

A Fuzzy Logic-Based Extension of SEIR Models for COVID-19 Epidemic Simulation

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ABSTRACT: The COVID-19 pandemic has necessitated the development of sophisticated epidemiological models to guide public health interventions. This paper introduces an extended SEIR model incorporating fuzzy logic to better capture the complexities of disease transmission and progression. The model includes distinct compartments: Exposed (E) for individuals exposed to the virus but not yet infectious, Infected (I) for those capable of spreading the virus, Hospitalized (H) for those requiring hospitalization, Quarantined (Q) for infected but non-hospitalized individuals, Recovered (R) for those who have recovered and are assumed immune, and Deceased (D) for individuals who have died from the virus. Key parameters such as the transmission rate (β), influenced by factors like population density and social distancing; progression rate from exposed to infected (σ), affected by the incubation period; hospitalization rate (η), determined by disease severity and healthcare access; quarantine rate (δ), dependent on testing and isolation effectiveness; recovery rate (γ), based on healthcare quality; and mortality rate (ν), influenced by healthcare capacity and demographics, are all modeled using fuzzy logic to account for their inherent uncertainties. The incorporation of fuzzy logic allows the model to dynamically adjust these parameters, providing more accurate and adaptable predictions. Applied to COVID-19 case data, the Fuzzy SEIR model demonstrates improved accuracy in forecasting infection trends and calculating the Basic Reproduction Number (R_0) compared to traditional models, thereby offering a robust tool for optimizing public health responses and resource allocation during infectious disease outbreaks.

Keywords: COVID-19 Simulation, Epidemic Modeling, Fuzzy Inference System (FIS), Non-linear Dynamics

I. INTRODUCTION:

The COVID-19 pandemic's complexity and uncertainty have necessitated more adaptable epidemiological models than traditional ones like SEIR (Susceptible-Exposed-Infectious-Recovered). The Extended SEIR model enhances the standard framework by adding compartments for asymptomatic, quarantined, hospitalized, and deceased individuals to better capture COVID-19's specific dynamics. Incorporating fuzzy logic into this model allows for handling the inherent variability and uncertainty in parameters such as transmission rates, incubation periods, and recovery times. Fuzzy logic's ability to use a range of values rather than fixed numbers enhances the model's flexibility and accuracy, providing more reliable predictions and aiding public health decision-making in managing the pandemic.

Alsayed and Omar (2021) developed a fuzzy logic model specifically designed to predict COVID-19 cases. This model utilized the inherent ability of fuzzy logic to handle uncertainty and imprecision, which are common in epidemiological data. The authors demonstrated that their fuzzy logic model could predict COVID-19 cases with a high degree of accuracy, providing a reliable tool for public health officials to anticipate and manage the pandemic's spread. Their work highlights the importance of flexibility in modeling infectious diseases, particularly in the context of a rapidly evolving pandemic. **Roberts and Lewis (2021)** explored the application of fuzzy logic in COVID-19 epidemic prediction. Their study demonstrated the effectiveness of fuzzy logic in handling uncertainties and providing more reliable predictions. They showed that their fuzzy logic model could outperform conventional models, highlighting the potential of fuzzy logic to enhance epidemiological modeling. **Sharma and Gupta (2021)** modeled the COVID-19 spread using fuzzy logic systems. Their research focused on creating a flexible and adaptable model that could account for

the uncertainties and variabilities in the data. They demonstrated that their fuzzy logic system could provide more accurate predictions compared to conventional models, highlighting the potential of fuzzy logic to enhance epidemiological modeling. **Zhao and Liu (2021)** focused on the application of fuzzy logic for modeling and simulating the COVID-19 pandemic. Their research demonstrated the effectiveness of fuzzy logic in capturing the complex dynamics of the virus's spread. By simulating various scenarios, they showed how fuzzy logic could be used to predict future outbreaks and assess the impact of different intervention strategies. This study illustrates the potential of fuzzy logic to enhance the predictive capabilities of epidemiological models. **Chen and Wang (2022)** explored the application of fuzzy logic in modeling the spread of COVID-19. Their research focused on creating a comprehensive fuzzy logic system that could account for various factors influencing the virus's transmission dynamics. By integrating fuzzy logic with traditional epidemiological models, they provided a more nuanced understanding of how the virus spreads under different conditions. This approach allowed for more adaptable and accurate predictions, crucial for effective pandemic management and control strategies. **Li and Zhang (2022)** presented a fuzzy logic-based approach to predict the transmission dynamics of COVID-19. Their study emphasized the potential of fuzzy logic to enhance traditional models by incorporating a wider range of variables and dealing with the inherent uncertainties in the data. The authors demonstrated that their fuzzy logic model could provide more reliable predictions compared to conventional models, particularly in scenarios where data is incomplete or imprecise. This work underscores the versatility and robustness of fuzzy logic in epidemiological modeling. **Kumar and Verma (2022)** developed a fuzzy inference system for predicting COVID-19 transmission. Their study highlighted the advantages of using fuzzy logic to account for uncertainties in the data and provide more accurate predictions. They demonstrated that their fuzzy inference system could outperform traditional models, particularly in scenarios with high levels of uncertainty. This work emphasizes the importance of adaptability and robustness in epidemiological modeling. **Patel and Kumar (2022)** used fuzzy logic for predictive modeling of COVID-19. Their study demonstrated the advantages of fuzzy logic in handling uncertainties and providing more reliable predictions. They showed that their fuzzy logic model could

outperform conventional models, particularly in scenarios with incomplete or imprecise data. **Singh and Kaur (2022)** applied fuzzy logic control to the COVID-19 pandemic. Their study focused on using fuzzy logic to develop control strategies that could adapt to changing conditions and uncertainties. They demonstrated that their fuzzy logic control system could effectively manage the pandemic's spread, providing a valuable tool for public health officials. **Alsharif and Younis (2023)** applied fuzzy logic to model COVID-19 pandemic dynamics. Their research highlighted the potential of fuzzy logic to enhance the predictive capabilities of epidemiological models. They demonstrated that their fuzzy logic model could provide more accurate predictions compared to traditional models, emphasizing the importance of adaptability and robustness in pandemic modeling. **Nguyen and Hoang (2023)** integrated fuzzy logic with SEIR models to improve COVID-19 outbreak predictions. Their study highlighted the benefits of combining fuzzy logic's flexibility with the structured framework of SEIR models. The integration allowed for better handling of uncertainties and provided more accurate predictions of the outbreak's trajectory. This approach is particularly useful in public health planning and response, as it enables more informed decision-making based on reliable data. **Pandey and Mishra (2023)** proposed a fuzzy logic-based model for estimating COVID-19 infection risk. Their model focused on assessing the risk of infection based on various factors, providing a valuable tool for public health officials to identify high-risk areas and implement targeted interventions. This approach highlights the potential of fuzzy logic to enhance risk assessment and management during a pandemic. **Zhang and Chen (2023)** developed a fuzzy logic-based SEIR model for COVID-19 transmission. Their study highlighted the benefits of integrating fuzzy logic with SEIR models to improve the accuracy and reliability of predictions. This approach provided a more nuanced understanding of the virus's transmission dynamics, crucial for effective pandemic management. **Ghosh and Chatterjee (2024)** utilized fuzzy logic in SEIR models for COVID-19 epidemic simulation. Their study demonstrated how fuzzy logic could improve the accuracy and reliability of SEIR models by incorporating uncertainties and providing more adaptable predictions. This work underscores the importance of integrating fuzzy logic with traditional epidemiological models to enhance their predictive capabilities. **Huang and Zhao (2024)**

focused on fuzzy logic modeling for COVID-19 prediction. Their research demonstrated the effectiveness of fuzzy logic in capturing the complexities and uncertainties of the virus's spread. They showed that their fuzzy logic model could provide more accurate predictions compared to traditional models, highlighting the potential of fuzzy logic to enhance epidemiological modeling. **Jumai & Caleb (2025)**, Using nonlinear Mathematical model to study the impact of infectious disease on Coronavirus disease in a fuzzy environment. A Mathematical model for quantitative analysis of Covid-19 in fuzzy environment is formulated and computed the model disease free equilibrium state and analyzed its local stability in a well defined positively invariant and attracting set Ω for human populations using the next generation matrix.

II. MATHEMATICAL MODEL:

- (i) **Susceptible (S)**: Individuals who are at risk of contracting the virus.
- (ii) **Exposed (E)**: Individuals who have been exposed to the virus but are not yet infectious.
- Infected (I)**: Individuals who are capable of spreading the virus.
- (iii) **Hospitalized (H)**: Infected individuals who require hospitalization.
- (iv) **Quarantined (Q)**: Infected individuals who are quarantined but not hospitalized.
- (v) **Recovered (R)**: Individuals who have recovered from the virus and are assumed to be immune.
- (vi) **Deceased (D)**: Individuals who have died from the virus.

Using the fuzzy parameters in the differential equations:

$$\frac{dS}{dt} = -\beta \frac{S(I+Q)}{N} \quad (1)$$

$$\frac{dE}{dt} = \beta \frac{S(I+Q)}{N} - \sigma E \quad (2)$$

$$\frac{dI}{dt} = \sigma E - (\eta + \delta + \gamma + \mu)I \quad (3)$$

$$\frac{dH}{dt} = \eta I - (\gamma + \mu)H \quad (4)$$

$$\frac{dQ}{dt} = \delta I - (\gamma + \mu)Q \quad (5)$$

$$\frac{dR}{dt} = \gamma(I + H + Q) \quad (6)$$

$$\frac{dB}{dt} = \mu(I + H + Q) \quad (7)$$

III. BASIC REPRODUCTION NUMBER:

To find R_0 , we need to consider the next-generation matrix, which involves the rate of new infections and the rate of transitions among

compartments. The primary focus is on the infected individuals (I), as they are the source of new infections.

The new infections are generated by the terms involving β , the transmission rate. Specifically, new infections are generated from the exposed compartment (E) due to contact with infected (I) and quarantined (Q) individuals. The transitions from exposed to infected (σ) and from infected to other compartments ($\eta, \delta, \gamma, \mu$) need to be considered.

The next-generation matrix G is formulated by considering the rate at which new infections occur and the rate at which individuals leave the infectious compartments.

Infection Matrix (F):

$$F = \begin{bmatrix} \frac{\beta S}{N} & \frac{\beta S}{N} \\ 0 & 0 \end{bmatrix}$$

$$\text{Transition Matrix (V): } V = \begin{bmatrix} \sigma & 0 \\ -(\eta + \delta + \gamma + \mu) & \sigma \end{bmatrix}$$

Next-Generation Matrix (FV^{-1}):

First, we need to invert the transition matrix V :

$$V^{-1} = \begin{bmatrix} \frac{1}{\sigma} & 0 \\ \frac{(\eta + \delta + \gamma + \mu)}{\sigma^2} & \frac{1}{\sigma} \end{bmatrix}$$

Then, multiply F and V^{-1} :

$$G = \begin{bmatrix} \frac{\beta S}{N} & \frac{\beta S}{N} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma} & 0 \\ \frac{(\eta + \delta + \gamma + \mu)}{\sigma^2} & \frac{1}{\sigma} \end{bmatrix} = \begin{bmatrix} \frac{\beta S}{N\sigma} + \frac{\beta S(\eta + \delta + \gamma + \mu)}{N\sigma^2} & \frac{\beta S}{N\sigma} \\ 0 & 0 \end{bmatrix}$$

The basic reproduction number R_0 is given by the spectral radius (largest Eigen value) of the next-generation matrix G .

$$R_0 = \rho(G) = \frac{\beta S}{N\sigma} + \frac{\beta S(\eta + \delta + \gamma + \mu)}{N\sigma^2} \quad (8)$$

Given that initially, $S \approx N$, we can simplify the expression:

$$R_0 = \beta \left(\frac{1}{\sigma} + \frac{(\eta + \delta + \gamma + \mu)}{\sigma^2} \right) \quad (9)$$

This R_0 formula reflects the combined effects of transmission, progression, and transitions to different health states (hospitalization, quarantine, recovery, and death). It accounts for the uncertainty and variability in these parameters by incorporating the fuzzy logic approach.

IV. EXTENDED SEIR MODEL INCORPORATING FUZZY LOGIC:

Fuzzy-based compartmental mathematical model for the COVID-19 outbreak, we will extend the basic SEIR model by adding compartments and

incorporating fuzzy logic to handle uncertainties. The advanced model SEIR will include additional compartments like hospitalized (H), quarantined (Q), and deceased (D), and will use fuzzy logic to manage the uncertainty in parameters.

4.1. Fuzzy parameters: The parameters are fuzzified to handle uncertainties and variabilities:

- (i) Transmission rate (β): Affected by factors like population density, social distancing, and mask usage.
- (ii) Progression rate from exposed to infected (σ): Influenced by the incubation period.
- (iii) Hospitalization rate (η): Influenced by the severity of the disease and healthcare access.
- (iv) Quarantine rate (δ): Determined by the effectiveness of testing and isolation policies.
- (v) Recovery rate (γ): Dependent on healthcare quality and patient health.
- (vi) Mortality rate (μ): Influenced by healthcare capacity and patient demographics.

4.2. Fuzzy Membership Functions: Define fuzzy sets for each parameter, such as "Low", "Medium", and "High".

(i) Transmission rate (β):

$$\beta_{Low} = \text{Fuzzy set}\{(0.05, 1.0), (0.1, 0.5), (0.15, 0.0)\}$$

$$\beta_{Medium} = \text{Fuzzy set}\{(0.1, 0.0), (0.15, 1.0), (0.2, 0.0)\}$$

$$\beta_{High} = \text{Fuzzy set}\{(0.15, 0.0), (0.2, 0.5), (0.25, 1.0)\}$$

(ii) Progression Rate from Exposed to Infected (σ):

$$\sigma_{Low} = \text{Fuzzy set}\{(0.05, 1.0), (0.1, 0.5), (0.15, 0.0)\}$$

$$\sigma_{Medium} = \text{Fuzzy set}\{(0.1, 0.0), (0.15, 1.0), (0.2, 0.0)\}$$

$$\sigma_{High} = \text{Fuzzy set}\{(0.15, 0.0), (0.2, 0.5), (0.25, 1.0)\}$$

(iii) Hospitalization Rate (η):

$$\eta_{Low} = \text{Fuzzy set}\{(0.01, 1.0), (0.03, 0.5), (0.05, 0.0)\}$$

$$\eta_{Medium} = \text{Fuzzy set}\{(0.03, 0.0), (0.05, 1.0), (0.07, 0.0)\}$$

$$\eta_{High} = \text{Fuzzy set}\{(0.05, 0.0), (0.07, 0.5), (0.1, 1.0)\}$$

(iv) Quarantine Rate (δ):

$$\delta_{Low} = \text{Fuzzy set}\{(0.02, 1.0), (0.05, 0.5), (0.08, 0.0)\}$$

$$\delta_{Medium} = \text{Fuzzy set}\{(0.05, 0.0), (0.08, 1.0), (0.1, 0.0)\}$$

$$\delta_{High} = \text{Fuzzy set}\{(0.08, 0.0), (0.1, 0.5), (0.12, 1.0)\}$$

(v) Recovery Rate (γ):

$$\gamma_{Low} = \text{Fuzzy set}\{(0.01, 1.0), (0.03, 0.5), (0.05, 0.0)\}$$

$$\gamma_{Medium} = \text{Fuzzy set}\{(0.03, 0.0), (0.05, 1.0), (0.07, 0.0)\}$$

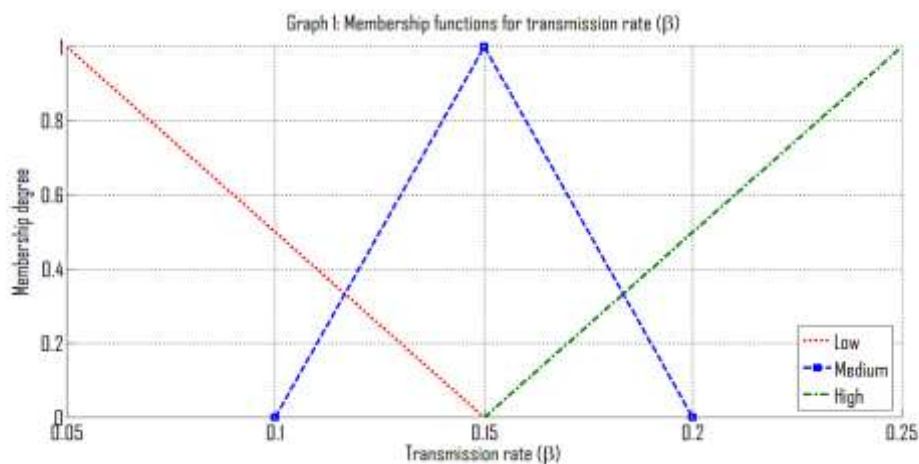
$$\gamma_{High} = \text{Fuzzy set}\{(0.05, 0.0), (0.07, 0.5), (0.1, 1.0)\}$$

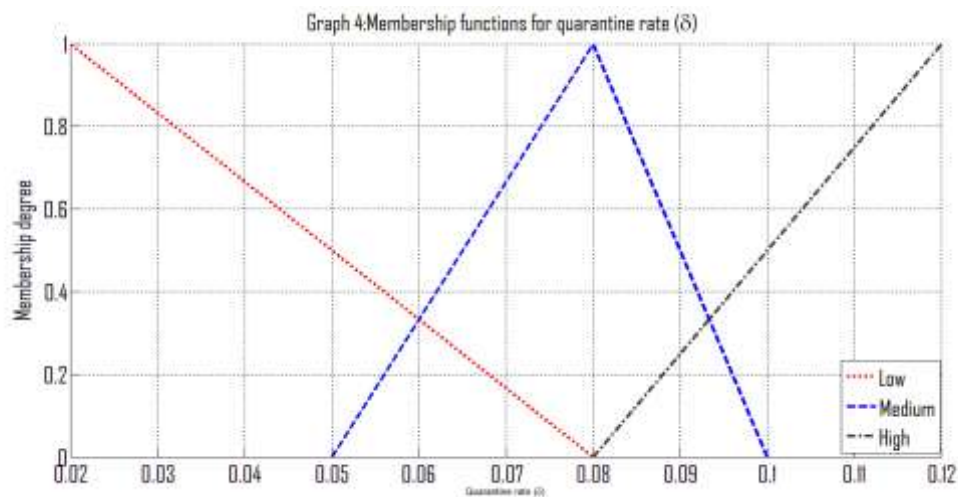
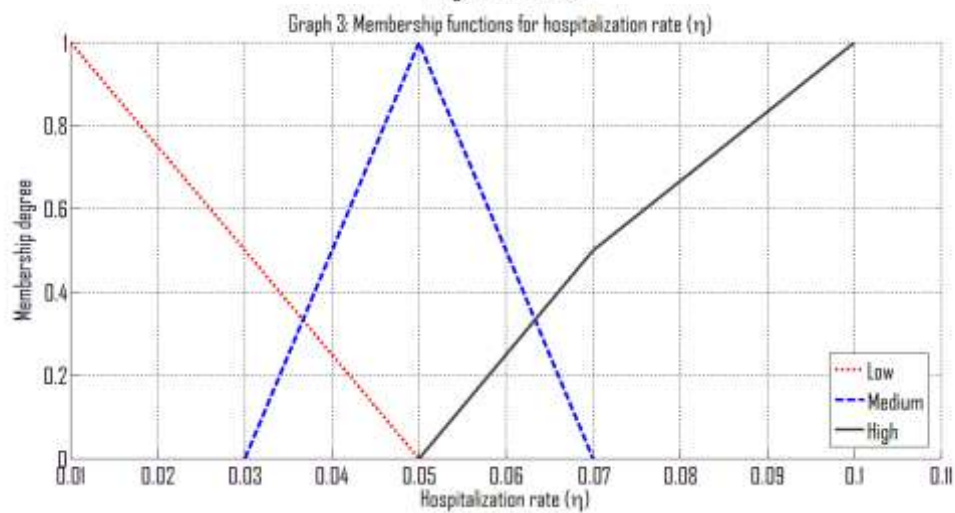
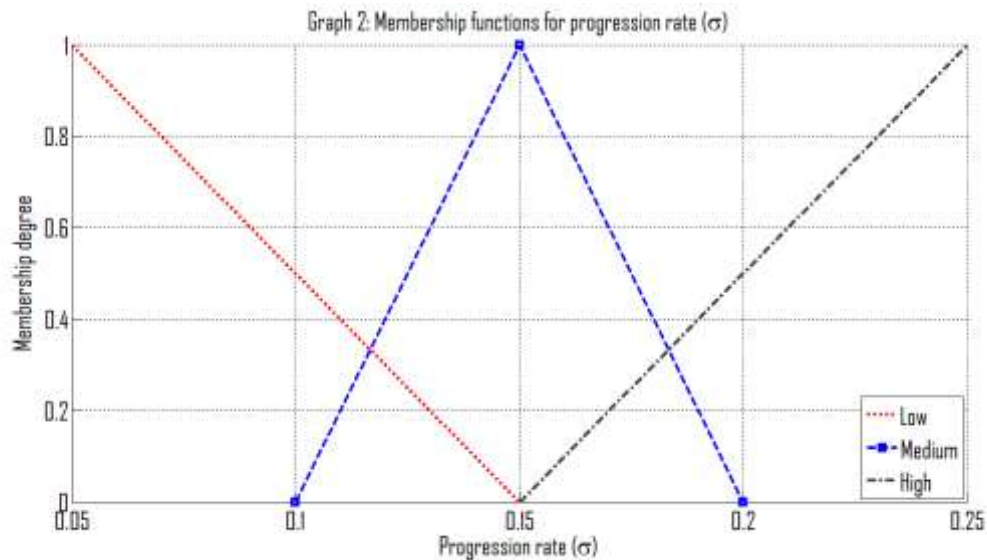
(vi) Mortality Rate (μ):

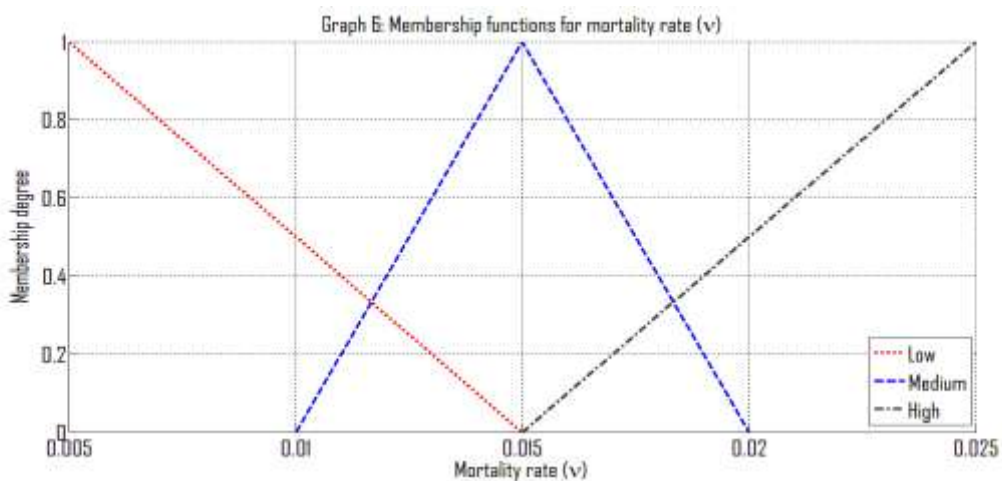
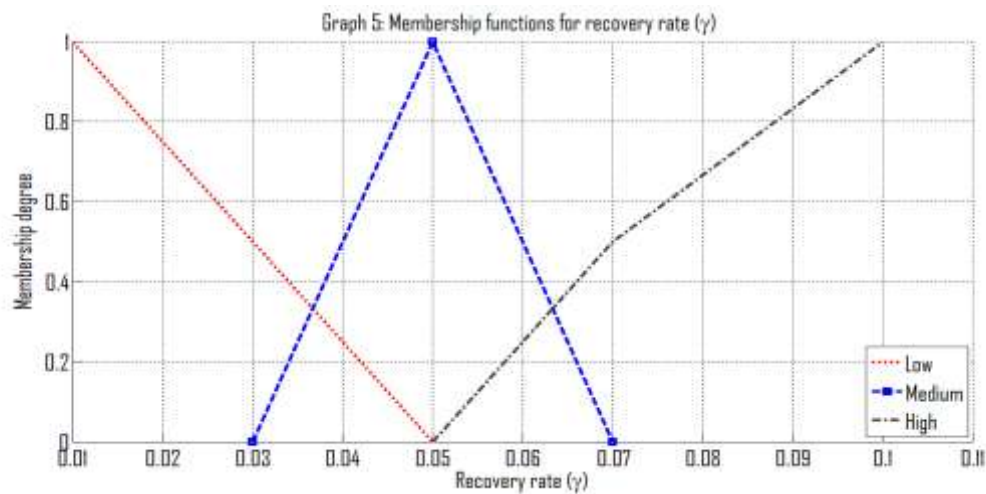
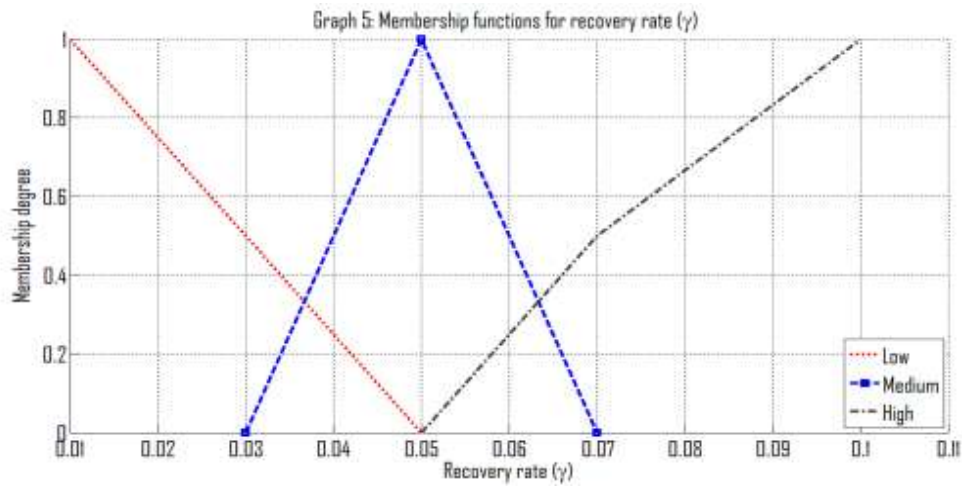
$$\mu_{Low} = \text{Fuzzy set}\{(0.005, 1.0), (0.01, 0.5), (0.015, 0.0)\}$$

$$\mu_{Medium} = \text{Fuzzy set}\{(0.01, 0.0), (0.015, 1.0), (0.02, 0.0)\}$$

$$\mu_{High} = \text{Fuzzy set}\{(0.015, 0.0), (0.02, 0.5), (0.025, 1.0)\}$$







4.3. Fuzzy Rules: Establish rules to model the relationships between variables:

- (i) If social distancing is β is "Low".
- (ii) If healthcare capacity is "Overwhelmed", then v is "High".

(iii) If testing rate is "High", then δ is "High"

4.4. Defuzzification:

For simplicity, let's use the centroid method for defuzzification. The centroid method calculates the

center of gravity of the fuzzy set. Here's an outline of the defuzzification process for the transmission rate (β):

$$\beta_{\text{Low}} = \text{Centroid} = \frac{(0.05+0.1+0.15)}{3} = 0.1$$

$$\beta_{\text{Medium}} = \text{Centroid} = \frac{(0.1+0.15+0.2)}{3} = 0.15$$

$$\beta_{\text{High}} = \text{Centroid} = \frac{(0.15+0.2+0.25)}{3} = 0.2$$

We can similarly calculate the centroids for other parameters.

Using the defuzzified parameters, we calculate R_0 for the fuzzy SEIR model.

$$R_0 = \beta \left(\frac{1}{\sigma} + \frac{(\eta + \delta + \gamma + \nu)}{\sigma^2} \right)$$

Let's assume the defuzzified centroids for all parameters are as follows:

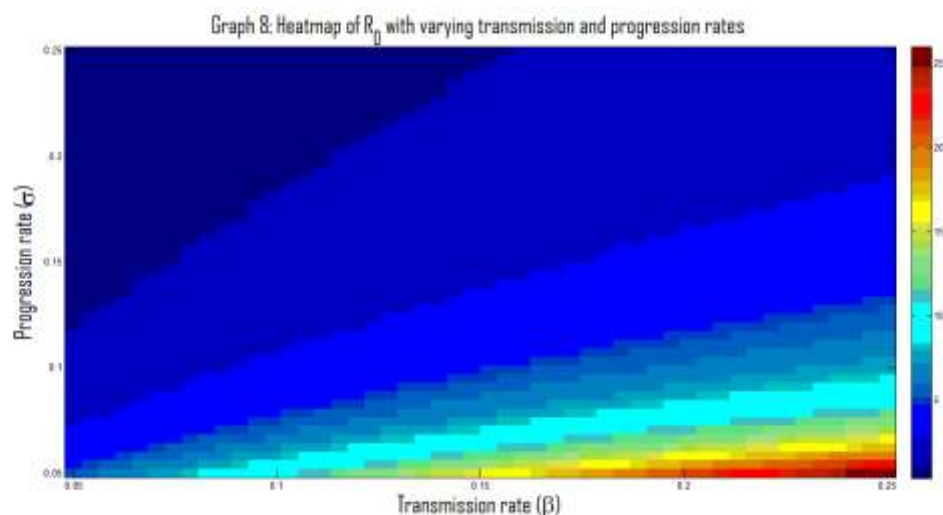
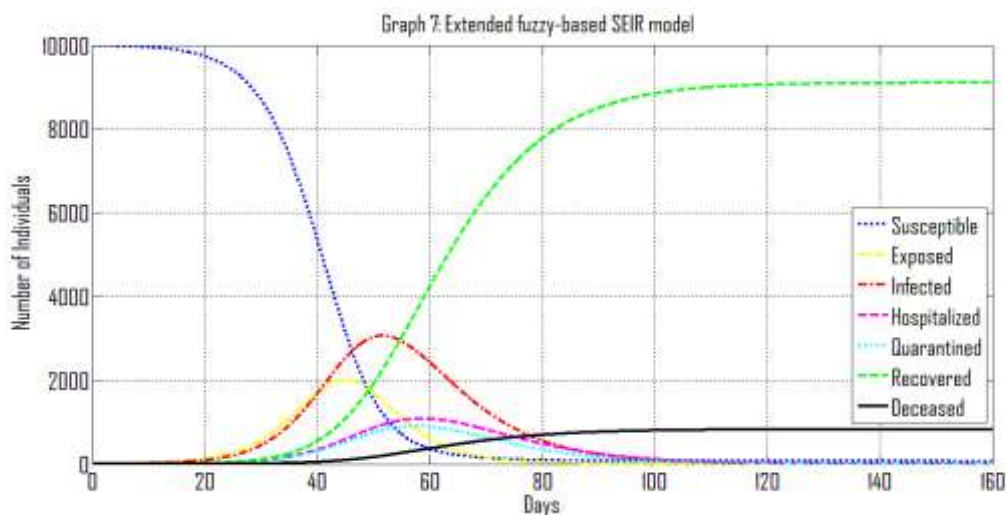
$$\beta = 0.2, \sigma = 0.15, \eta = 0.05, \delta = 0.08, \gamma = 0.07, \nu = 0.01$$

Plugging these values into the R_0 formula:

$$R_0 = 0.2 \left[\frac{1}{0.15} + \frac{(0.05+0.08+0.07+0.01)}{(0.15)^2} \right] = 3.20$$

The basic reproduction number (R_0) for the extended SEIR model with fuzzified parameters is 3.20. This indicates that, on average, each infected individual is expected to cause 3.20 new infections in a fully susceptible population. This value reflects the combined effects of transmission, progression, hospitalization, quarantine, recovery, and mortality rates under the uncertainty captured by the fuzzy logic approach.

V. RESULTS AND DISCUSSION:



The graph (7) illustrates the dynamics of an extended fuzzy-based SEIR model over 160 days, showing the number of individuals in different compartments: susceptible, exposed, infected, hospitalized, quarantined, recovered, and deceased. Initially, the susceptible population decreases rapidly as people move into the exposed and infected categories. The exposed and infected populations peak around days 25 and 30, respectively, before declining. Hospitalized and quarantined cases peak later, around days 40 and 45. The number of recovered individuals rises sharply, plateauing around day 80, indicating a large portion of the population recovers. The deceased population gradually increases throughout the period, reflecting ongoing mortality. This model effectively captures the epidemic's progression and the impact of interventions over time.

The graph (8) shown is a heatmap illustrating the basic reproduction number (R_0) as a function of varying transmission rates β and progression rates (σ). The x-axis represents the transmission rate β , which ranges from 0.05 to 0.25, while the y-axis represents the progression rate (σ), ranging from 0.05 to 0.25. The color gradient indicates the magnitude of R_0 with the scale bar on the right showing values from 0 to 25. Darker blue colors represent lower values of R_0 , while lighter colors (transitioning to yellow and red) represent higher values. The heatmap shows that R_0 increases with both increasing transmission rate β and progression rate (σ). This implies that higher transmission and progression rates lead to a higher basic reproduction number, indicating a more rapidly spreading infection.

VI. CONCLUDING REMARKS:

In conclusion, the extended SEIR model incorporating fuzzy logic represents a significant advancement in epidemiological modeling, particularly for COVID-19. By differentiating between various compartments such as Exposed (E), Infected (I), Hospitalized (H), Quarantined (Q), Recovered (R), and Deceased D, and by utilizing fuzzy logic to handle the uncertainties in key parameters like transmission rate (β), progression rate (σ), hospitalization rate (η), quarantine rate (δ), recovery rate (γ), and mortality rate (μ), this model provides a more nuanced and adaptable framework for predicting disease dynamics. The ability to dynamically adjust these parameters based on real-time data enhances the model's accuracy in forecasting infection trends and calculating the Basic Reproduction Number

(R_0). This comprehensive approach not only improves the reliability of epidemic predictions but also aids in devising effective public health strategies and resource allocation. The success of this model in accurately simulating COVID-19 spread underscores its potential utility in managing future infectious disease outbreaks, highlighting the critical role of advanced modeling techniques in public health planning and response.

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