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An Analytical View of Arcs in Intuitionistic Fuzzy Dombi Graphs

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ABSTRACT

Arcs in a graph plays a significant role in modelling a real time network problems. The arcs of an Intuitionistic Fuzzy Dombi Graph (IFDG), like in the case of crisp, are crucial to its construction. A two-dimensional perspective of an arcs in IFDG is went through and are categorized into α-strong , β-strong and δ-weak based on its strength to examine the formation of complete IFDG and constant IFDG. Types such as sturdy arc, feeble arc and δ^* weak arc are also examined along with their properties.

I. INTRODUCTION

Rosenfeld [5] presented fuzzy graphs ten years after Zadeh published his seminal study ”Fuzzy Sets.” Many modern scientific and technological domains, including information theory, neural networks, expert systems, cluster analysis, medical diagnostics, control theory, etc., are finding extensive use for fuzzy graph theory. In order to establish some of the features of these fuzzy counterparts, Rosenfeld produced the fuzzy equivalents of numerous fundamental graph-theoretic concepts, including bridges, pathways, cycles, trees, and connectivity [10]. Since every arc in graph theory is strong in the sense of [4], arc analysis is not very essential. However, determining the nature of arcs in fuzzy graphs is crucial, and there isn’t any analysis of arcs like this in the literature [8], with the exception of the classification of arcs as strong and non-strong. Mathew and Sunitha studied about types of arcs in fuzzy graphs [6].

Atanassov introduced the concept of intuitionistic fuzzy relations and defined intuitionistic fuzzy graphs (IFGs) using six types of cartesian products. IFG is a model that extends theory of fuzzy graphs with a new component, namely, degree of non-membership in the definition of fuzzy graph. K. Atanassov and A. Shannon first proposed the idea of an intuitionistic fuzzy graph in 1994, and since then, much research has been done in this field (see, for example, [1, 2], etc.). Parvathi et al., in [5], defined arcs, bridges and cutnodes of an intuitionistic fuzzy graph. Zadeh’s conventional T-operators min and max have been used in every application of fuzzy logic in decision-making process and fuzzy graph theory. Ashraf [3] introduced the concept of Dombi fuzzy graph using Dombi operator. Sunitha, Kanagavalli and N. Sangeetha [4] introduced the notion of intuitionistic fuzzy dombi graph and its properties. Nivethana et.al (2015)discussed about Complement of intuitionistic fuzzy graphs and arc analysis in intuitionistic fuzzy graphs [7, 9].

In IFDGs, it is necessary to identify the nature of edges and no analysis on edges is available. The classification of edges highlights the significance of each edge, which minimize the cost and improve the effectiveness of the system network problems. In this paper, a two dimensional approach for degree of membership and non-membership is carried out for the arcs in intuitionistic fuzzy dombi graph. Any arc of an intuitionistic fuzzy dombi graph is categorized under sturdy arc or feeble arc or δ^* weak arc and their properties are also studied. The firm path and the infirm path are introduced which can be used as an ideal application of intuitionistic fuzzy dombi graph in decision making problem. We define intuitionistic fuzzy bridges and cutnodes for IFDG and their characteristic are studied. We also analyse the connectivity of arcs in IFDG’s. The article is categorized as given. In section 2, preliminaries required for the study are given. Section 3 presents several types of arcs in IFDGs. A few properties and theorem on strong path are discussed. IF bridge in IFDG is defined in section 4. The paper is concluded in section 5.

II. PRELIMINARIES

Definition 2.1

Minmax intuitionistic fuzzy graph (IFG) is of the form G = (V, E), where
(i) $V = \{v_1, v_2, \ldots v_n\}$ such that $\mu_i: V \to [0, 1]$ and $\nu_i: V \to [0, 1]$ denote the degrees of membership and non-membership of the element $v_i \in V$ respectively and $0 \le \mu_i + \nu_i \le 1$, for every $v_i \in V$ (i = 1, 2, . . . , n). (ii) $E \subset V \times V$ where $\mu_{ij} : V \times V \to [0, 1]$ and $\nu_{ij} : V \times V \to [0, 1]$ are such that

$$\mu_{ij} \le \min[\mu_i, \mu_j]$$
$$\nu_{ij} \le \max[\nu_i, \nu_j]$$

and $0 \le \mu_{ij} + \nu_{ij} \le 1$ for every $e_{ij} \in E$.

Here the triple (v_i, μ_i, ν_i) denotes the degrees of membership and non-membership of the vertex v_i. The triple $(e_{ij}, \mu_{ij}, \nu_{ij})$ denotes the degrees of membership and non-membership of the edge $e_{ij} = (v_i, v_j)$ on

$$V \times V .$$

For each IFG G, the degree of hesitancy (hesitation degree) of the vertex $v_i \in V$ is $\Pi_i = 1 - \mu_i - \nu_i$ and the degree of hesitancy of an edge $e_{ij} \in E$ is $\Pi_{ij} = 1 - \mu_{ij} - \nu_{ij}$.

Definition 2.2

An intuitionistic fuzzy dombi graph(IFDG) is of the form $G = (V, E, \eta, \zeta)$ where $\eta = (\eta^\mu, \eta^\nu)$, $\zeta = (\zeta^\mu, \zeta^\nu)$ and

- $\eta^\mu : V \to [0, 1]$ and $\eta^\nu: V \to [0, 1]$ denotes the degrees of membership and non membership of the element $\chi \in V$ respectively and $0 \le \eta^\mu x + \eta^\nu(x) \le 1$ for every $x \in V$.
- $E \subseteq V \times V$, where $\zeta^\mu : V \times V \to [0, 1]$ and $\zeta^\nu: V \times V \to [0, 1]$ such that

$$\zeta(xy) \le \left(\frac{\eta^\mu(x)\eta^\mu(y)}{\eta^\mu(x)+\eta^\mu(y)-\eta^\mu(x)\eta^\mu(y)}, \frac{\eta^\nu(x)+\eta^\nu(y)-2\eta^\nu(x)\eta^\nu(y)}{1-\eta^\nu(x)\eta^\nu(y)}\right)$$

where η denotes the intuitionistic fuzzy dombi vector set of G and ζ denotes the intuitionistic fuzzy dombi edge set of G.

Definition 2.3

Minmax Intuitionistic Fuzzy Dombi Graph(IFDG) is of the form $G = (V, E, \eta, \zeta)$ where • V is a finite non-empty set of vertices such that $\eta^\mu : V \to [0, 1]$ and $\eta^\nu: V \to [0, 1]$ denotes the degree of membership and non-membership of the element $x \in V$ respectively and $0 \le \eta^\mu x + \eta^\nu(x) \le 1$ for every $x \in V$.

• $E \subseteq V \times V$ where $\zeta^\mu : V \times V \to [0, 1]$ and $\zeta^\nu: V \times V \to [0, 1]$ are such that

$$\zeta^\mu(xy) \le min[\frac{\eta^\mu(x)\eta^\mu(y)}{\eta^\mu(x)+\eta^\mu(y)-\eta^\mu(x)\eta^\mu(y)}]$$
$$\zeta^\nu(xy) \le max[\frac{\eta^\nu(x)+\eta^\nu(y)-2\eta^\nu(x)\eta^\nu(y)}{1-\eta^\nu(x)\eta^\nu(y)}]$$

and $0 \le \zeta^\mu x + \zeta^\nu(x) \le 1$ for every $(x, y) \in E$.

Note: Edge (x, y) is represented by (xy) whose membership function in $\zeta^\mu(xy)$ and non-membership in $\zeta^\nu(xy)$.

Definition 2.4

An arc (x, y) in IFDG is strong if both

$$\zeta^\mu(xy) = min[\frac{\eta^\mu(x)\eta^\mu(y)}{\eta^\mu(x)+\eta^\mu(y)-\eta^\mu(x)\eta^\mu(y)}]$$
$$\zeta^\nu(xy) = max[\frac{\eta^\nu(x)+\eta^\nu(y)-2\eta^\nu(x)\eta^\mu(y)}{1-\eta^\nu(x)\eta^\nu(y)}]$$

Definition 2.5

A path $v_i - v_j$ is an IFDG is the sequence of distinct vertices $v_1, v_2, \cdots , v_n$ for all (i, j = 1, 2, · · · , n) such that either one of the following conditions is satisfied.

- $\zeta^\mu(v_i, v_j) > 0$ and $\zeta^\nu(v_i, v_j) = 0$ for some i and j
- $\zeta^\mu(v_iv_j) = 0$ and $\zeta^\nu(v_iv_j) > 0$ for some i and j
- $\zeta^\mu(v_iv_j) > 0$ and $\zeta^\nu(v_iv_j) > 0$ for some i and j

Definition 2.6

μ-strength of a path, $S_\mu(xy)$ is defined as the least value of degree of membership of all the arcs in the path.

Definition 2.7

ν-strength of a path, $S_\nu(xy)$ is defined as the maximum value of degree of non-membership of all the arcs in the path.

Definition 2.8

μ-strength of connectedness between two nodes x and y is defined as the maximum of μ-strength of all the paths between x and y excluding the arc joining x and y.

Definition 2.9

ν-strength of connectedness between two nodes x and y is defined as the minimum of ν-strength of all the paths between x and y excluding the arc joining x and y.

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Definition 2.10
Total μ-strength of connectedness denoted by $T_{CONN\mu}(xy)$ is defined as the maximum of μ-strength of all the paths between x and y including the arc joining x and y.

Definition 2.11
Total ν-strength of connectedness denoted by $T_{CONN\nu}(xy)$ is defined as the minimum of ν-strength of all the paths between x and y including the arc joining x and y.

Note:
1. In a IFDG, (xy) ∨ $\zeta^{\mu}(xy) = T_{CONN\mu}(xy)$ and (xy) ∧ $\zeta^{\nu}(xy) = T_{CONN\nu}(xy)$
2. If $T_{CONN\mu}(xy)$ = (xy) and $T_{CONN\nu}(xy)$ = (xy) then either $\zeta^{\mu}(xy)$ < (xy) and $\zeta^{\nu}(xy)$ > (xy) or there is no arc joining the nodes x and y.
of connectedness of the above graph are tabulated in Table . The arcs are classified into α-strong, β-

III. TYPES OF ARCS IN IFDGS AND ITS PROPERTIES

Definition 3.1
An arc (x, y) in G with membership $\zeta^{\mu}(xy)$ and non-membership $\zeta^{\nu}(xy)$ is called • α − μ strong arc if $\zeta^{\mu}(xy)$ > (xy)
• α − ν strong arc if $\zeta^{\nu}(xy)$ < (xy)
• β − μ strong arc if $\zeta^{\mu}(xy)$ = (xy)
• β − ν strong arc if $\zeta^{\nu}(xy)$ = (xy)
• δ − μ weak arc if $\zeta^{\mu}(xy)$ < (xy)
• δ − ν weak arc if $\zeta^{\nu}(xy)$ > (xy)

Example 3.1:
Consider the following IF DG in Figure 1.
By repeated computation the values of strength of the paths, strength of connectedness and total strength 4

strong and δ*-weak based on strength.

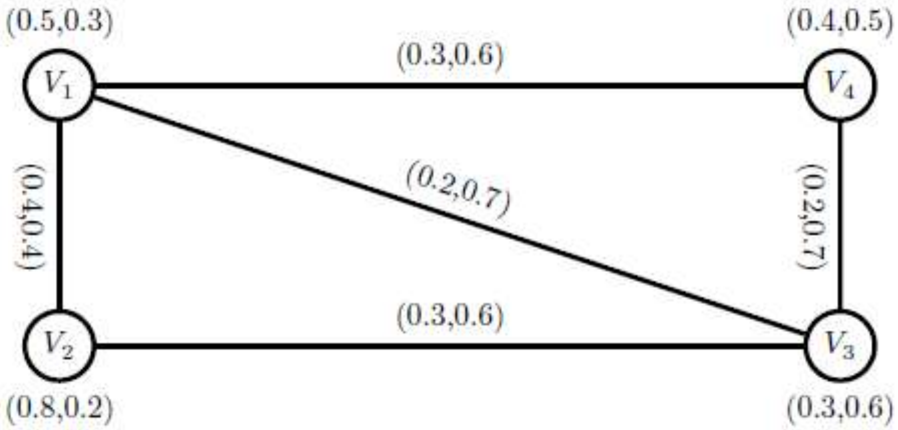

Figure 1: Intuitionistic fuzzy dombi graph

End Nodes	Paths	$S_\mu(xy)$	$S_\nu(xy)$	μ strength	ν strength	$T_{CONN\mu}(xy)$	$T_{CONN\nu}(xy)$
v_1v_2	$v_1 - v_2$	0.4	0.4	0.2	0.7	0.4	0.4
	$v_1 - v_3 - v_2$	0.2	0.7				
	$v_1 - v_4 - v_3 - v_2$	0.2	0.7				
v_1v_3	$v_1 - v_3$	0.2	0.7	0.3	0.6	0.3	0.6
	$v_1 - v_2 - v_3$	0.3	0.6				
	$v_1 - v_4 - v_3$	0.2	0.7				
v_1v_4	$v_1 - v_4$	0.3	0.6	0.2	0.7	3*0.3	0.6

	$v_1 - v_3 - v_4$	0.2	0.7				
	$v_1 - v_2 - v_3 - v_4$	0.2	0.7				
v_2v_3	$v_2 - v_3$	0.3	0.6	0.2	0.7	3*0.3	0.6
	$v_2 - v_1 - v_3$	0.2	0.7				
	$v_2 - v_1 - v_4 - v_3$	0.2	0.7				
v_2v_4	$v_2 - v_1 - v_4$	0.3	0.6	0.3	3*0.6	0.3	0.6
	$v_2 - v_3 - v_4$	0.2	0.7				
	$v_2 - v_1 - v_3 - v_4$ $v_2 - v_3 - v_1 - v_4$	0.2 0.2	0.7 0.7				
v_3v_4	$v_3 - v_4$	0.2	0.7	0.3	0.6	0.3	0.6
	$v_3 - v_1 - v_4$	0.2	0.7				
	$v_3 - v_2 - v_1 - v_4$	0.3	0.6				

Definition 3.2
An arc (v_i, v_j) is called a μ-strong arc if it is $\alpha - \mu$ or $\beta - \mu$ strong.

Definition 3.3
An arc (v_i, v_j) is called a α-strong arc if it is $\alpha - \nu$ or $\beta - \nu$ strong.

Definition 3.4
An arc (v_i, v_j) is called a sturdy arc if it is both μ-strong and ν-strong.

Definition 3.5
An arc (v_i, v_j) is called a feeble arc if it is either $\delta - \mu$ weak or $\delta - \nu$ weak.

Definition 3.6
An arc (v_i, v_j) is called as a δ^* weak arc if it is $\delta - \mu$ weak or $\delta - \nu$ weak.

Definition 3.7
A path P is firm path if it contains only the sturdy arc.

Definition 3.8
A path P is infirm path if it contains only the δ^* weak arc.

Definition 3.9
A path $P : x \to y$ is called a strong path if its strength equals $T_{CONN\mu}(xy)$ and $T_{CONN\nu}(xy)$. (i.e) $S_\mu(xy) = T_{CONN\mu}(xy)$ and $S_\nu(xy) = T_{CONN\nu}(xy)$.
From Example 3.1 it can be observed that the path $v_2 - v_1 - v_4$ is a strong path. Also, the arcs $(v_1, v_2)(v_1, v_4)(v_2, v_3)$ are sturdy arcs and arcs (v_1, v_3) and (v_3, v_4) are feeble arcs. Proposition 3.1
An arc (x, y) is sturdy iff $\zeta^\mu(xy) = T_{CONN\mu}(xy)$ and $\zeta^\nu(xy) = T_{CONN\nu}(xy)$.

Proof
Let the arc (x, y) be the sturdy arc, then $\zeta^\mu(xy) \geq (xy)$ and $\zeta^\nu(xy) \leq (xy)$. From proposition 2.2, we have $T_{CONN\mu}(xy) = \zeta^\mu(xy)$ and $T_{CONN\nu}(xy) = \zeta^\nu(xy)$.

Conversely, if $\zeta^\mu(xy) = T_{CONN\mu}(xy)$ and $\zeta^\nu(xy) = T_{CONN\nu}(xy)$ then by proposition 2.2, we have $(xy) \leq \zeta^\mu(xy)$. Hence, the arc (x, y) must be either $\alpha - \mu$ strong or $\beta - \nu$ strong arc. Hence, (x, y) is a μ-strong arc. Similarly, it can be shown that the arc (x, y) is a ν-strong arc. Hence, (x, y) must be sturdy arc.

Proposition 3.2
A strong path has only sturdy arcs.
Proof
Let $P : v_1, v_2, \cdots, v_n$ be the strong path. Consider

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the arc (v_1, v_2) in the path P. Let $\zeta^\mu(v_1v_2)$ has the least membership value in the path P. Hence, $S_\mu(v_1v_n) = \zeta^\mu(v_1v_2)$. Hence, for all arcs in the path P,

$$\zeta^\mu(v_iv_j) \geq S_\mu(v_1v_n) = \zeta^\mu(v_1v_2) \quad (1)$$

for all i, j = 1, 2, · · · , n
Since P is strong $S_\mu(v_1v_n) = T_{CONN\mu}(v_1v_n)$. By proposition 2.2, we have $S_\mu(v_1v_n) \geq (v_1v_n)$. Hence from(1) $\zeta_\mu(v_iv_j) \geq (v_1v_n)$ for i, j = 1, 2, · · · , n. Therefore, from definition 3.1 every arc in the path must be $\alpha - \mu$ strong or $\beta - \mu$ strong. Similarly, repeating the argument for ν values, it can be shown that all the arcs in the path P must be $\alpha - \nu$ strong or $\alpha - \nu$ strong. Hence, P has only μ-strong and ν-strong arcs. (i.e) it has only sturdy arcs.

Corollary 3.1
A strong path is a firm path but not conversely.
Proof
From proposition 3.2 it is obviously true.
Conversely, from Example 3.1, the path $v_2 - v_3$ is a firm path but it is not strong path.

Proposition 3.3
An arc to the end vertex is a sturdy arc iff its non-membership value is zero.
Proof
Let v_n be the end vertex and so the only arc containing v_n be (v_m, v_n). Hence, the strength of con nectedness $(v_mv_n) = 0$ and $(v_mv_n) = 0$, since v_n is end vertex, there is no other path from v_m to v_n. ∴ $\zeta^\mu(v_mv_n) \geq (v_mv_n) = 0$ and $\zeta^\nu(v_mv_n) \leq (v_mv_n) = 0$

∴ The arc is $\alpha - \mu$ strong arc.
Case (i): If $\zeta^\mu(v_mv_n) = 0$, then $\zeta^\mu(v_mv_n) = (v_mv_n)$ (i.e) the arc v_mv_n is $\beta - \nu$ strong arc. Hence the arc v_mv_n is a sturdy arc.
Case (ii): If $\zeta^\mu(v_mv_n) = 0$, then $\zeta^\nu(v_mv_n) > (v_mv_n)$. By definition, the arc v_mv_n is $\delta - \nu$ weak arc. Then the arc is a feeble arc.

Proposition 3.4
If there is more than one strong path between a pair of vertices v_i and v_j, then all the paths are of equal strength.

Proof
Let P_1 and P_2 be two strong paths between vertices v_i and v_j. If not, let the strength of P_1 < the strength of P_2. Since both P_1 and P_2 are strong, for $P_1 \rightarrow S_\mu(v_iv_j) = T_{CONN\mu}(v_iv_j)$ and for $P_2 \rightarrow S_\mu(v_iv_j) = T_{CONN\mu}(v_iv_j)$.
Comparing P_1 and P_2, $T_{CONN\mu}(v_iv_j) < T_{CONN\mu}(v_iv_j)$ is meaningless. Hence, we arrive at a contra diction.
∴ The strength of paths P_1 and P_2 are equal.

IV. IF BRIDGE IN IFDGS

Definition 4.1
An arc (v_i, v_j) is said to be a bridge in G if the deletion of the arc (v_iv_j) reduces the total μ-strength of connectedness and increases the total ν-strength of connectedness between some pair of vertices at the same time.
(i.e) $CONN_{\mu(G)-(vi,v_j)}(v_i, v_j) \leq CONN_{\mu(G)(vi,v_j)}$
$CONN_{\nu(G)-(vi,v_j)}(v_i, v_j) > CONN_{\nu(G)(vi,v_j)}$

Example 4.1
In figure 2.1, the arc (v_1, v_2) is an IF bridge. Since the removal of the arc (v_1, v_2) reduces the 7 $T_{CONN\mu}(v_1v_2)$ and increases $T_{CONN\nu}(v_1v_2)$ at the same time between the nodes v_1 and v_2. Also the arc (v_1v_3) is not a bridge once removal of (v_1v_3) does not reduce $T_{CONN\mu}(v_1v_3)$ between nodes v_1 and v_3.

Definition 4.2
A node(vertex) is an intuitionistic fuzzy cutnode of an IFDG if the removal of it reduces the total μ-strength of connectedness and increases the total ν-strength of connectedness at the same time between some other pair of nodes.

Note
In Example 3.1, the node v_1 is the IF cutnode. Since, if v_1 is removed $T_{CONN\mu}(v_2v_4) = 0.2 < 0.3$ and $T_{CONN\nu}(v_2v_4) = 0.7 < 0.6$.

Proposition 4.1
In an IF DG, the arc (a, b) is an IF-bridge, then $\zeta^\mu(ab) = T_{CONN\mu}(ab)$ and $\zeta^\nu(ab) = T_{CONN\nu}(ab)$.

Proof
By definition of IF bridge, $ab < \zeta^\mu(ab)$
∴ $(ab) \wedge \zeta^\mu(ab) = \zeta^\mu(ab)$
∴ From Proposition 2.2, $T_{CONN\mu}(ab) = \zeta^\mu(ab)$ and similarly we can prove that $T_{CONN\nu}(ab) = \zeta^\nu(ab)$.

Proposition 4.2
Every bridge is a sturdy arc, but a sturdy arc need not be a bridge.

V. CONCLUSION

In this paper, arcs are classified into α-strong, β-strong and δ*-weak based on strength. Some fasci nating properties of firm paths , infirm paths and strong paths are also discussed. The author intend to work on the application of intuitionistic fuzzy dombi graph in decision making scenarios which can also be extended to artificial intelligence, networking areas.

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