, but as people search for a better life and a higher standard of living, they tend to consume more goods and generate more waste. Consequently, society is searching for improved methods of waste management and ways to reduce the amount of waste that needs to be landfilled or otherwise (see Figure 1).

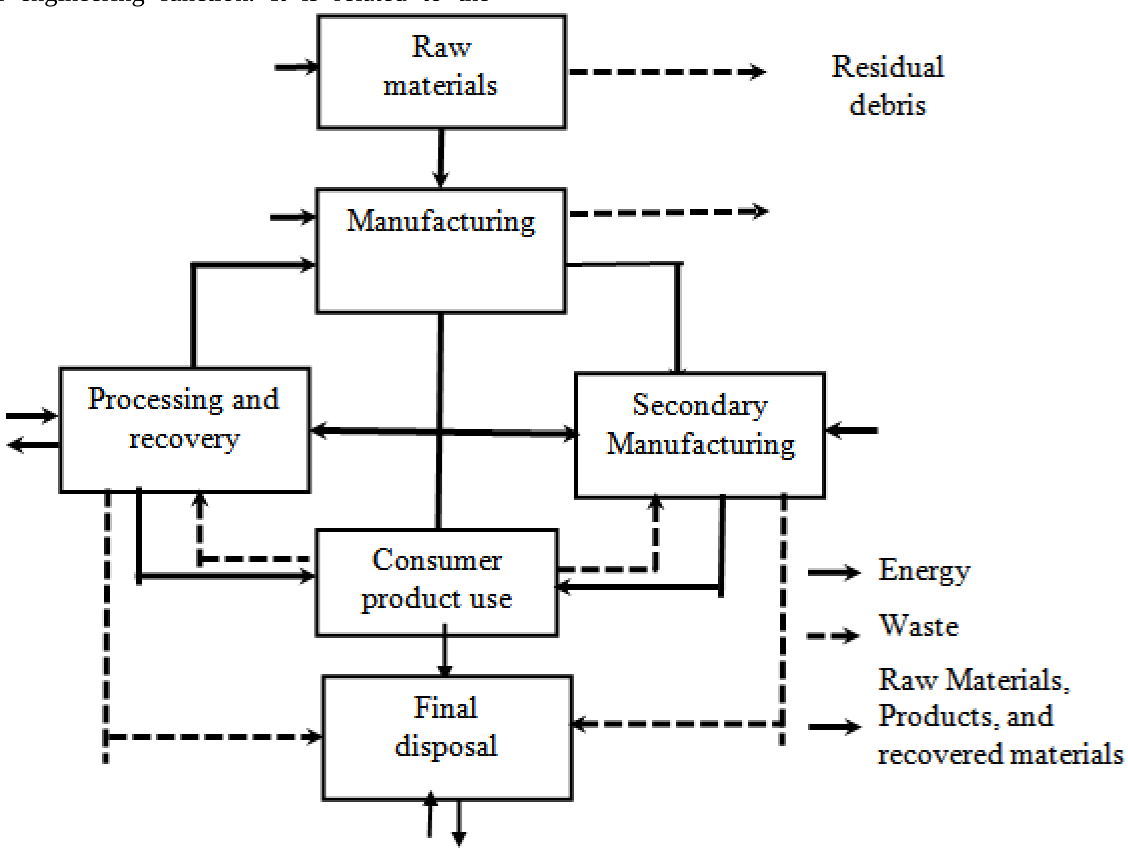


Figure 1: Flow of materials and waste in an industrial society

**Source**: Pichtel (2005)

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### 1.1 Aim and Objectives of the Study

The aim of this study is to develop optimization solution strategies for solid waste management in Enugu urban using Jacobi's Iteration model.

The objectives of this study include the following:

i. To quantify solid wastes volumes in Enugu urban.
ii. To determine the characteristics of solid wastes generated in Enugu urban
iii. To apply Jacobi's Iteration Model as a better waste management system for Enugu urban,
iv. To develop the optimization modeling cost to model the cost effective solid waste management system and planning for Enugu urban
v. To convert waste to wealth.

## II. LITERATURE REVIEW

The literature review is based on the concept of Jacobi's method which is used to determine the optimization model for solid waste generation in Enugu.

### 2.1 Jacobi'sMethod

The method is named after Carl Gustav Jacob Jacobi. The Jacobi's method is an iterative algorithm for determining the solutions of a strictly diagonally dominantsystem of linear equations in numerical linear algebra. It consists of solving for each diagonal element before an approximate value is plugged in. The process is then iterated until it converges. This algorithm is a stripped-down version of the Jacobi transformation method of matrix diagonalization.

**Description**

Let, $A_X = b$ be a square system of n linear equations, where: (2.1)

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_1 \end{bmatrix}, \quad (2.2)$$

Then A can be decomposed into a diagonal component D, and the remainder R:

$$A = D + R \text{ where } D = \begin{bmatrix} a_{11} & 0 & \cdots & 0 \\ 0 & a_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_{nn} \end{bmatrix}, \text{and } R = \begin{bmatrix} 0 & a_{22} & \cdots & a_{1n} \\ a_{11} & 0 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & 0 \end{bmatrix}, \quad (2.3)$$

The solution is then obtained iteratively via

$$x^{(k+1)} = D^{-1}(b - Rx^{(k)}), \quad (2.4)$$

where $x^{(k)}$ is the kth approximation or iteration of x and $x^{(k+1)}$ is the next or k + 1 iteration of x. The element-based formula is thus:

$$x_i^{(k+1)} = \frac{1}{a_{ii}}\left(b_i - \sum_{j \neq i} a_{ij} x_j^{(k)}\right), i = 1,2, \ldots, n \quad (2.5)$$

The computation of $x_i^{(k+1)}$ requires each element in $\mathbf{x}^{(k)}$ except itself. Unlike the Gauss–Seidel method, we can't overwrite $x_i^{(k)}$ with $x_i^{(k+1)}$, as that value will be needed by the rest of the computation. The minimum amount of storage is two vectors of size n.

**Convergence:** The standard convergence condition (for any iterative method) is when the spectral radius of the iteration matrix is less than 1:

$$\rho(D^{-1}R) < 1. \quad (2.6)$$

A sufficient (but not necessary) condition for the method to converge is that the matrix A is strictly or irreducibly diagonally dominant. Strict row diagonal dominance means that for each row, the absolute value of the diagonal term is greater than the sum of absolute values of other terms:

$$|a_{ii}| > \sum_{j \neq i} |a_{ij}|.$$

The Jacobi method sometimes converges even if these conditions are not satisfied.

Note that the Jacobi method does not converge for every symmetric positive-definite matrix. For example

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$$A = \begin{pmatrix} 29 & 2 & 1 \\ 2 & 6 & 1 \\ 1 & 1 & \frac{1}{5} \end{pmatrix} \to D^{-1}R = \begin{pmatrix} 0 & \frac{2}{29} & \frac{1}{29} \\ \frac{1}{3} & 0 & \frac{1}{6} \\ 5 & 5 & 0 \end{pmatrix} \Rightarrow$$

$\rho(D^{-1}R) \approx 1.0661$ (2.8)

Example : A linear system of the form Ax = bwith initial estimate $x^{(0)}$

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 7 \end{bmatrix}, \quad b = \begin{bmatrix} 11 \\ 13 \end{bmatrix} and x^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We use the equation, $x^{(k+1)} = D^{-1}(b - Rx^{(k)})$, described above, to estimate x. First, we rewrite the equation in a more convenient form D $^{-1}$ (b – R $x^{(k)}$) = T $x^{(k)}$ + C

where T= $D^{-1}R$ $and$ C = D $^{-1}$ b. Note that $R = L + U$ where L and U are the strictly lower and upper parts of A. From the known values

$$D^{-1} = \begin{bmatrix} 1/2 & 1 \\ 0 & 1/7 \end{bmatrix}, L = \begin{bmatrix} 0 & 0 \\ 5 & 0 \end{bmatrix} and\ U = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

$$we\ determine\ T = -D^{-1}(L + U)\ as$$

$$T = \begin{bmatrix} 1/2 & 1 \\ 0 & 1/7 \end{bmatrix} \left\{ \begin{bmatrix} 0 & 0 \\ -5 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 0 & -1/2 \\ -5/7 & 0 \end{bmatrix}$$

Further, C is found as

$$C = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/7 \end{bmatrix} \begin{bmatrix} 11 \\ 13 \end{bmatrix} = \begin{bmatrix} 11/2 \\ 13/7 \end{bmatrix}.$$

With$T$ and $C$ calculated, we estimate as $x^{(1)}$ and T$x^{(0)} + C$:

$$x^{(1)} = \begin{bmatrix} 0 & -1/2 \\ -5/7 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 11/2 \\ 13/7 \end{bmatrix} = \begin{bmatrix} 5.0 \\ 8/7 \end{bmatrix} \approx \begin{bmatrix} 5 \\ 1.143 \end{bmatrix}.$$

The next iteration yields

$$x^{(2)} = \begin{bmatrix} 0 & -1/2 \\ -5/7 & 0 \end{bmatrix} \begin{bmatrix} 5.0 \\ 8/7 \end{bmatrix} = \begin{bmatrix} 11/2 \\ 13/7 \end{bmatrix} = \begin{bmatrix} 69/14 \\ -12/7 \end{bmatrix}$$

$$\approx \begin{bmatrix} 4.929 \\ -1.714 \end{bmatrix}$$

$This\ process\ is\ repeated\ until\ convergence\ (i.e., until\ \|Ax^{(n)} - b\|\ is\ small).$

$$The\ solution\ after\ 25\ iterations\ is x = \begin{bmatrix} 7.111 \\ -3.222 \end{bmatrix}$$

Another Example: Suppose we are given the following linear system:

$$10x_1 - x_2 + 2x_3 = 6\,,$$
$$-x_1 + 11x_2 - x_3 + 3x_4 = 25$$
$$2x_1 - x_2 + 10x_3 - x_4 = -11\,,$$
$$3x_2 - x_3 + 8x_4 = 15$$

If we choose (0, 0, 0, 0) as the initial approximation, then the first approximate solution is given by

$$x_1 = (6 + 0 - (2 * 0))/10 = 0.6,$$
$$x_2 = (25 + 0 + 0 - (3 * 0))/11 = 25/11 = 2.2727,$$
$$x_3 = (-11 - (2 * 0) + 0 + 0)/10 = -1.1,$$
$$x_4 = (15 - (3 * 0) + 0)/8 = 1.875$$

Using the approximations obtained, the iterative procedure is repeated until the desired accuracy has been reached. The following are the approximated solutions after five iterations as shown in Table 2.16.

**Table 2.16: Approximated Solution after five iterations in Jacobi's Method.**

| $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|
| 0.6 | 2.27272 | -1.1 | 1.875 |
| 1.04727 | 1.7159 | -0.80522 | 0.88522 |
| 0.93263 | 2.05330 | -1.0493 | 1.13088 |
| 1.01519 | 1.95369 | -0.9681 | 0.97384 |
| 0.98899 | 2.0114 | -1.0102 | 1.02135 |

The exact solution of the system is (1, 2, -1, 1)

**Weighted Jacobi method:** The weighted Jacobi iteration uses a parameter$\omega$to compute the iteration as

$X^{(k-1)} =$
$\omega D^{-1}(b - Rx^{(k)}) +$
$(1 - \omega)x^{(k)}$ (2.9)

With ω = 2 / 3 being the usual choice.

**Convergence in the Symmetric Positive Definite Case:** In case that the system matrix A is of symmetric positive-definite type one can show convergence.

Let $C = C_\omega$ = I − ωD$^{-1}$$A$be the iteration matrix. Then, convergence is guaranteed for

$$\rho(C_w) < 1 \Leftrightarrow 0 < \omega < \frac{2}{\lambda_{max}\,(D^{-1}A)}, \quad (2.10)$$

where $\lambda_{max}$ is the maximal eigenvalue.

The spectral radius can be minimized for a particular choice of $\omega = \omega_{opt}$ as follows

$$min\,\rho(C_w) = \left(C_{\omega_{opt}}\right) = 1 - \frac{2}{k(D^{-1}A)+1}\ for\ \omega_{opt} := \omega\ \omega\ \frac{2}{\lambda_{max}\,(D^{-1}A)+\lambda_{max}\,(D^{-1}A)}$$

Where, $k$ is the matrix condition number.

## III. METHODOLOGY

The methodology for the analysis is based on the projected population of Enugu urban to determine the volume of Solid Waste generated in the area.

### 3.1 Population Projection of Enugu Urban in the Next Fifty Years from 2006 Census

The demographic data of population in Nigeria does not follow a uniform trend. Census figure supposed to be obtained every ten (10) years but the period from the available data the National Population Commission (NPC) shows that Enugu Urban Population rose from 3,170 in 1921 to 12,959 in 1931, 62,764 in 1953 to 138,874 in 1964; 385,735 in 1983. The census figures of 407,756 in 1991 to 722,664 in 2006. However, there is a sporadic increase in population of Enugu Urban due to religious, ethnic and political crises in different parts of the country especially in Northern Nigeria. Most people of Igbo extraction living in the North and West decided to relocate to Enugu as a safe choice. This is because of its position as the capital of the former Eastern region of Nigeria. This makes it difficult to forecast the population growth rate based on decades estimation.

Therefore, Geometric method using the method of assumed growth rate as adopted at 17% per decade for fast growing city like Enugu with 2006 population figure as a base year was used for the estimation.

$$Using\ the\ formular,\ P_n = P_o\left(1 + \frac{r}{100}\right)^n \quad (4.1)$$

Where $P_o$ = Initial population i.e. the population at the end of last known census

$P_n$ = Future population after $n$ decades

$r$ = Assumed growth rate (%)

$n$ = Number of decades

It is pertinent here, to forecast the population of Enugu Urban in current year 2020 and project to year 2056 with 2006 population data. This will help to forecast the volume or tons of solid waste generated, with a view of planning and managing them for effective Solid Waste Management in Enugu Urban.

Referring to the formulae above, for year 2020,

$$P_o = P_{2006} = 722664, r = 17\%, \quad n = 1.4$$

$$P_n = P_{2020} = 722664, \left(1 + \frac{17}{100}\right)^{1.4} = 722664\ (1.17)^{1.4} = 900{,}319$$

$$for\ year\ 2056\ i.e.\ forecasting\ for\ the\ next\ 5\ decades$$

$$P_n = P_{2056} = 722664(1.17)^5 = 1{,}584{,}403$$

Agbaeze et al. (2014) estimated that Enugu Urban generated 150 metric tons per day.

In order to estimate the population based on 2006

$$P_n = P_{2014} = P_{2006}\ (1.17)^{0.8} = 722664\ (1.17)^{0.8} = 819{,}380$$

Forecasting using this research model, in 2020 we have, $\frac{900319}{819{,}380} \times 150 = 165\ tons$. However, this model has not taken into account the total waste generated in Enugu urban but on the capacity of daily disposal vehicles /Equipment by Enugu State Waste Management Authority (ESWAMA). A lot of solid wastes generated are lying uncollected at alternate days while those from industries, agricultural and allied institutions were not captured because their collection route did not extend to those areas. Some of these companies use their private vehicles to dispose their Solid wastes to convenient waste disposal sites or incinerate them.

Today the waste generation has increased tremendously that ESWAMA employs the services of individual private vehicles to improve the solid waste collection and disposal. The current strategy is not allowing wastes to stay long on dumpsites before collection and disposal. The information from ESWAMA Landfill site at Enugu reveals that as at date from their records, an average of 2400 tons per day was deposited at the site.

## IV. RESULT AND DISCUSSION

### 4.1 Analysis of Total Solid Waste Generated in Enugu Urban Using Forecast for the Period Generated

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The total waste generated from the four (4) locations as at January, 2020 = 2,400,000 kg/day= 2,400 tons/day.
The population projection based on 2006 Census of 900319 persons generates

$$\frac{2{,}400{,}000 kg/day}{900319\ persons} = 2.666 kg/person/day$$

With the projected population of 1,584,403 in 2056 based on next 50 years from 2006 Census adjusted for each of the four locations we have 1055894 kg/day as stated above. The total waste generated as per projected population of 2056 is

$$\frac{1584403}{900319} \times 2{,}400{,}000 kg/day$$
$$= 4223577.6\ kg/day \frac{4223577.6\ kg}{4}$$
$$= 1055894\ kg/day$$

We use this to derive the Table 4.1 below for the quantity of options/day/tons.

**Table 1: Quantity of Options/Day/Ton**

| Group | E-waste ($X_1$) | Plastics ($X_2$) | Ceramics ($X_3$) | Metal ($X_4$) | Total Available Wastes |
|---|---|---|---|---|---|
| A | $\frac{1055894}{864} \times \frac{100.2}{1000}$ = 122.46 tons | $\frac{1055894}{864} \times \frac{88.5}{1000}$ = 108.16 tons | $\frac{1055894}{864} \times \frac{1}{1000}$ = 2 tons | $\frac{1055894}{864} \times \frac{33.5}{1000}$ = 40.94 tons | $\frac{1055894}{864} \times \frac{93.4}{1000}$ = 1142 tons |
| B | $\frac{1055894}{864} \times \frac{31.6}{1000}$ = 38.62 tons | $\frac{1055894}{864} \times \frac{142.7}{1000}$ = 174.4tons | $\frac{1055894}{864} \times \frac{142.5}{1000}$ = 94.4 tons | $\frac{1055894}{864} \times \frac{29.2}{1000}$ = 35.68 tons | $\frac{1055894}{864} \times \frac{1080}{100}$ = 1320 tons |
| C | $\frac{1055894}{864} \times \frac{90.6}{1000}$ = 111 tons | $\frac{1055894}{864} \times \frac{87.6}{1000}$ = 107.2 tons | $\frac{1055894}{864} \times \frac{96.4}{1000}$ = 117.8 tons | $\frac{1055894}{864} \times \frac{48.5}{1000}$ = 59.2 tons | $\frac{1055894}{864} \times \frac{1130}{1000}$ = 1381 tons |
| D | $\frac{1055894}{864} \times \frac{122.2}{1000}$ = 149.4 tons | $\frac{1055894}{864} \times \frac{40.1}{1000}$ = 49 tons | $\frac{1055894}{864} \times \frac{89.3}{1000}$ = 109.2 tons | $\frac{1055894}{864} \times \frac{142.5}{1000}$ = 174.2 tons | $\frac{1055894}{864} \times \frac{1200}{1000}$ = 1467 tons |

Table 1 above shows the relationship with maximum available wastes and the various sample waste at location A, B, C, and D. extracted from information on the location where the samples of the solid waste deposited were sampled and characterized for the analysis. These figures are derived based on the population projection for 2056 and will be used to formulate the constraints equation in the linear programming model.

**Table 2: Cost of the Recycling Options**

| S/N | Material type/Option | Cost/kg (₦) | Cost per ton (₦ ton) |
|---|---|---|---|
| 1 | e-waste = 2350/kg×1.5 ($X_1$) = ₦3525 | $\frac{1055894}{864} \times 3525$ = ₦ 4308349.6 | 4308 |
| 2 | Plastics =77×1.5 ($X_2$) = ₦116 | $\frac{1055894}{864} \times 116$ = ₦ 141778.32 | 141.8 |
| 3 | Ceramics = 880×1.5 ($X_3$) = ₦1320 | $\frac{1055894}{864} \times 1320$ = ₦ 1613339.44 | 1613.3 |

| | | | |
|---|---|---|---|
| 4 | Metals = 846×1.5<br>($X_4$) = ₦1269 | $\frac{1055894}{864} \times 1269$<br>= ₦ 1551005.88 | 1551 |

The Cost per tons as Stated in Table 2 above will be used to formulate objective function for Profit maximization. Using the forgoing data, the optimization problem can be written in the form

$$\text{Maximize, } Z = ₦4308\,X_1 + ₦142\,X_2 + ₦1613\,X_3 + ₦1551\,X_4 \qquad \text{(Objective Function)}$$

$$\text{Subject to:}\quad \left.\begin{array}{ll} 122\,X_1 + 108\,X_2 + 2\,X_3 + 41\,X_4 \le 1142 & \text{(i)} \\ 39\,X_1 + 174\,X_2 + 94\,X_3 + 36\,X_4 \le 1320 & \text{(ii)} \\ 111\,X_1 + 107\,X_2 + 118X_3 + 59\,X_4 \le 1381 & \text{(iii)} \\ 149\,X_1 + 49\,X_2 + 109X_3 + 174\,X_4 \le 1467 & \text{(iv)} \end{array}\right\} \text{System constraints}$$

$$X_1 \ge 0,\ X_2 \ge 0,\ X_3 \ge 0,\ X_4 \ge 0, \quad - \quad \text{Non-Negativity condition.}$$

Where; Z = Profit/day/ton/1,584,403 (Population)

$X_1$= tons of optimal quantity of e-waste

$X_2$ = tons of optimal quantity of Plastics

$X_3$ = tons of optimal quantity of Ceramics

$X_4$ = tons of optimal quantity of Metals.

This Linear Programming can be formulated using Jacobi's Method (Dass, 2000). We start with an approximation to the true solution and by applying the method repeatedly we get better and better approximation till accurate solution is achieved. This is referred to as **"Iterative Method or Indirect Methods**". There are two iterative methods for solving the simultaneous equations;

**(1). Jacobi's Method (Method of Simultaneous Correction)**
**(2). Gaus-Seidel Method (Method of Successive Correction).**

However, the **Jacobi's iterative Method** is used for this analysis.

Note: Condition for using the iterative methods is that the coefficients in the leading diagonal are large compared to the other. If are not so, then on interchanging the equation, we can make the leading diagonal dominant diagonal.

**4.2 The Jacobi's Iteration Optimization Model for Optimal Solution**

To solve the Linear Programming Model using Jacobi's Iterative Method, the system constraints equation can be written as;

$$122\,X_1 + 108\,X_2 + 2\,X_3 + 41\,X_4 \le 1142 \qquad \text{(i)}$$

$$39\,X_1 + 174\,X_2 + 94\,X_3 + 36\,X_4 \le 1320 \qquad \text{(ii)}$$

$$111\,X_1 + 107\,X_2 + 118X_3 + 59\,X_4 \le 1381 \qquad \text{(iii)}$$

$$149\,X_1 + 49\,X_2 + 109X_3 + 174\,X_4 \le 1467 \qquad \text{(iv)}$$

After division of suitable constraints and transposition, the equations can be written as;

For, $122\,X_1 = 1142 - 108\,X_2 - 2X_3 - 41\,X_4$

Divide through by 122,

$$\Rightarrow X_1 = \frac{1142}{122} - \frac{108}{122}X_2 - \frac{2}{122}X_3 - \frac{41}{122}X_4 \qquad (i)$$

For, $174\,X_2 = 1320 - 39\,X_1 - 94X_3 - 36\,X_4$

Divide through by 174,

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$$\Rightarrow \quad X_2 = \frac{1320}{174} - \frac{39}{174}X_1 - \frac{94}{174}X_3 - \frac{36}{174}X_4 \qquad (ii)$$

For, 118 $X_3$ = 1381 – 111 $X_1$ – 107$X_2$ – 59 $X_4$

Divide through by 118,

$$\Rightarrow X_3 = \frac{1381}{118} - \frac{111}{118}X_1 - \frac{107X_2}{118} - \frac{59}{118}X_4 \qquad (iii)$$

For, 174 $X_4$ = 1467 – 149 $X_1$ – 49$X_2$ – 109 $X_4$

$$\Rightarrow X_4 = \frac{1467}{174} - \frac{149}{174}X_1 - \frac{49X_2}{174} - \frac{109}{174}X_3 \qquad (iv)$$

$$Therefore,\ X_1 = \frac{1142}{122} - \frac{108}{122}X_2 - \frac{2}{122}X_3 - \frac{41}{122}X_4 \qquad (i)$$

$$X_2 = \frac{1320}{174} - \frac{39}{174}X_1 - \frac{94}{174}X_3 - \frac{36}{174}X_4 \qquad (ii)$$

$$X_3 = \frac{1381}{118} - \frac{111}{118}X_1 - \frac{107}{118}X_2 - \frac{59}{118}X_4 \qquad (iii)$$

$$X_4 = \frac{1467}{174} - \frac{149}{174}X_1 - \frac{49}{174}X_2 - \frac{109}{174}X_3 \qquad (iv)$$

**OR,** $X_1 = 9.36 - 0.89X_2 - 0.02X_3 - 0.34\,X_4 \qquad (i)$

$X_2 = 7.59 - 0.22\,X_1 - 0.54X_3 - 0.21\,X_4 \qquad (ii)$

$X_3 = 11.70 - 0.94\,X_1 - 0.91X_2 - 0.5\,X_4 \qquad (iii)$

$X_4 = 8.43 - 0.86\,X_1 - 0.28X_2 - 0.63X_3 \qquad (iv)$

**First (1st) Iteration**

When, $X_1$ = 0, $X_2$= 0, $X_3$ = 0, $X_4$= 0

Substituting the values in RHS of the above equations (i) to (iv)

We have, $X_1$ = 9.36; $X_2$= 7.59; $X_3$ = 11.70; $X_4$= 8.43

These values are used for the second iteration.

**Second (2nd) iteration**

Substituting the new values of $X_1$ = 9.36; $X_2$= 7.59; $X_3$ = 11.70; $X_4$= 8.43 in equation (i) to (iv), we have,

$$X_1 = 9.36 - 0.89(7.59) - 0.02(11.70) - 0.34\,(8.43) = \mathbf{-0.50}$$

$$X_2 = 7.59 - 0.22\,(9.36) - 0.54(11.70) - 0.21\,(8.43) = \mathbf{-2.56}$$

$$X_3 = 11.70 - 0.94\,(9.36) - 0.91(7.59) - 0.5\,(8.43) = \mathbf{-8.22}$$

$$X_4 = 8.43 - 0.86\,(9.36) - 0.28(7.59) - 0.63(11.70) = \mathbf{-9.12}$$

**Discussion on the result of the 2nd Iteration**

In this 2nd iteration, the values of $X_1$ = –0.50; $X_2$= –2.56; $X_3$ = –8.22; and $X_4$= –9.12. These values are used for the third (3rd) iteration.

**Third (3rd) Iteration**

Again, substituting the new values of $X_1$ = –0.50; $X_2$= –2.56; $X_3$ = –8.22; $X_4$= –9.12 in equations (i) to (iv), we have,

$$X_1 = 9.36 - 0.89(-2.56) - 0.02(-8.22) - 0.34\,(-9.12) = \mathbf{14.90}$$

$$X_2 = 7.59 - 0.22\,(-0.50) - 0.54(-8.22) - 0.21\,(-9.12) = \mathbf{14.05}$$

$$X_3 = 11.70 - 0.94\,(-0.50) - 0.91(-2.56) - 0.5\,(-9.12) = \mathbf{19.06}$$

$$X_4 = 8.43 - 0.86\,(-0.50) - 0.28(-2.56) - 0.63(-8.22) = \mathbf{14.76}$$

**Discussion on the results of iteration 3**

The result of the 3rd iteration shows that the values of the variables are; $X_1$ = 14.90, $X_2$= 14.05, $X_3$ = 19.06, and $X_4$= 14.76 in the right hand side (RHS) of equations (i) to (iv). These values are used for the 4th iteration.

**Fourth (4th) Iteration**

Substituting the new values of $X_1$ = 14.90, $X_2$= 14.05, $X_3$ = 19.06, and $X_4$= 14.76 in equations (i) to (iv), we obtain,

$$X_1 = 9.36 - 0.89(14.05) - 0.02(19.06) - 0.34\,(14.76) = \mathbf{-8.54}$$
$$X_2 = 7.59 - 0.22\,(14.90) - 0.54(19.06) - 0.21\,(14.76) = \mathbf{-9.08}$$
$$X_3 = 11.70 - 0.94\;(14.90) - 0.91(14.05) - 0.5\;(14.76) = \mathbf{-22.47}$$
$$X_4 = 8.43 - 0.86\;(14.90) - 0.28(14.05) - 0.63(19.06) = \mathbf{-20.33}$$

**Discussion on the results of fourth (4th) iteration**

The result of the fourth (4th) iteration shows that the values of the variables are; $X_1$ = –8.54, $X_2$= –9.08, $X_3$ = –22.47, $X_4$= –20.33. These values are used for the 5th iteration.

**Fifth (5th) Iteration**

Substituting the new values of $X_1$ = –8.54, $X_2$= –9.08, $X_3$ = –22.47, and $X_4$= –20.33 in the RHS of equations (i) to (iv), we obtain the following;

$$X_1 = 9.36 - 0.89(-9.08) - 0.02(-22.47) - 0.34\,(-20.33) = \mathbf{24.80}$$
$$X_2 = 7.59 - 0.22\,(-8.54) - 0.54(-22.47) - 0.21\,(-20.33) = \mathbf{25.87}$$
$$X_3 = 11.70 - 0.94\;(-8.54) - 0.91(-9.08) - 0.5\;(-20.33) = \mathbf{38.15}$$
$$X_4 = 8.43 - 0.86\;(-8.54) - 0.28(-9.08) - 0.63(-22.47) = \mathbf{32.47}$$

**Discussion on the results of fifth (5th) iteration**

The results of the fifth iteration show that the new values of the variables are; $X_1$ = 24.80, $X_2$= 25.87, $X_3$ = 38.15, and $X_4$= 32.47. These values are used for the sixth (6th) iteration.

**Sixth (6th) Iteration**

Substituting the new values of $X_1$ = 24.80, $X_2$= 25.87, $X_3$ = 38.15, and $X_4$= 32.47 in the RHS of equations (i) to (iv), we obtain the following;

$$X_1 = 9.36 - 0.89(25.87) - 0.02(38.15) - 0.34\,(32.47) = -25.47$$
$$X_2 = 7.59 - 0.22\,(24.80) - 0.54(38.15) - 0.21\,(32.47) = -25.29$$
$$X_3 = 11.70 - 0.94\;(24.80) - 0.91(25.87) - 0.5\;(32.47) = -51.39$$
$$X_4 = 8.43 - 0.86\;(24.80) - 0.28(25.87) - 0.63(38.15) = -44.18$$

**Discussion on the results of sixth (6th) iteration**

The result of the sixth (6th) iteration shows that the values of the variables are; $X_1$ = –25.47, $X_2$= –25.29, $X_3$ = –51.39, and $X_4$= –44.18. These values are used for the seventh (7th) iteration.

**Seventh (7th) Iteration**

Substituting the new values of $X_1$ = –25.47, $X_2$= –25.29, $X_3$ = –51.39, and $X_4$= –44.18 in the RHS of equations (i) to (iv), we have the following;

$$X_1 = 9.36 - 0.89(-25.29) - 0.02(-51.39) - 0.34\,(-44.18) = 47.92$$
$$X_2 = 7.59 - 0.22\,(-25.47) - 0.54(-51.39) - 0.21\,(-44.18) = 50.22$$
$$X_3 = 11.70 - 0.94\;(-25.47) - 0.91(-25.29) - 0.5\;(-44.18) = 80.75$$
$$X_4 = 8.43 - 0.86\;(-25.47) - 0.28(-25.29) - 0.63(-51.39) = 69.79$$

**Discussion on the results of seventh (7th) iteration**

The result of the seventh (7th) iteration shows that the values of the variables are; $X_1$ = 47.92, $X_2$= 50.22, $X_3$ = 80.75, and $X_4$= 69.79. These values are used for the eighth (8th) iteration.

**Eighth (8th) Iteration**

Substituting the new values of $X_1$ = 47.92, $X_2$= 50.22, $X_3$ = 80.75, and $X_4$= 69.79 in the RHS of equations (i) to (iv), we have the following;

$$X_1 = 9.36 - 0.89(50.22) - 0.02(80.75) - 0.34\,(69.79) = -60.68$$
$$X_2 = 7.59 - 0.22\,(47.92) - 0.54(80.75) - 0.21\,(69.79) = -61.21$$

$$X_3 = 11.70 - 0.94\ (47.92) - 0.91(50.22) - 0.5\ (69.79) = -113.94$$

$$X_4 = 8.43 - 0.86\ (47.92) - 0.28(50.22) - 0.63(80.75) = -97.72$$

**Discussion on the results of Eighth (8th) iteration**

The result of the eighth (8th) iteration shows that the values of the variables are; $X_1$ = –60.68, $X_2$= –61.21, $X_3$ = –113.94, and $X_4$= –97.72. These values are used for the 9th iteration.

**Ninth (9th) Iteration**

Substituting the new values of $X_1$ = –60.68, $X_2$= –61.21, $X_3$ = –113.94, and $X_4$= –97.72 in the RHS of equations (i) to (iv), we have the following;

$$X_1 = 9.36 - 0.89(-61.21) - 0.02(-113.94) - 0.34\ (-97.72) = 99.34$$

$$X_2 = 7.59 - 0.22\ (-60.68) - 0.54(-113.94) - 0.21\ (-97.72) = 102.99$$

$$X_3 = 11.70 - 0.94\ (-60.68) - 0.91(-61.21) - 0.5\ (-97.72) = 173.30$$

$$X_4 = 8.43 - 0.86\ (-60.68) - 0.28(-61.21) - 0.63(-113.94) = 149.54$$

**Discussion on the results of ninth (9th) iteration**

The result of the ninth iteration shows that the new values of the variables are; $X_1$ = 99.34, $X_2$= 102.99, $X_3$ = 173.30, and $X_4$= 149.54. These values are used for the tenth (10th) iteration.

**Tenth (10th) Iteration**

Substituting the new values of $X_1$ = 99.34, $X_2$= 102.99, $X_3$ = 173.30, and $X_4$= 149.54 in the RHS of equations (i) to (iv), we have the following;

$$X_1 = 9.36 - 0.89(102.99) - 0.02(173.30) - 0.34\ (149.54) = -136.61$$

$$X_2 = 7.59 - 0.22\ (99.34) - 0.54(173.30) - 0.21\ (149.54) = -139.25$$

$$X_3 = 11.70 - 0.94\ (99.34) - 0.91(102.99) - 0.5\ (149.54) = -250.17$$

$$X_4 = 8.43 - 0.86\ (99.34) - 0.28(102.99) - 0.63(173.30) = -215.01$$

**Discussion on the results of Tenth (10th) iteration**

The result of the tenth (10th) iteration shows that the values of the variables are; $X_1$ = –136.61, $X_2$= –139.25, $X_3$ = –250.17, and $X_4$= –215.01. These values are used for the eleventh (11th) iteration.

**Eleventh (11th) Iteration**

Substituting the new values of $X_1$ = –136.61, $X_2$= –139.25, $X_3$ = –250.17, and $X_4$= –215.01 in the RHS of equations (i) to (iv), we have the following result;

$$X_1 = 9.36 - 0.89(-139.25) - 0.02(-250.17) - 0.34\ (-215.01) = 211.40$$

$$X_2 = 7.59 - 0.22\ (-136.61) - 0.54(-250.17) - 0.21\ (-215.01) = 217.89$$

$$X_3 = 11.70 - 0.94\ (-136.61) - 0.91(-139.25) - 0.5\ (-215.01) = 374.34$$

$$X_4 = 8.43 - 0.86\ (-136.61) - 0.28(-139.25) - 0.63(-250.17) = 322.51$$

**Discussion on the results of Eleventh (11th) iteration**

The result of the eleventh (11th) iteration shows that the values of the variables are; $X_1$ = 211.40, $X_2$= 217.89, $X_3$ = 374.34, and $X_4$= 322.51. These values are used for the twelfth (12th) iteration.

**Twelfth (12th) Iteration**

Substituting the new values of $X_1$ = 211.40, $X_2$= 217.89, $X_3$ = 374.34, and $X_4$= 322.51 in the RHS of equations (i) to (iv), we have the following result;

$$X_1 = 9.36 - 0.89(217.89) - 0.02(374.34) - 0.34\ (322.51) = -301.70$$

$$X_2 = 7.59 - 0.22\ (211.40) - 0.54(374.34) - 0.21\ (322.51) = -308.79$$

$$X_3 = 11.70 - 0.94\ (211.40) - 0.91(217.89) - 0.5\ (322.51) = -546.55$$

$$X_4 = 8.43 - 0.86\ (211.40) - 0.28(217.89) - 0.63(374.34) = -470.22$$

**Discussion on the results of Twelfth (12th) iteration**

The result of the twelfth (12th) iteration shows that the values of the variables are; $X_1$ = –301.70, $X_2$= –308.79, $X_3$ = –546.55, and $X_4$= –470.22. These values are used for the thirteenth (13th) iteration.

**Thirteenth (13th) iteration**

Substituting the new values of $X_1$ = –301.70, $X_2$= –308.79, $X_3$ = –546.55, and $X_4$= –470.22 in the RHS of equations (i) to (iv), we have the following result;

$$X_1 = 9.36 - 0.89(-308.79) - 0.02(-546.55) - 0.34\ (-470.22) = 454.99$$

$$X_2 = 7.59 - 0.22\ (-301.70) - 0.54(-546.55) - 0.21\ (-470.22) = 467.85$$

$$X_3 = 11.70 - 0.94\ (-301.70) - 0.91(-308.79) - 0.5\ (-470.22) = 811.41$$

$$X_4 = 8.43 - 0.86\ (-301.70) - 0.28(-308.79) - 0.63(-546.55) = 740.92$$

**Discussion on the results of Thirteenth (13th) iteration**

The result of the Thirteenth (13th) iteration shows that the values of the variables are; $X_1$ = 454.99, $X_2$= 467.85, $X_3$ = 811.46, and $X_4$= 740.92. These values are used for the fourteenth (14th) iteration.

**Fourteenth (14th) iteration**

Substituting the new values of $X_1$ = 454.99, $X_2$= 467.85, $X_3$ = 811.46, and $X_4$= 740.92 in the RHS of equations (i) to (iv), we have the following result;

$$X_1 = 9.36 - 0.89(467.85) - 0.02(811.41) - 0.34\ (740.92) = -675.17$$

$$X_2 = 7.59 - 0.22\ (454.99) - 0.54(811.41) - 0.21\ (740.92) = -686.26$$

$$X_3 = 11.70 - 0.94\ (454.99) - 0.91(467.85) - 0.5\ (740.92) = -1212.19$$

$$X_4 = 8.43 - 0.86\ (454.99) - 0.28(467.85) - 0.63(811.41) = -1025.05$$

**Discussion on the results of Fourteenth (14th) iteration**

The result of the fourteenth (14th) iteration shows that the values of the variables are; $X_1$ = –675.17, $X_2$= –686.26, $X_3$ = –1212.19, and $X_4$= –1025.05. These values are used for the fifteenth (15th) iteration.

**Fifteenth (15th) iteration**

Substituting the new values of $X_1$ = –675.17, $X_2$= –686.26, $X_3$ = –1212.19, and $X_4$= –1025.05 in the RHS of equations (i) to (iv), we have the following result;

$$X_1 = 9.36 - 0.89(-686.26) - 0.02(-1212.19) - 0.34\ (-1025.05) = 992.89$$

$$X_2 = 7.59 - 0.22\ (-675.17) - 0.54(-1212.19) - 0.21\ (-1025.05) = 1025.97$$

$$X_3 = 11.70 - 0.94\ (-675.17) - 0.91(-686.26) - 0.5\ (-1025.05) = 1783.38$$

$$X_4 = 8.43 - 0.86\ (-675.17) - 0.28(-686.26) - 0.63(-1212.19) = 1544.91$$

**Discussion on the results of Fifteenth (15th) iteration**

The result of the Fifteenth (15th) iteration shows that the values of the variables are; $X_1$ = 992.89, $X_2$= 1025.97, $X_3$ = 1783.38, and $X_4$= 1544.91. These values are used for the sixteenth (16th) iteration.

**Sixteenth (16th) iteration**

Substituting the new values of $X_1$ = 992.89, $X_2$= 1025.97, $X_3$ = 1783.38, and $X_4$= 1544.91in the RHS of equations (i) to (iv), we have the following result;

$$X_1 = 9.36 - 0.89(1025.97) - 0.02(1783.38) - 0.34\ (1544.91) = -1464.69$$

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$$X_2 = 7.59 - 0.22\,(992.89) - 0.54(1783.38) - 0.21\,(1544.91) = -1498.30$$

$$X_3 = 11.70 - 0.94\ (992.89) - 0.91(1025.97) - 0.5\ (1544.91) = -2627.70$$

$$X_4 = 8.43 - 0.86\ (992.89) - 0.28(1025.97) - 0.63(1783.38) = -2256.25$$

**Discussion of results in Sixteenth (16th) iteration**

The result of the Sixteenth (16th) iteration shows that the values of the variables are; $X_1 = -1464.69$, $X_2 = -1498.30$, $X_3 = -2627.70$, and $X_4 = -2256.26$. These values will be used for the seventeenth (17th) iteration. It is worthy to note that values cannot converge on negative values hence, the need for the next iteration.

**Seventeenth (17th) iteration**

Substituting the new values of $X_1 = -1464.69$, $X_2 = -1498.30$, $X_3 = -2627.70$, and $X_4 = -2256.26$ in the RHS of equations (i) to (iv), we have the following result;

$$X_1 = 9.36 - 0.89(-1498.30) - 0.02(-2627.70) - 0.34\,(-2256.26) = 2162.03$$

$$X_2 = 7.59 - 0.22\,(-1464.69) - 0.54(-2627.70) - 0.21\,(-2256.26) = 2222.09$$

$$X_3 = 11.70 - 0.94\ (-1464.69) - 0.91(-1498.30) - 0.5\ (-2256.26) = 3880.09$$

$$X_4 = 8.43 - 0.86\ (-1464.69) - 0.28(-1498.30) - 0.63(-2627.70) = 3343.04$$

**Discussion on the results of Seventeenth (17th) iteration**

The result of the Seventeenth (17th) iteration shows that the values of the variables are; $X_1 = 2162.03$, $X_2 = 2222.09$, $X_3 = 3880.09$, and $X_4 = 3343.04$. The iteration shows that it has reached a convergence point where the respective values of $X_1$, $X_2$, $X_3$, $X_4$have attained the optimal solution. Therefore, the optimal solutions for the variables are actual values which are; $X_1 = 2162$, $X_2 = 2222$, $X_3 = 3880$, and $X_4 = 3343$.

**Table 3: Summary of Results of Jacobi's Iteration Model**

| S/N | Iterations | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $X_4=9.36-0.89X_2-0.02X_3-0.34X_4$ | 0 | 9.36 | −0.50 | 14.90 | −8.54 | 24.80 | −25.47 | 47.92 | −60.68 | 99.34 | −136.61 |
| 2 | $X_2=7.59-0.22X_1-0.34X_3-0.21X_4$ | 0 | 7.59 | −2.56 | 14.05 | −9.08 | 25.87 | −25.29 | 50.22 | −61.21 | 102.99 | −139.25 |
| 3 | $X_3=11.70-0.94X_1-0.91X_2-0.50X_4$ | 0 | 11.70 | −8.22 | 19.06 | −22.47 | 38.15 | −51.39 | 80.75 | −113.94 | 173.30 | −250.17 |
| 4 | $X_4=8.43-0.86X_1-0.28X_2-0.63X_3$ | 0 | 8.43 | −9.12 | 14.76 | −20.33 | 32.47 | −44.18 | 69.79 | −97.72 | 149.54 | −215.01 |

These values of $X_1$, $X_2$, $X_3$, and X4are now substituted in the objective function.

Maximize $Z = 4308X_1 + 142X_2 + 1613\,X_3 + 1551X_4$

Therefore, the optimal solution is

$Z = 4308\times 2162 + 142\times 2222 + 1613\times 3880 + 1551\times 3343$.

**= ₦ 21,072,853 per ton per day**,

**= ₦ 21.07 million per ton per day**,

In a week, the waste generated will yield a revenue of ₦ 21,072,853.00 × 7 days = **₦147,509,971.00 = ₦147.51 million per ton per week**

Then, the Annual cost will be

₦147,509,971.00 × 52 weeks = **₦7, 670,518,492 per ton per annum**

**₦7.67 billion per ton per annum**

However, **The Total Revenue generated** are as follows:

**For a day:**

Total revenue = ₦ 21,072,853.00 × 2400 tons =**₦ 50,574,847,200.00**

=**₦ 50.575 billion daily**

**For a week:**

Total revenue = ₦ 147,509,971.00 × 2400 tons = **₦ 354,023,930,400.00**

=**₦ 354.024 billion weekly**

**In a year:**

Total revenue = ₦7, 670,518,492 × 2400 tons = ₦18,409,244,380,000.00
= **₦ 18.409 trillion annually**.

## V. CONCLUSION AND RECOMMENDATIONS

The result from the Jacobi's Iteration optimization model has revealed that with waste generation capacity of 2400 metric tons per day, Enugu urban can generate revenue of ₦21, 072, 853. 00 per ton per day. This will amount to ₦147, 509, 791.00 per ton per week or ₦7, 670, 518, 492.00per ton per annum. The total revenue that will be generated per 2400 tons per day will be₦ 50,574, 847, 200.00 i.e. ₦ 50.575 billion daily with 2056 projected population. This amounts to ₦ 354, 023, 930, 400.00 i.e. ₦ 354.024 billion weekly and ₦ 18, 409, 244, 380, 000.00 i.e. ₦ 18.409 trillion per annum. This will reduce drastically the quantity of waste generated in Enugu urban because it will be on demand and a scarce commodity.

Waste recycling will be a profitable venture because it will help to grow the economy of Enugu urban. It will help to create employment among the youths and increase the standard of living of the people.

Source reduction backed by effective legislation will encourage companies to use materials that are less hazardous for packaging their products thereby reducing waste and encourage recycling of packages for manufactured products.

It is in this regards that this study here suggest the following recommendations;

i. There is need to provide solid waste management in the yearly budget with a separate head for the purpose of adequate revenue allocation, implementation and monitoring.
ii. In order to enhance environmental education program and public participation as it affects solid waste management, it should be provided not only through the radio, television and print media but also through grassroots enlightenment campaigns via the chiefs, community leaders.
iii. To champion the course of effective solid waste management, the involvement, participation and cooperation of local communities and the government is of utmost importance.
iv. There is dire need to encourage Public, Private, Partnership (PPP) in Solid Waste Planning and Environmental Management.
v. There should be serious commitment on the part of Enugu State Government to sponsor more research projects into the reduction of solid waste at source, collection and efficient disposal.
vi. Solid waste management should be integrated in the curricula of primary, secondary and tertiary schools as a way of general enlightenment.
vii. The government should enact a comprehensive environmental legislation that will encourage source reduction of wastes, environmental sanitation, and other associated issues that will fortify proper implementation. Competent penal institution should be established for reprimand or to convict the offenders.
viii. There is need for access road to the entire street around the metropolis to be constructed and put in good condition to aid accessibility of the waste collection trucks to all the streets and compound in the area.
ix. The procurement of more Compaction vehicles will ease the problem of collection to disposal location.
x. The optimization programme modeled in this research work has shown that recyclingof solid wastes, especially if embarked on industrially, is a very good and big profitmaking business.
xi. Since this research has created adequate awareness on the economic value of solidwaste, the demand for it will be high, therefore eliminating unhealthy dumping of solidwaste to appreciable value of 42 percent.

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