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Time-Frequency Analysis of Seismic Signals Based on WOA-Optimized VMD

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ABSTRACT: This paper proposes a novel approach for seismic signal processing using Whale Optimization Algorithm-based Variational Mode Decomposition (WOA-VMD). The method adaptively optimizes key VMD parameters, including the number of modes and the penalty factor, to achieve effective mode decomposition and noise suppression. Experimental results on both synthetic and real seismic data demonstrate that WOA-VMD outperforms conventional time-frequency analysis methods in terms of mode separation quality and time-frequency resolution. The proposed method provides a reliable tool for high-resolution seismic interpretation and offers significant potential for applications in hydrocarbon exploration.

KEYWORDS: Seismic Signal Processing, Variational Mode Decomposition (VMD), Whale Optimization Algorithm (WOA), Time-Frequency Analysis.

I. INTRODUCTION

Seismic data serves as a vital source of information in the exploration of oil and natural gas. Enhancing the resolution of seismic data represents a crucial task in seismic signal processing [1,2]. Such data are primarily acquired through seismic exploration, wherein a series of techniques are employed to generate seismic waves that provide essential information for characterizing subsurface geological structures and locating mineral resources such as oil and gas [3].Time-frequency analysis methods are widely used in seismic data processing and interpretation due to their ability to effectively reveal how the frequency content of seismic signals evolves over time. Conventional time-frequency analysis techniques—including the Short-Time Fourier Transform (STFT), Wavelet Transform (WT), and Wigner-Ville Distribution (WVD)—have been successfully applied in seismic studies, yet each is constrained by inherent limitations in joint time-frequency resolution [4,5].

More recently, time-frequency analysis methods based on Empirical Mode Decomposition (EMD) have emerged as a promising alternative, offering higher time-frequency concentration and adaptability [6]. These methods have shown practical value in seismic interpretation by highlighting fine structural features, enhancing hydrocarbon indicators, and facilitating noise suppression. Although EMD can recursively decompose a multi-component seismic trace into multiple Intrinsic Mode Functions (IMFs), it still suffers from notable limitations, such as sensitivity to noise and sampling, as well as a lack of rigorous mathematical foundation [7,8].

In this study, the Whale Optimization Algorithm-based Variational Mode Decomposition (WOA-VMD) is employed to process seismic signals. This approach effectively overcomes the limitations of conventional EMD by incorporating a variational framework and adaptive parameter optimization. The WOA algorithm automatically optimizes two critical VMD parameters—the number of modes K and the penalty factor α—ensuring a data-driven and robust decomposition process. As a result, the WOA-VMD method achieves superior time-frequency localization, improves mode separability, and significantly enhances the signal-to-noise ratio, thereby providing more reliable support for high-resolution seismic interpretation.

II. THEORY

A. Whale Optimization Algorithm

The WOA is a metaheuristic optimization technique inspired by the foraging behavior of humpback whales in nature [9]. It simulates the collective hunting strategies of whales, including encircling, pursuing, and attacking prey, to perform global optimization search.The algorithm begins by randomly generating an initial population ofNwhales within the search space. During the evolutionary process, each whale updates its position based on either the current best whale or a randomly selected individual from the population. Finally, depending on a randomly generated valuep, each whale performs either a spiral updating maneuver or a shrinking encircling action. This process repeats iteratively until a satisfactory solution is found.

The position update under the shrinking encircling mechanism is described by the following equation:

$$D = \left| C \cdot X^*(t) - X(t) \right|$$
$$X(t+1) = X^*(t) - A \cdot D \quad (1)$$

where t denotes the current iteration number, X^*represents the position vector of the prey, andA, Care coefficient vectors. The vectorsAandCare defined as follows:

$$A = 2a \cdot r_1 - a$$
$$C = 2 \cdot r_2 \quad (2)$$

where$r_1$and$r_2$are random vectors uniformly distributed in [0,1], andais the convergence factor, which decreases linearly from 2 to 0 over the course of iterations as follows:

$$a = 2 - \frac{2t}{t_{\max}} \quad (3)$$

where t_{max} denotes the maximum number of iterations.

In the spiral-updating position method, which simulates the spiral movement of whales as they approach their prey, the position update is formulated as follows:

$$X(t+1) = \begin{cases} X^*(t) - A \cdot D & p < 0.5 \\ D' \cdot e^{bl} \cdot \cos(2\pi l) + X^*(t) & p \ge 0.5 \end{cases}$$
(4)

where D denotes the distance between thei-th whale and the prey,bis a constant that defines the shape of the logarithmic spiral, andlis a random number uniformly distributed in the interval [−1,1].To simulate this behavior during the optimization process, the algorithm selects either the shrinking encircling mechanism or the spiral updating position with equal probability of 0.5.

B. Variational Mode Decomposition

VMD is an adaptive, non-recursive, and quasi-orthogonal signal decomposition method that decomposes a signal into a finite number of sub-signals, referred to as IMFs[10]. Each IMF is a band-limited mode with a specific sparsity property, oscillating around its own center frequency. To constrain the bandwidth of each IMF within a targeted frequency range, the decomposition is achieved by addressing the following constrained variational problem:

The procedure involves three key steps for eachIMF:
(1) Compute the unilateral frequency spectrum via the Hilbert transform;
(2) Shift the frequency band of each mode to baseband using an exponential tuning term;
(3) Estimate the bandwidth of each IMF by squaring the L_2-norm of the gradient.
Thus, the resulting constrained variational problem can be formulated as:

$$\min_{\{u_k\},\{\omega_k\}} \left\{ \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-j\omega_k t} \right\|_2^2 \right\}$$
$$\text{s.t.} \quad \sum_k u_k = f$$
(5)

where u_k denotes thek-th mode of thesignal, $\{u_k\}$ represents the set of all modes $\{u_1, u_2, \cdots, u_k\}$, ω_k stands for the centre frequency of thek-th mode,$\{W_k\}$indicates the set of all centerfrequencies,fis the input signal to be decomposed, andδ(t)refers to the Dirac delta function.

To transform the constrained variational problem into an unconstrained one, a quadratic penalty term and the augmented Lagrangian function are introduced to optimize the constrained solution. The corresponding formulation is given as follows:

$$L(u_k, \omega_k, \lambda) = \alpha \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * u_k(t) \right] e^{-j\omega_k t} \right\|_2^2$$
$$+ \left\| f - \sum_k u_k \right\|_2^2 + \left\langle \lambda(t), f - \sum_k u_k \right\rangle.$$
(6)

where αdenotes the balancing parameter of the data-fidelity constraint.

Subsequently, the Alternating Direction Method of Multipliers (ADMM) is employed to iteratively update each IMF component u_k and its corresponding center frequency ω_k . The update equations are formulated as follows:

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$$\hat{u}_k^{n+1}(\omega) = \frac{\hat{f}(\omega) - \sum_{i \neq k} \hat{u}_i(\omega) + \frac{\hat{\lambda}(\omega)}{2}}{1 + 2\alpha(\omega - \omega_k)^2}. \quad (7)$$

The center frequency ω_k^{n+1} is updated as follows:

$$\omega_k^{n+1} = \frac{\int_0^{\infty} \omega |\hat{u}_k(\omega)|^2 \, d\omega}{\int_0^{\infty} |\hat{u}_k(\omega)|^2 d\omega}. \quad (8)$$

where ω_k represents the center of gravity of the power spectrum of the k-th IMF.

Therefore, each IMF can be regarded as a monocomponent signal, for which the instantaneous amplitude and instantaneous frequency can be calculated using the following expressions:

$$\begin{cases} A(t) = \sqrt{R(u(t))^2 + I(u(t))^2} \\ F(t) = \frac{1}{2\pi} \frac{R(u(t)) I(u(t))' - R(u(t))' I(u(t))}{R(u(t))^2 + I(u(t))^2} \end{cases}$$

(9)

where $A(t)$ and $F(t)$ denote the instantaneous amplitude and instantaneous frequency, respectively; $R(\cdot)$ and $I(\cdot)$ represent the real and imaginary parts of the analytic signal, and R' and I' are their respective derivatives with respect to time t.

The performance of the VMD algorithm depends on several input parameters: the balancing parameter for the data-fidelity constraint α, the time-step for the dual ascent τ, the number of modes to be extracted K, the convergence tolerance Tol, and the initialization of the center frequencies $Init$. Not all parameters are elaborated in this paper, as some are primarily related to the optimization solver (ADMM); a comprehensive description of these can be found in Dragomiretskiy & Zosso (2014).

Among these, α and K are the two most critical parameters in VMD, while the others have relatively minor influence on the decomposition results. An excessively large value of K may lead to mode mixing, whereas an insufficient K can adversely affect the focus of the time-frequency representation, thereby hindering the accurate capture of each mode's center frequency. Meanwhile, the parameter α controls the data fidelity constraint. An inappropriate choice of α may compromise the preservation of modal components.

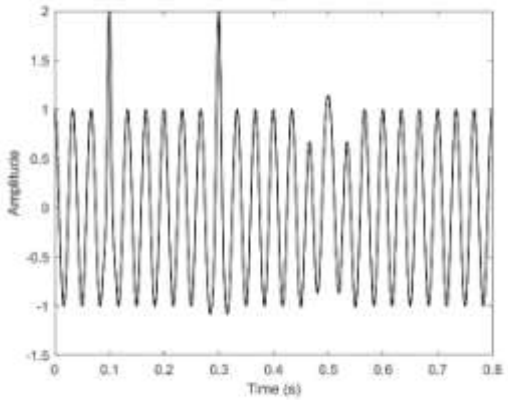

Figure 1 Synthetic signal

III. EXPERIMENT

In this section, we first compare different decomposition levels of VMD using synthetic data to investigate the impact of an inappropriate selection of the decomposition level on the results. Subsequently, real seismic data are employed to determine the optimal parameters, followed by a time-frequency analysis.

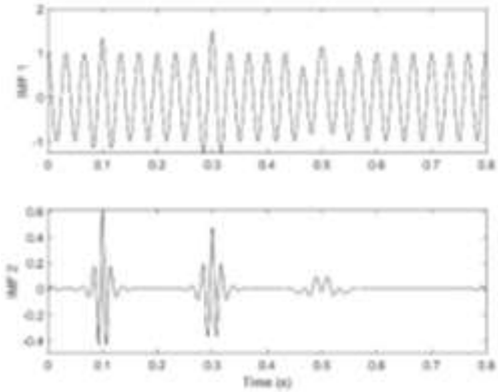

Figure 2 VMD result of the synthetic data. The decomposition level is set to 2.

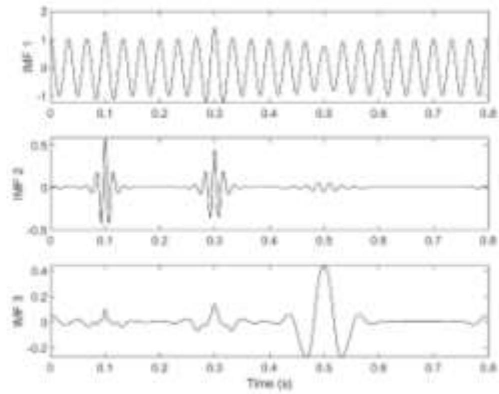

Figure 3 VMD result of the synthetic data. The decomposition level is set to 3.

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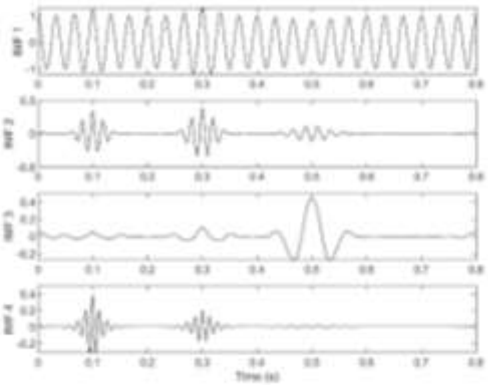

Figure 4 VMD result of the synthetic data. The decomposition level is set to 4.

The synthetic signal shown in Figure 1 consists of an initial 30 Hz cosine wave, superimposed with an 80 Hz Ricker wavelet at 0.1 s, a 60 Hz Ricker wavelet at 0.3 s, and two additional 30 Hz Ricker wavelets located at 0.49 s and 0.51 s.

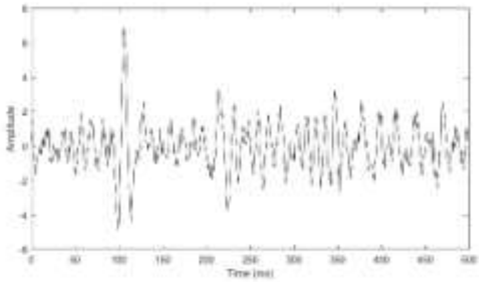

Figure 5 Time domain waveform of a single seismic trace.

Figures 2 to 4 present the decomposition results of the synthetic signal using different decomposition levels of the VMD method. Among these, the most appropriate decomposition level is 3, as illustrated in Figure 3. In this case, the background cosine wave is primarily captured in IMF1, while IMF2 mainly contains the high-frequency Ricker wavelets, specifically reflecting the 80 Hz wavelet at 0.1 s and the 60 Hz wavelet at 0.3 s. IMF3 accurately represents the two low-frequency 30 Hz Ricker wavelets located at 0.49 s and 0.51 s. In contrast, the result with a decomposition level of 2 (Figure 2) fails to effectively separate the high- and low-frequency Ricker wavelets, leading to mode mixing and aliasing of signal components, which adversely affects subsequent processing. On the other hand, the over-decomposition observed in Figure 4 (using a higher decomposition level) not only demands greater computational resources and time but also introduces information redundancy. Specifically, IMF2 and IMF4 contain repetitive and overlapping information, which undermines the efficiency and clarity of signal analysis and interpretation.

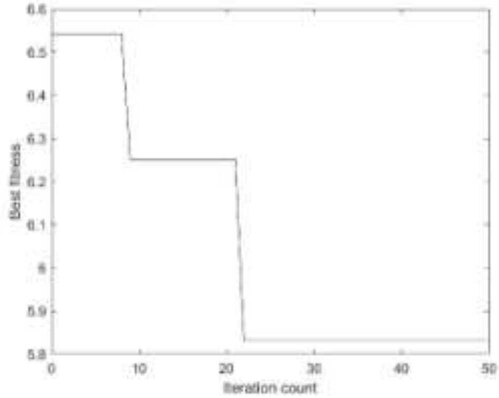

Figure 6 WOA optimization results for VMD parameters.

A real seismic trace is extracted, as depicted in Figure 5. The WOA is employed to optimize two critical parameters of the VMD method: the number of modesKand the penalty factorα. The optimal parameter values obtained are [K,α]=[3,1500]. The iterative optimization process is illustrated in Figure 6, and the corresponding decomposition results are shown in Figure 7. It can be observed that the random noise is predominantly captured in IMF3, enabling effective noise removal while preserving valuable stratigraphic information.

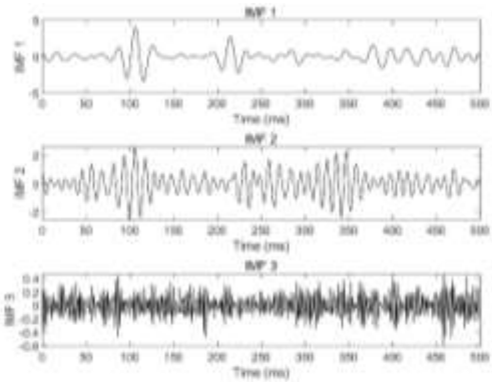

Figure 7 Decomposition results of VMD for a single seismic trace.

Subsequently, the time-frequency representation of the seismic trace obtained through WOA-VMD is compared with that derived from the STFT, as shown in Figure 8. The results demonstrate that the time-frequency distribution provided by WOA-VMD allows for more precise localization of spectral anomalies, thereby facilitating further interpretation.

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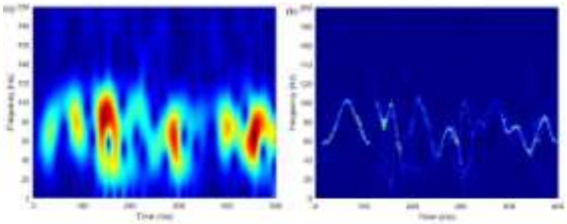

Figure 8 Time-frequency representation of a single-channel seismic signal(a. STFT b.WOA-VMD)

IV. INTRODUCTION

This study demonstrates that the WOA-optimized VMD method effectively enhances seismic signal decomposition by adaptively determining the optimal mode number and penalty factor. The approach outperforms conventional techniques in mode separation and noise suppression, providing superior time-frequency localization for improved interpretation.

The proposed method offers a more robust and adaptive alternative to traditional EMD and other time-frequency analyses, showing significant potential for high-resolution seismic processing. Future work will focus on extending its application to 3D seismic data and optimizing computational efficiency for practical implementation.

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