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Analysis of Rectangular Plate on Winkler's Foundation using Second Degree Characteristic Orthogonal Polynomials.

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ABSTRACT:
This study examines rectangular plates on Winkler's foundation using second-degree characteristic orthogonal polynomials. It explores plates with three distinct edge conditions: simply supported on all sides (SSSS), clamped on all sides (CCCC), and clamped on two opposite edges with the others simply supported (CSCS). Using the work principle and the total minimum energy method, the research derives the exact displacement and bending moment functions for plates subjected to uniformly distributed in-plane loading along their longitudinal edges. The process involves solving the governing differential equations of plates to obtain the deflection coefficient W_{uv} while considering the plate's aspect ratio. The resulting bending moments are calculated by substituting the shape functions into the moment equation, using non-dimensional parameters and aspect ratios.
The midspan deflection results for the SSSS plate, with aspect ratios ranging from 1.0 to 2.0, show values like 0.0000625900, 000062748, 0.000062859, 0.000062939, 0.000063000, 0.000063082, 0.000063111, 0.000063135, 0.000063154. These findings indicate that second-degree characteristic orthogonal polynomials yield accurate results, particularly for higher-degree polynomials, compared to Timoshenko and Woinowsky values. Similarly, results for the CCCC plate exhibit midspan deflection values such as 0.000065330,0.000065680,0.000065923,0.000066098,0.000066227,0.000066325,0.000066401,0.000066460,0.000066508,0.000066546, while the CSCS plate shows values like 0.0011881,0.0011898,0.0011910,0.0011917,0.0011923,0.0011927,0.0011930,0.0011932,0.0011934,0.0011935. These results highlight the effectiveness of the analytical approach using second-degree characteristic orthogonal polynomials for plates on Winkler's foundation under various edge conditions.
KEYWORDS: Rectangularplates, two dimensional orthogonal plates, winkler's foundation, deflection..

I. INTRODUCTION

Thin plates serve as integral structural elements characterized by initially flat surfaces bounded by two parallel planes called faces, alongside either flat or cylindrical edges. The separation between these faces defines the thickness (h) of the plate. These plates exhibit either isotropic or anisotropic mechanical attributes. Classical elasticity theory assumes homogeneity and isotropy within the material, implying uniform mechanical properties across all orientations and locations. Materials like steel and aluminum typically conform to this model. However, certain materials, including wood, plywood, and fiber-reinforced plastics, display anisotropic behavior, with mechanical properties varying by direction.

Rectangular plates, prevalent in civil and mechanical engineering, feature four flat edges and are defined by three primary dimensions: length (a), width (b), and thickness (h). The length-to-thickness ratio (a/h) classifies plates into different categories: thick, stiff, thin, or membrane. A ratio below 10 designates a thick plate, while a ratio between 10 and 100 indicates a thin plate. A ratio exceeding 100 suggests a membrane configuration.

This text focuses on isotropic thin rectangular plates with consistent thickness. These plates are identified by their four edges, which are labeled in Figure 1.1.

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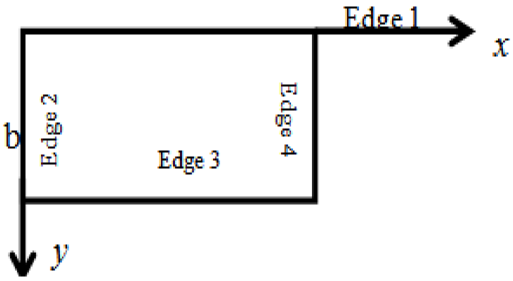

Figure 1.1: Rectangular plate with edge numbering

This research has:

a) Shown unsuitability of Ritz and Galerkin methods in plate's continua problem, as it involves multi – degree of freedom deflection functional problems
b) Further explained and demonstrated the application of work error method (Work principle technique) and its suitability over trigonometric series and variational methods in analysis of plates and shells problems.
c) Through adjusting the characteristic orthogonal polynomial theorem contributed in addressing problem of finding an adequate and accurate approximate shape functional for the plate problems.
d) Exposed the potentials of the use of characteristic orthogonal polynomial theorem as against trigonometric series in analysis of plates and shells problems.
e) Provided reliable approximate solution for the plate's problems, which compares favorably with the exact solution, through the characteristic orthogonal polynomial approach.
f) Provided comprehensive design data: deflections and moments solution for SSSS, CCCC, and CSCS types of plate.
g) Shown unsuitability of Ritz and Galerkin methods in plate's continua problem, as it involves multi – degree of freedom deflection functional problems.

II. MATERIALS AND METHODS.

i) PLATE ON ELASTIC FOUNDATION

Regarding the historical development of plates on elastic foundations, the work by Winkler in 1867 is well-known for its contribution to modeling soil-structure interaction. This model as expressed below assumes that the vertical displacement of the soil at a point depends on the contact pressure at that point and a proportionality constant, k (Straughan, 1990). i.e.

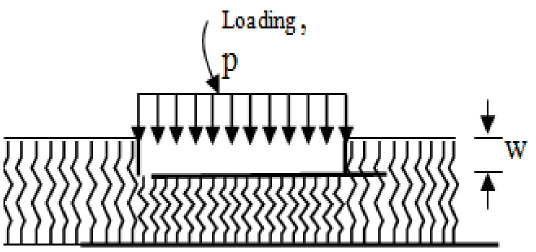

Figure 2.1. Deformation of uniformly loaded plate on Winkler's model

where k is the proportionality constant also regarded as sub-grade reaction modulus proven by Vesic (1961) and depends on the soil stiffness and the structure, so that similar different size stuctures of stiffnesses will yield values of k differently for the same applied load .

$$k = \frac{0.65E_s}{(1-v_s{}^2)}\sqrt[12]{\frac{E_s B^4}{EI}} \quad \mathbf{2.2}$$

Where B = beam width,
E = modulus of elasticity of the beam,
E_s, = modulus of elasticity of the soil,
v_s, = Poisson's ratio of the soil,
I = moment of inertia of the beam.
Using the fourth-order differential equation solution i..e equation 2.2, to apply on Winkler's model, (Vesic (1961)).
Thus,

$$D\nabla^4 w + kw = q \quad \mathbf{2.3}$$

Where, D = the plate flexural rigidity,
q = the uniformly distributed load on the plate and,
K = the modulus of subgrade reaction proven by Vesic (1961)

ii. BOUNDARY CONDITIONS.

The boundary conditions for these edges can be simplely support (S), clamped support (C), or free support (F). The unique configuration of these edge conditions defines each rectangular plate, which may have uniform or mixed boundary conditions, such as SSSS, CCCC, CSCS, and so on. The naming convention for these plates follows the specific order of edge arrangements as depicted in Figure 1.1. above.

iii. TWO-PARAMETER MODEL

To address the limitations in the Winkler model, several researchers proposed modifications to create a more accurate representation of soil continuum by introducing interactions among the spring elements. Filonenko-Borodich (1940) designed a new approach that enhanced the Winkler

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model by linking an elastic membrane with the tops of the spring, T under a constant tension. This adjustment introduced a layer of interconnection among the springs, impacting the behavior of the system. This revised model redefined the modulus of subgrade reaction, allowing for a more realistic representation of the soil-structure interaction.

$p = kw - T\nabla^2 w$ **2.4**

In this updated model as expressed by equation 2.14 above, Hetenyi (1946 and 1950) introduced interaction among the foundation's springs by embedding another plate with flexural rigidity, denoted as D, into the Winkler foundation as depicted in Figure 2.2

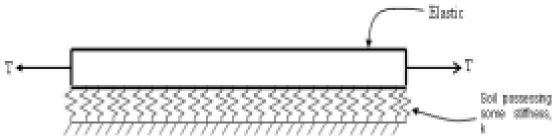

Figure 2.2 Illustration of the assumption of various methods for providing for interaction among the foundationplate spring elements

In this setup, the modulus of subgrade response is defined by incorporating the effects of this additional plate, which alters the behavior of the springs and enhances the foundation's ability to handle varying loads.

$p = kw + D^*\nabla^2\nabla^2 w$ **2.5**

The Vlasov and Leont'ev (1966) two-parameter model, commonly referred to as the "Vlasov model", establishes the modulus of subgrade response as expressed in equation 2.6. This model accounts for shear strains within the soil continuum, leading to a domain equation that describes the interaction between the elastic foundation and the plate. More comprehensive approach incorporates the soil's internal shear characteristics, offering a refined understanding of how plates on elastic foundations behave.

$D\nabla^4 w - 2t\nabla^2 w + kw = q$ 2.6

An iterative method was designed by Vallabhan and Das (1988b) to calculate the λ parameter for beams resting on an elastic foundation as shown in equation 2.7. It was observed that for a beam subjected to a uniform load on an elastic foundation, the λ parameter is influenced by the ratio of the soil stratum's depth relative to the structure's geometry and other model-specific variables. The foundation's elastic characteristics are described by interdependent parameters: k, 2t, and λ. These parameters depend on the load distribution, material properties, and structural design. The behavior of the beam on an elastic foundation is characterized through a specific equation that takes all these factors into account.

$EI\nabla^4 w - 2t\nabla^2 w + kw = q$ 2.7

METHODS OF ANALYZING THIN PLATES

Threeprimary methods are typically utilized for analyzing thin rectangular plates that is The equilibrium (Euler) approach, the energy (approximate) approach, and the numerical approach are the three main methods.

i. The Euler method seeks to solve the governing differential equation by directly integrating it, ensuring that the boundary conditions are satisfied for all four edges of the plate.
ii. The numerical approach presents an alternative solution strategy, employing techniques such as truncated double Fourier series, finite difference, finite strip, Runge-Kutta, and finite element methods. These numerical techniques can handle plates with various boundary conditions, often yielding results that closely align with those obtained through exact methods (Ventsel and Krauthammer, 2001). However, a limitation of numerical solutions is their accuracy, which is contingent upon the computational effort expended.
iii. The energy approach constitutes another method for analyzing thin plates. Diverging from the Euler and numerical approaches, it concentrates on minimizing the total potential energy. Examples of this approach include the Ritz, ,Garlekin, Raleigh-Ritz, and minimum potential energy methods. These methods, categorized as variational methods, strive to ascertain the optimal solution by minimizing the total potential energy functional, which is contingent on the plate's deflection. The shape function approximated is introduced into the total potential energy functional, and the resulting expression is partially differentiated. To determine the optimal solution, the partial derivatives are equated to zero, indicating that the solution has been minimized (Iyengar, 1988). The energy approach can be segmented into direct and indirect methods. In the direct variational approach, the energy functional is minimized to deduce the force equilibrium equations, wherein differentiation with respect to displacement yields a force function that, when equated to zero, indicates that the energy functional has been minimized. Conversely, in the indirect variational approach, energy conversion is not transformed into a force function; instead, it relies on the conservation of energy principle, wherein the total energy in a system in static equilibrium is perpetually

zero. Common examples of direct energy variational methods include the Rayleigh and Ritz methods (Ritz, W. (1909)). Indirect energy variational approaches encompass the finite difference method, boundary collocation methods, boundary detail methods, and the Galerkin method.

2.1.2.2. CHARACTERISTIC ORTHOGONAL POLYNOMIALS SHAPE FUNCTIONS.

Characteristic orthogonal polynomials (COPs) can be generated using the Gram-Schmidt process, and these polynomials of high degrees are utilized as deflection functions in the Galerkin and Ritz energy methods. The orthogonality of these polynomials facilitates easy and direct evaluation, thereby avoiding potential issues related to problem ill-conditioning. Traditionally, the assumed deflection shapes were crafted through inspection or trial and error until Bhat (1985a, 1985b) introduced a systematic approach to construct such functions in the COPs format. The criteria for these series are as follows:

i. They must adhere to the geometrical boundary conditions.
ii. They need to be complete.
iii. They should not inherently violate the natural boundary conditions.

When these conditions are met, numerical solutions converge to the accurate answer, depending also on the number of terms included in the acceptable series. Various types of series, such as trigonometric, hyperbolic, or polynomial, yield different outcomes for the same number of terms, and the effectiveness of the solution partly relies on the series type chosen. The selection of functions F(x) or G(y) can encompass algebraic, trigonometric, hyperbolic functions, or combinations thereof, primarily dictated by the boundary conditions of the plate. For instance, if the two opposite edges of the plate at x = 0 and x = a are simply supported, the deflected surface on the x-z plane may be adequately represented by a trigonometric function in the form of a sine or cosine series.

$$\mathbf{F(x)} = \sum_{m=1}^{\infty} \mathbf{F_m \sin\frac{m\pi x}{a}} \quad \mathbf{2.8}$$

$$\mathbf{G(y)} = \sum_{n=1}^{\infty} \mathbf{G_n \sin\frac{n\pi y}{b}} \quad \mathbf{2.9}$$

In which, F_m and G_n re the coefficients (like F_1, F_2 . . . F_m) to be determined. Thus, the displacement function for the rectangular plate is therefore assumed as a product of two functions; one of which is a pure function of x and the other is of y so that:

$$\mathbf{W(x,y) = F(x)\cdot G(y)} \quad \mathbf{2.10}$$

$$\mathbf{W(x,y)} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \mathbf{C_{mn} Sin\left(\frac{m\pi x}{a}\right) Sin\left(\frac{n\pi y}{b}\right)} \quad \mathbf{2.11a}$$

Where,

$$\mathbf{C_{mn} = F_m G_n} \quad \mathbf{2.11b}$$

$$\mathbf{W(x,y)} = \sum_{m=1}^{\infty} \mathbf{F_m \sin\frac{m\pi x}{a}} \cdot \sum_{n=1}^{\infty} \mathbf{G_n \sin\frac{n\pi y}{b}} \quad \mathbf{2.12}$$

The double Fourier series is a suitable solution method for plates that are simply supported on all four edges. This approach was developed by Navier (Navier 1823). The Characteristic Orthogonal Polynomial (COP) method provides a simple yet approximate technique for quickly analyzing and designing rectangular plates. Here's the general procedure for using the COP method to find shape functions for these plates:

Consider a rectangular plate with dimension a along the x-axis and b along the y-axis. If the deflection pattern along the x-axis is assumed to behave like a beam strip, the beam characteristic along the x-axis is represented by a function F(x). Similarly, for the y-axis, the corresponding beam function is represented by G(y).

For prismatic beams with constant flexural rigidity (EI) and spanning lengthwise along the x-axis, the general solution is as follows:

$$W(x) = F(x) = \sum_{m=1}^{\infty} X_m x^m \quad \mathbf{2.13}$$

Similarly, technique in the y-direction we shall obtain:

$$W(y) = G(y) = \sum_{n=1}^{\infty} Y_n y^n \quad \mathbf{2.14}$$

Where, X_m and $Y_n$are constant parameters in x and y directions respectively. m and n are series to infinity limit.

Expressing Equations 2.13 and 2.14 in the form of non-dimensional parameters, say R and Q for x and y directions respectively:

$$x = aR \quad \mathbf{2.15}$$

$$y = bQ \quad \mathbf{2.16}$$

Then

$$W(y) = G(Q) = \sum_{n=1}^{\infty} B_n Q^n \quad \mathbf{2.17}$$

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$W(x) = F(R) = \sum_{m=1}^{\infty} A_m R^m$ **2.18**

If $A_m = X_m a^m$; $B_n = Y_n b^n$ **2.19**

The order m and n of the shape function of Equation 2.17 and 2.18 is dependent of the type of loading of the plate whether it is uniformly distributed, varying or point.

2.1.2.3. ORDER OF CHARACTERISTIC ORTHOGONAL POLYNOMIALS SHAPE FUNCTIONS FOR A UNIFORMLY VARYING LOAD

Imagine a beam with any support configuration, exposed to a uniformly distributed load from a chosen direction, as illustrated in Figure 3.1. As a result of this load, reactive forces, moments, and reactions will occur at the beam's support points.

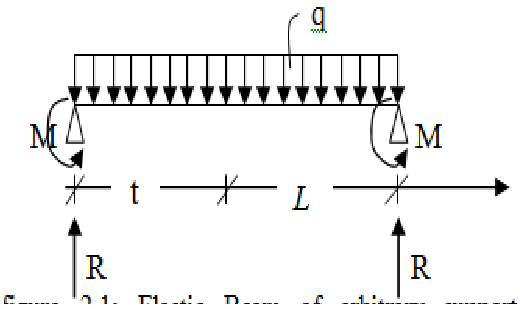

Figure 2.1: Elastic Beam of arbitrary support condition subjected to uniformly distributed load.

The Equation of the moment of the beam at a section say t would be given as: w

$M_t = R_1 \cdot t - \frac{q \cdot t^2}{2} - M_1$ **2.20**

Then, employing the elastic beam equation:

$M_t = -D\frac{d^2W}{dt^2}$ **2.21**

Hence, equating Equations 2.20 and 2.21 and obtaining the deflection function equated expression by integrating twice with respect to the arbitrary direction, t:

$W_t = C_o + C_1 \cdot t + C_2 t^2 + C_3 \cdot t^3 + C_4 \cdot t^4$ **2.22**

Where, C_o and C_1 are constants of integration and;

$C_4 = \frac{q}{24D}$; $C_3 = \frac{-R}{6D}$; $C_2 = \frac{M_1}{2D}$ **2.23**

Then, the maximum value of m and n in Equations 2.17 and 2.18 must be equal to 4 (Onyeyili, 2012). Expanding Equations 2.17 and 2.18 to the 4th series where the constants, of the series along R and Q directions, are denoted by A_m and B_n respectively.

$W(x) = F(R) = \sum_{m=1}^{4} A_m R^m = (A_0 + A_1 R + A_2 R^2 + A_3 R^3 + A_4 R^4)$ **2.24**

$W(y) = G(Q) = \sum_{n=1}^{\infty} B_n Q^n = (B_0 + B_1 Q + B_2 Q^2 + B_3 Q^3 + B_4 Q^4$ **2.25**

The coefficients A_m and B_n in the series are derived from the boundary conditions at the plate's edges. The polynomial functions obtained in equations 2.24 and 2.25 are combined through multiplication, and the resulting product is subjected to boundary conditions to obtain the shape function. This shape function's second-degree freedom functions are specified in equations, 2.27, 2.28 and 2.29 for the SSSS, CCCC, and CSCS plates, respectively, as analyzed in this thesis.

2.1.2.3. ORDER OF CHARACTERISTIC ORTHOGONAL POLYNOMIALS SHAPE FUNCTIONS FOR A UNIFORMLY VARYING LOAD

Imagine a beam with any support configuration, exposed to a uniformly distributed load from a chosen direction, as illustrated in Figure 3.1. As a result of this load, reactive forces, moments, and reactions will occur at the beam's support points.

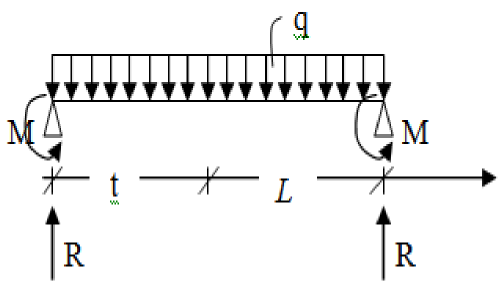

Figure 2.1: Elastic Beam of arbitrary support condition subjected to uniformly distributed load.

The Equation of the moment of the beam at a section say t would be given as: w

$M_t = R_1 \cdot t - \frac{q \cdot t^2}{2} - M_1$ **2.20**

Then, employing the elastic beam equation:

$M_t = -D\frac{d^2W}{dt^2}$ **2.21**

Hence, equating Equations 2.20 and 2.21 and obtaining the deflection function equated expression

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by integrating twice with respect to the arbitrary direction, t:

$W_t = C_o + C_1 \cdot t + C_2 t^2 + C_3 \cdot t^3 + C_4 \cdot t^4$ **2.22**

Where, $C_o \, and \, C_1$ are constants of integration and;

$C_4 = \frac{q}{24D}; \quad C_3 = \frac{-R}{6D}; \quad C_2 = \frac{M_1}{2D}$ **2.23**

Then, the maximum value of m and n in Equations 2.17 and 2.18 must be equal to 4 (Onyeyili, 2012). Expanding Equations 2.17 and 2.18 to the 4th series where the constants, of the series along R and Q directions, are denoted by A_m and B_n respectively.

$W(x) = F(R) = \sum_{m=1}^{4} A_m R^m = (A_0 + A_1 R + A_2 R^2 + A_3 R^3 + A_4 R^4$ **2.24**

$W(y) = G(Q) = \sum_{n=1}^{\infty} B_n Q^n = (B_0 + B_1 Q + B_2 Q^2 + B_3 Q^3 + B_4 Q^4$ **2.25**

The coefficients A_m and B_n in the series are derived from the boundary conditions at the plate's edges. The polynomial functions obtained in equations 2.24 and 2.25 are combined through multiplication, and the resulting product is subjected to boundary conditions to obtain the shape function. This shape function's second-degree freedom functions are specified in equations, 2.27, 2.28 and 2.29 for the SSSS, CCCC, and CSCS plates, respectively, as analyzed in this thesis.

After obtaining the shape function, it is substitutedinto the work-done expression. According to the theory of characteristic orthogonal polynomials, the displacement function for a rectangular plate is assumed to be the product of two independent functions, R and Q, as provided.

$\boldsymbol{W(R, Q) = W(R) \times W(Q)}$ **2.26**

2.1.2.2.1. ALL ROUND SIMPLY SUPPORTED RECTANGULAR PLATE (TYPE SSSS)

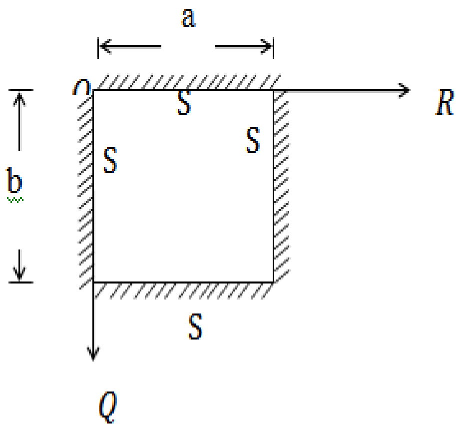

Figure 2.2: All Round Simply Supported Rectangular Plate (SSSS)

PLATE STRIP ALONG R - DIRECTION

From the polynomials Equation 2.24, the shape function along R direction is deduced as:

$\boldsymbol{W(R) = A_o + A_1 R + A_2 R^2 + A_3 R^3 + A_4 R^4}$ **2.27**

Boundary conditions along R direction

$(a) \quad W(R, Q) = \begin{cases} 0, & i) \, R, Q = 0 \\ 0, & ii) \, R, Q = 1 \end{cases} \quad b) \, \frac{d^2W}{dR^2}(R) = \begin{cases} 0, & i) \, R, = 0 \\ 0, & ii) \, R, = 1 \end{cases}$ **2.28**

Boundary conditions along Q – direction

$(a) \quad W(Q) = \begin{cases} 0, & i.) \, Q = 0 \\ 0, & ii) \, Q = 1 \end{cases} \quad (b) \frac{d^2W}{dQ^2}(Q) = \begin{cases} 0, & i.) \, Q = 0 \\ 0, & ii) \, Q = 1 \end{cases}$ **2.29**

Applying the four Boundary conditions contained in equations 2.28 and 2.29 stated above on equation 2.27 we will arrive at the single degree of freedom functional shown as equations 2.30 and its corresponding second degree of freedom functional identified as 2.31.

Single degree of freedom functional

$W(R, Q) = K_o(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4)$ **2.30**

Second degree of freedom functional

$$W(R, Q) = k_0(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4) + k_1 (R - 2R^3 + R^4)(Q - 2Q^3 + Q^4) R^2 + k_2 (R - 2R^3 + R^4)(Q - 2Q^3 + Q^4) Q^2$$ 3.20

2.31

2.1.2.2.2. ALL ROUND CLAMPED RECTANGULAR PLATE (CCCC TYPE) AND THIN RECTANGULAR PLATE CLAMPED ON TWO OPPOSITE SHORT EDGES AND SIMPLY SUPPORTED ON TWO OPPOSITE LONG EDGES (TYPE CSCS)

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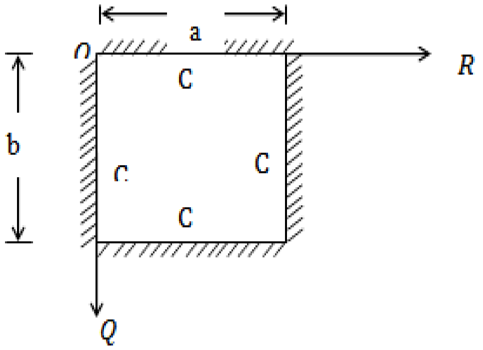

Figure 2.3 shows a thin rectangular plate whose all edges R = 0, 1; and Q = 0, 1 are clamped.

PLATE STRIP ALONG DIRECTIONS

CCCC Boundary conditions along R – direction

$(a)\ \ W(R,Q) = \begin{cases} 0, & i)\ R,Q = 0 \\ 0, & ii)\ R,Q = 1 \end{cases}$ $\quad b)\ \frac{dW}{dR}(R) =$

$(R) = \begin{cases} 0, & i)\ R = 0 \\ 0, & ii)\ R = 1 \end{cases}$ **2.32**

CCCC Boundary conditions along Q – direction

$(\boldsymbol{a})W(Q) = \begin{cases} 0, & i)\ Q = 0 \\ 0, & ii)\ Q = 1 \end{cases}$ $\quad (b)\frac{dW}{dQ}(Q) =$

$\begin{cases} 0, & i)\ Q = 0 \\ 0, & ii)\ Q = 1 \end{cases}$ **2.33**

Applying the four Boundary conditions of equations 2.32 and 2.33 stated above on equation 2.27, we will arrive at the single degree of freedom functional shown as equations 2.34 and its corresponding second degree of freedom functional identified as 2.35.

Single degree of freedom functional

$$W(R,Q) = K_o K(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) \qquad \mathbf{2.34}$$

Second degree of freedom functional for CCCC

$W(R,Q) = k_0(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) + k_1(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)\,R^2 +$
$k_2(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)Q^2$

$W(R,Q) = k_0(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 +$
$Q4 + k1\ (R4 - 2R5 + R6)(Q2 - 2Q3 + Q4) +$
$k_2(R^2 - 2R^3 + R^4)(Q^4 - 2Q^5 + Q^6)$**2.35**

2.1.2.2.2 THIN RECTANGULAR PLATE CLAMPED ON TWO OPPOSITE SHORT EDGES AND SIMPLY SUPPORTED ON TWO OPPOSITE LONG EDGES (TYPE CSCS)

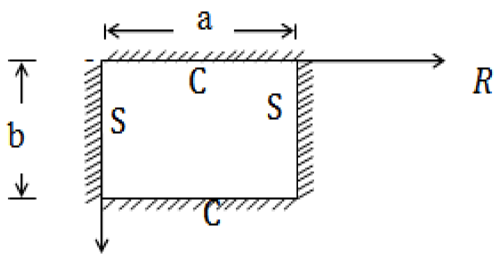

Figure 2.4 shows a thin rectangular plate whose all edges R = 0, 1; and Q = 0, 1 are clampedon two opposite short edges and simply supported on two opposite long edges.

CSCS Boundary conditions along, R – direction

$(a)\ \ W(R,Q) = \begin{cases} 0, & i)\ R,Q = 0 \\ 0, & ii)\ R,Q = 1 \end{cases}$ $\quad b)\ \frac{d^2W}{dR^2}(R) =$
$\begin{cases} 0, & i)\ R, = 0 \\ 0, & ii)\ R, = 1 \end{cases}$ **2.36**

CSCS Boundary conditions along, Q – direction

$(a)\quad W(Q) = \begin{cases} 0, & i)\ Q = 0 \\ 0, & ii)\ Q = 1 \end{cases}$

$(b)\frac{dW}{dQ}(Q) = \begin{cases} 0, & i)\ Q = 0 \\ 0, & ii)\ Q = 1 \end{cases}$**2.37**

Applying the four Boundary conditions of equations 2.36 and 2.37 stated above on equation 2.27, we will arrive at the single degree of freedom functional shown as equations 2.38 and its corresponding second degree of freedom functional identified as 2.39.

Single degree of freedom functional

$W(R,Q) = k_1(R - 2R^3 + R^4)(Q^2 - 2Q^3 +$
$Q4$ 2.38

Second degree of freedom functional

$W(R,Q) = k_0(R - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) + k_1(R - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4)\,R^2 +$
$k_2(R - 2R^3 + R^4\,)(Q^2 - 2Q^3 + Q^4)Q^2$
$W(R,Q) = k_0(R - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) + k_1(R^3 - 2R^5 + R^6)(Q^2 - 2Q^3 + Q^4) +$
$k_2(R - 2R^3 + R^4)(Q^4 - 2Q^5 + Q^6)$**2.39**

2.1.2.3. FORMULATION OF THE GENERAL EXPRESSION FOR LATERAL LOADS OF THIN RECTANGULAR PLATES USING WORK PRINCIPLE TECHNIQUE

The governing differential equilibrium equation for pure bending of elastic isotropic thin rectangular

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plate on Winkler's foundation is given as equation 2.40 as stated by Ibearugbulem et al 2014.

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + kw = \frac{q}{D} \quad \mathbf{2.40}$$

This will further be expressed in the form of non-dimensional coordinates related with the aspect ratio;

$$p = \frac{b}{a}; b = pa \mathbf{2.41}$$

In his book, Oguaghamba (2018), it is highlighted that if a material is elastic and adheres to Hooke's Law, where the load-displacement curve is linear, the displacement is minimal, and the applied load is steady, then the work done by the system as defined by Oguaghamba (2018), are as follows:

$$W = \frac{1}{2}Px \quad 2.42$$

Where, P is the applied load, x is the virtual (small) displacement

$$\Delta W = \frac{1}{2}qw = \frac{1}{2}\frac{D}{a^4}\left(\frac{\partial^4 W}{\partial R^4} + 2\frac{\partial^4 W}{\partial R^2 \partial Q^2}\frac{1}{P^2} + \frac{\partial^4 W}{\partial Q^4}\frac{1}{P^4} + kwa4 \mathbf{2.43}$$

Hence, the total work performed by the plate is the sum of the elementary work functional equation of the plate. This is obtained by integrating Equation 2.43 over the entire plate's domain
($0 \leq x \leq a$; $0 \leq x \leq b$).

Thus, the total work in purely non – dimensional coordinates become:

$$\frac{1}{2}\frac{D}{a^4}\int_0^1\int_0^1\left(\frac{\partial^4 w}{\partial R^4}\cdot w + 2\frac{\partial^4 W}{\partial R^2 \partial Q^2}\frac{1}{P^2}\cdot w + \frac{\partial^4 W}{\partial Q^4}\frac{1}{P^4}\cdot w + kw2a4abdRdQ = q20101wabdRdQ\mathbf{2.44}$$

Dividing through by ab, we have:

$$\frac{1}{2}\frac{D}{a^4}\int_0^1\int_0^1\left(\frac{\partial^4 w}{\partial R^4}\cdot w + 2\frac{\partial^4 W}{\partial R^2 \partial Q^2}\frac{1}{P^2}\cdot w + \frac{\partial^4 W}{\partial Q^4}\frac{1}{P^4}\cdot w + kw^2 a^4\right)dRdQ = \frac{q}{2}\int_0^1\int_0^1 w\,dRdQ \quad 2.45$$

To justify this claim, Oguaghamba (2018) disclosed the principle of virtual work which states that "a necessary and sufficient condition for static equilibrium of a system of particles is that the work done by the internal forces plus the work done by the external loads during any virtual displacement is zero".

That is;

$$\sum W = \sum W_{int} + \sum W_{ext} = 0 \quad \mathbf{2.46}$$

Here, the left-hand side of Equation 2.46 is the internal work done while the right-hand side is the external work done by the system. Therefore, transferring the right-hand side to the left-hand side, we obtain the sum of all work done by all applied and internal forces is zero for all virtual displacement.

Equation 2.35 becomes;

$$\frac{1}{2}\frac{D}{a^4}\int_0^1\int_0^1\left(\frac{\partial^4 w}{\partial R^4}\cdot w + 2\frac{\partial^4 W}{\partial R^2 \partial Q^2}\frac{1}{P^2}\cdot w + \frac{\partial^4 W}{\partial Q^4}\frac{1}{P^4}\cdot w + kw2a4dRdQ - q20101wdRdQ = \Omega R,Q\mathbf{2.47}$$

where, $\Omega(R,Q)$ is a constant proportionality representing the sum of the work errors, ϵ_i.

Minimizing the total work done Equation 2.47 withrespect to the undetermined coefficients, K_0, K_1 and K_2; we have:

$$\frac{\partial \Omega(R,Q)}{\partial k_i} = 0; for\ i = 1,2,3$$

$$\Pi_{0,0}k_0 + \Pi_{1,1}k_1 + \Pi_{1,2}k_2$$
$$\Pi_{1,0}k_0 + \Pi_{1,1}k_1 + \Pi_{1,2}k_2 \quad \mathbf{2.48}$$
$$\Pi_{2,0}k_0 + \Pi_{2,1}k_1 + \Pi_{2,2}k_2$$

Rearranging equation 2.37 in matrix form to solve for the coefficients

$$\begin{bmatrix} k_o \\ k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} \Pi_{0,0} & \Pi_{0,1} & \Pi_{0,2} \\ \Pi_{1,0} & \Pi_{1,1} & \Pi_{1,2} \\ \Pi_{2,0} & \Pi_{2,1} & \Pi_{2,2} \end{bmatrix}^{-1} \begin{bmatrix} e \\ f \\ g \end{bmatrix} \frac{q}{D} a^4 2.49$$

`Where e, f,g are numerical figures after collecting the like terms.

2.1.2.4. THIN RECTANGULAR PLATES SECOND – DEGREE OF FREEDOM OF BENDING MOMENTS EXPRESSIONS

From, the moments equations are given above:

$$M_x = -D\left[\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right] \mathbf{2.50}$$

$$M_y = -D\left[\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right] \mathbf{2.51}$$

Expressing these moments in terms of non – dimensional parameters R and Q and aspect ratio, P.
Where, x = aR, y = bQ, b/a = P

$$M_x = -\frac{D}{a^2}\left[\frac{\partial^2 w}{\partial R^2} + \mu\frac{\partial^2 w}{P^2 \partial Q^2}\right] \mathbf{2.52}$$

Thus $M_{xi} = qa^2 \sum_{i=0}^{2} \psi_{xi}(R,Q,\mu) = qa^2\psi_x(R,Q,\mu)$ 2.53

Similarly,

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$M_y = -\frac{D}{a^2}\left[\mu\frac{\partial^2 w}{\partial R^2} + \frac{\partial^2 w}{P^2\partial Q^2}\right]$ **2.54**

$M_{yi} = qa^2\sum_{i=0}^{2}\psi_{yi}(R,Q,\mu)$ **2.55**

Where, ψ_{xi} and ψ_{yi} are the bending moment coefficients along x and y directions respectively

The shape functions obtained in Equations 2.31, 2.35, and 2.39 for different plate support conditions shall be substituted into the non – dimensional moment Equations 2.52 and 2.53 so as to get the corresponding second-degree moments for SSSS, CCCC and CSCS Rectangular Plate.

Thus, the moment M_{yi}, and M_{xi} for SSSS Rectangular Plates can be gotten using equations 2.54 2.55, 2.56 and 2.57 thus can be gotten as;

$$M_{yi} = -\frac{D}{a^2}[\mu[12k_0(-R+R^2)(Q-2Q^3+Q^4) + 4k_1(1.5R-10R^3+7.5R^4)(Q-2Q^3+Q^4) + 12k_2(-R+R^2)(Q^3-2Q^5+Q^6)] + \frac{1}{P^2}[12k_0(R-2R^3+R^4)(-Q+Q^2) + 12k_1(R^3-2R^5+R^6)(-Q+Q^2) + 4k_2(R-2R^3+R^4)(1.5Q-10Q^3+7.5Q^4)]]$$

2.57

$$M_{xi} = -\frac{D}{a^2}[[12k_0(-R+R^2)(Q-2Q^3+Q^4) + 4k_1(1.5R-10R^3+7.5R^4)(Q-2Q^3+Q^4) + 12k_2(-R+R^2)(Q^3-2Q^5+Q^6)] + \frac{\mu}{P^2}[12k_0(R-2R^3+R^4)(-Q+Q^2) + 12k_1(R^3-2R^5+R^6)(-Q+Q^2) + 4k_2(R-2R^3+R^4)(1.5Q-10Q^3+7.5Q^4)]] \times \frac{q}{D}a^4$$

$$M_{xi} = -[[12k_0(-R+R^2)(Q-2Q^3+Q^4) + 4k_1(1.5R-10R^3+7.5R^4)(Q-2Q^3+Q^4) + 12k_2(-R+R^2)(Q^3-2Q^5+Q^6)] + \frac{\mu}{P^2}[12k_0(R-2R^3+R^4)(-Q+Q^2) + 12k_1(R^3-2R^5+R^6)(-Q+Q^2) + 4k_2(R-2R^3+R^4)(1.5Q-10Q^3+7.5Q^4)]]qa^2$$

2.58

The bending moment coefficients along x and y directions, ψ_{xi} and ψ_{yi} will be

$$\psi_{xi} = -[[12k_0(-R+R^2)(Q-2Q^3+Q^4) + 4k_1(1.5R-10R^3+7.5R^4)(Q-2Q^3+Q^4) + 12k_2(-R+R^2)(Q^3-2Q^5+Q^6)] + \frac{\mu}{P^2}[12k_0(R-2R^3+R^4)(-Q+Q^2) + 12k_1(R^3-2R^5+R^6)(-Q+Q^2) + 4k_2(R-2R^3+R^4)(1.5Q-10Q^3+7.5Q4 \quad 3.134d$$

2.59

$$\psi_{yi} = -[\mu[12k_0(-R+R^2)(Q-2Q^3+Q^4) + 4k_1(1.5R-10R^3+7.5R^4)(Q-2Q^3+Q^4) + 12k_2(-R+R^2)(Q^3-2Q^5+Q^6)] + \frac{1}{P^2}[12k_0(R-2R^3+R^4)(-Q+Q^2) + 12k_1(R^3-2R^5+R^6)(-Q+Q^2) + 4k_2(R-2R^3+R4)1.5Q-10Q3+7.5Q4 \quad 3.135d$$

2.60

Similarly, For CCCC Rectangular shaped plate, the moment can be expressed as,

$$M_{xi} = -\frac{D}{a^2}[[2k_0(1-6R+6R^2)(Q^2-2Q^3+Q^4) + 4k_1(3R^2-10R^3+7.5R^4)(Q^2-2Q^3+Q^4) + 2k_2(1-6R+6R^2)(Q^4-2Q^5+Q^6)] + \frac{\mu}{P^2}[2k_0(R^2-2R^3+R^4)(1-6Q+6Q^2) + 2k_1(R^4-2R^5+R^6)(1-6Q+6Q^2) + 4k_2(R^2-2R^3+R^4)(3Q^2-10Q^3+7.5Q^4)]] \times \frac{q}{D}a^4$$

$$M_{xi} = -[[2k_0(1-6R+6R^2)(Q^2-2Q^3+Q^4) + 4k_1(3R^2-10R^3+7.5R^4)(Q^2-2Q^3+Q^4) + 2k_2(1-6R+6R^2)(Q^4-2Q^5+Q^6)] + \frac{\mu}{P^2}[2k_0(R^2-2R^3+R^4)(1-6Q+6Q^2) + 2k_1(R^4-2R^5+R^6)(1-6Q+6Q^2) + 4k_2(R^2-2R^3+R^4)(3Q^2-10Q^3+7.5Q^4)]]qa^2$$

2.61

$$M_{yi} = -\frac{D}{a^2}[\mu[2k_0(1-6R+6R^2)(Q^2-2Q^3+Q^4)+4k_1(3R^2-10R^3+7.5R^4)(Q^2-2Q^3+Q^4)+2k_2(1-6R+6R^2)(Q^4-2Q^5+Q^6)]+\frac{1}{P^2}[2k_0(R^2-2R^3+R^4)(1-6Q+6Q^2)+2k_1(R^4-2R^5+R^6)(1-6Q+6Q^2)+4k_2(R^2-2R^3+R^4)(3Q^2-10Q^3+7.5Q^4)]]$$

$$M_{yi} = -[\mu[2k_0(1-6R+6R^2)(Q^2-2Q^3+Q^4)+4k_1(3R^2-10R^3+7.5R^4)(Q^2-2Q^3+Q^4)+2k_2(1-6R+6R^2)(Q^4-2Q^5+Q^6)]+\frac{1}{P^2}[2k_0(R^2-2R^3+R^4)(1-6Q+6Q^2)+2k_1(R^4-2R^5+R^6)(1-6Q+6Q^2)+4k_2(R^2-2R^3+R^4)(3Q^2-10Q^3+7.5Q^4)]]qa^2$$

2.62

The CCCC rectangular plate bending moment coefficients along x and y directions, ψ_{xi} and ψ_{yi} will be;

$$\psi_{xi} = -[[2k_0(1-6R+6R^2)(Q^2-2Q^3+Q^4)+4k_1(3R^2-10R^3+7.5R^4)(Q^2-2Q^3+Q^4)+2k_2(1-6R+6R^2)(Q^4-2Q^5+Q^6)]+\frac{\mu}{P^2}[2k_0(R^2-2R^3+R^4)(1-6Q+6Q^2)+2k_1(R^4-2R^5+R^6)(1-6Q+6Q^2)+4k_2(R^2-2R^3+R^4)(3Q^2-10Q^3+7.5Q^4)]]$$

2.63

$$\psi_{yi} = -[\mu[2k_0(1-6R+6R^2)(Q^2-2Q^3+Q^4)+4k_1(3R^2-10R^3+7.5R^4)(Q^2-2Q^3+Q^4)+2k_2(1-6R+6R^2)(Q^4-2Q^5+Q^6)]+\frac{1}{P^2}[2k_0(R^2-2R^3+R^4)(1-6Q+6Q^2)+2k_1(R^4-2R^5+R^6)(1-6Q+6Q^2)+$$

Similarly, For CSCS Rectangular shaped plate, the moment can be expressed as,

$$M_{xi} = -\frac{D}{a^2}[[12k_0(-R+R^2)(Q^2-2Q^3+Q^4)+4k_1(1.5R-10R^3+7.5R^4)(Q^2-2Q^3+Q^4)+12k_2(-R+R^2)(Q^4-2Q^5+Q^6)]+\frac{\mu}{P^2}[2k_0(R-2R^3+R^4)(1-6Q+6Q^2)+2k_1(R^3-2R^5+R^6)(1-6Q+6Q^2)+4k_2(R-2R^3+R^4)(3Q^2-10Q^3+7.5Q^4)]]$$

$$M_{xi} = -[[12k_0(-R+R^2)(Q^2-2Q^3+Q^4)+4k_1(1.5R-10R^3+7.5R^4)(Q^2-2Q^3+Q^4)+12k_2(-R+R^2)(Q^4-2Q^5+Q^6)]+\frac{\mu}{P^2}[2k_0(R-2R^3+R^4)(1-6Q+6Q^2)+2k_1(R^3-2R^5+R^6)(1-6Q+6Q^2)+4k_2(R-2R^3+R^4)(3Q^2-10Q^3+7.5Q^4)]]qa^2$$

2.64

$$M_{yi} = -\frac{D}{a^2}[\mu[12k_0(-R+R^2)(Q^2-2Q^3+Q^4)+4k_1(1.5R-10R^3+7.5R^4)(Q^2-2Q^3+Q^4)+12k_2(-R+R^2)(Q^4-2Q^5+Q^6)]+\frac{1}{P^2}[2k_0(R-2R^3+R^4)(1-6Q+6Q^2)+2k_1(R^3-2R^5+R^6)(1-6Q+6Q^2)+4k_2(R-2R^3+R^4)(3Q^2-10Q^3+7.5Q^4)]]$$

$$M_{yi} = -[\mu[12k_0(-R+R^2)(Q^2-2Q^3+Q^4)+4k_1(1.5R-10R^3+7.5R^4)(Q^2-2Q^3+Q^4)+12k_2(-R+R^2)(Q^4-2Q^5+Q^6)]+\frac{1}{P^2}[2k_0(R-2R^3+R^4)(1-6Q+6Q^2)+2k_1(R^3-2R^5+R^6)(1-6Q+6Q^2)+4k_2(R-2R^3+R^4)(3Q^2-10Q^3+7.5Q^4)]]qa^2$$

2.65

The CSCS rectangular plate bending moment coefficients along x and y directions, ψ_{xi} and ψ_{yi} will be;

$$\psi_{xi} = -[[12k_0(-R+R^2)(Q^2-2Q^3+Q^4)+4k_1(1.5R-10R^3+7.5R^4)(Q^2-2Q^3+Q^4)+12k_2(-R+R^2)(Q^4-2Q^5+Q^6)]+\frac{\mu}{P^2}[2k_0(R-2R^3+R^4)(1-6Q+6Q^2)+2k_1(R^3-2R^5+R^6)(1-6Q+6Q^2)+4k_2(R-2R^3+R^4)(3Q^2-10Q^3+7.5Q^4)]]$$

2.66

$$\psi_{yi} = -[\mu[12k_0(-R+R^2)(Q^2-2Q^3+Q^4)+4k_1(1.5R-10R^3+7.5R^4)(Q^2-2Q^3+Q^4)+12k_2(-R+R^2)(Q^4-2Q^5+Q^6)]+\frac{1}{P^2}[2k_0(R-2R^3+R^4)(1-6Q+6Q^2)+2k_1(R^3-2R^5+R^6)(1-6Q+6Q^2)+4k_2(R-2R^3+R^4)(3Q^2-10Q^3+7.5Q^4)]]$$ **2.67**

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III. RESULTS

In this section, typical expressions for the coefficients, k_0, k_1 and k_2 for deflections of the three cases of thin rectangular plates on elastic foundation subjected to uniformly distributed load, q are obtained using the solution of differential equations in Equations 2.38. With these coefficients different support conditions are obtained.

3.1. Stiffness and Degree of Freedom Coefficients

Stiffness, $\Pi_{i,i}$ and Degree of Freedom k_iCoefficients for various aspect ratios were obtained for aspect ratio range $1.0 \leq p \leq 2.0$ for SSSS, CCCC and CSCS plates respectively. With these determined values of stiffness coefficients, $\Pi_{i,i}$; simultaneous Equations were solved and the degree of freedom coefficients, k_i for various aspect ratios determined.

3.1.1 Coefficients for all Edges Simply Supported Thin Rectangular Plate (SSSS)

The stiffness coefficients will after application on SSSS plates and substitution effectively done in equation 2.49 above

$$\Pi_{0,0} = \Big[2.3619 \times 10^{-1} + 4.7184 \times 10^{-1}\frac{1}{P^2} + 2.3619 \times 10^{-1}\frac{1}{P^4} + 2.4213 \times 10^{-3} ka^4\Big]$$

$$\Pi_{0,1} = \Big[-2.8118 \times 10^{-2} + 1.3415 \times 10^{-1}\frac{1}{P^2} + 6.6840 \times 10^{-2}\frac{1}{P^4} + 6.8520 \times 10^{-4} ka^4\Big]$$

$$\Pi_{0,2} = \Big[6.6840 \times 10^{-2} + 1.3415 \times 10^{-1}\frac{1}{P^2} - 2.8118 \times 10^{-2}\frac{1}{P^4} + 6.8520 \times 10^{-4} ka^4\Big]$$

$$\Pi_{1,0} = \Big[-2.8118 \times 10^{-2} + 1.3415 \times 10^{-1}\frac{1}{P^2} + 6.6840 \times 10^{-2}\frac{1}{P^4} + 6.8520 \times 10^{-4} ka^4\Big]$$

$$\Pi_{1,1} = \Big[2.8118 \times 10^{-2} + 1.0920 \times 10^{-1}\frac{1}{P^2} + 2.7066 \times 10^{-2}\frac{1}{P^4} + 2.7747 \times 10^{-4} ka^4\Big]$$

$$\Pi_{1,2} = \Big[-7.9571 \times 10^{-3} + 3.8141 \times 10^{-2}\frac{1}{P^2} - 7.9571 \times 10^{-3}\frac{1}{P^4} + 1.9390 \times 10^{-4} ka^4\Big]$$

$$\Pi_{2,0} = \Big[6.6840 \times 10^{-2} - 7.9571 \times 10^{-3}\frac{1}{P^2} + 2.7066 \times 10^{-2}\frac{1}{P^4} + 6.8520 \times 10^{-4} ka^4\Big]$$

$$\Pi_{2,1} = \Big[1.3415 \times 10^{-2} + 3.8141 \times 10^{-2}\frac{1}{P^2} + 1.0920 \times 10^{-1}\frac{1}{P^4} + 1.9390 \times 10^{-4} ka^4\Big]$$

$$\Pi_{2,2} = \Big[-2.8118 \times 10^{-2} - 7.9571 \times 10^{-3}\frac{1}{P^2} + 2.8118 \times 10^{-2}\frac{1}{P^4} + 1.1905 \times 10^{-2} ka^4\Big]$$

There giving us matrix equation,

$$\begin{bmatrix} k_o \\ k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} \Pi_{0,0} & \Pi_{0,1} & \Pi_{0,2} \\ \Pi_{1,0} & \Pi_{1,1} & \Pi_{1,2} \\ \Pi_{2,0} & \Pi_{2,1} & \Pi_{2,2} \end{bmatrix}^{-1} \begin{bmatrix} 4.0000 \times 10^{-2} \\ 1.1905 \times 10^{-2} \\ 1.1905 \times 10^{-2} \end{bmatrix} \frac{q}{D} a^4 \qquad 3.1$$

$\Pi_{i,i}$ is a variable dependent on the aspect ratio, p of the plate, modulus of subgrade reaction, k and the length of the plate in the R direction, a. For example, Using variables, we obtained values for stiffness coefficients using Short span length, a = 5 m; thickness of plate; h = 0.15m, modulus of elasticity of plate; E = 205 GPa, modulus of subgrade reaction;
k = 40 N/m^3 (5700 psi) for clayey medium dense sand
soil taken from reference Timoshenko, (1959).

Table 3.1: Stiffness Coefficients Values for SSSS Plate at varying aspect ratio

Aspect ratio, P	$\Pi_{i,i}$	0	1	2
1	0	61.47584	17.30279	17.30279
	1	17.30279	7.10101	4.86984
	2	17.30279	4.86984	297.78343
1.1	0	61.31908	17.25832	17.28842
	1	17.25832	7.07348	4.86574
	2	17.28842	4.86574	297.75557

1.2	0	61.20938	17.22719	17.27636
	1	17.22719	7.05363	4.86231
	2	17.27636	4.86231	297.73551
1.3	0	61.12970	17.20458	17.26629
	1	17.20458	7.03883	4.85944
	2	17.26629	4.85944	297.72057
1.4	0	61.07003	17.18764	17.25788
	1	17.18764	7.02750	4.85705
	2	17.25788	4.85705	297.70915
1.5	0	61.02417	17.17462	17.25082
	1	17.17462	7.01862	4.85504
	2	17.25082	4.85504	297.70020
1.6	0	60.98816	17.16440	17.24487
	1	17.16440	7.01153	4.85334
	2	17.24487	4.85334	297.69306
1.7	0	60.95936	17.15622	17.23981
	1	17.15622	7.00577	4.85190
	2	17.23981	4.85190	297.68727
1.8	0	60.93594	17.14957	17.23548
	1	17.14957	7.00103	4.85067
	2	17.23548	4.85067	297.68250
1.9	0	60.91664	17.14409	17.14409
	1	17.14409	6.99707	4.84961
	2	17.23176	4.84961	297.67852
2.0	0	60.90053	17.13951	17.22854
	1	17.13951	6.99374	4.84870
	2	17.22854	4.84870	297.67517

The degree of freedom coefficients, Ki (K_o, K_1 and K_2)
for any aspect ratio of the SSSS plate were then obtained by substituting these stiffness coefficients, $\Pi_{i,i}$
for that aspect ratio into Equation 2.49
For example, at aspect ratio, P = 1.0; the stiffness coefficients $\Omega_{i,i}$ were given (see Table 3.1).
Substituting these stiffness coefficients into

Equation 2.48 and subsequently solving the resulting canonical equation, gave;

$$\begin{bmatrix} k_o \\ k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} \Pi_{0,0} & \Pi_{0,1} & \Pi_{0,2} \\ \Pi_{1,0} & \Pi_{1,1} & \Pi_{1,2} \\ \Pi_{2,0} & \Pi_{2,1} & \Pi_{2,2} \end{bmatrix}^{-1} \begin{bmatrix} 4.0000 \times 10^{-2} \\ 1.1905 \times 10^{-2} \\ 1.1905 \times 10^{-2} \end{bmatrix} \frac{q}{D} a^4$$

$$\begin{bmatrix} k_o \\ k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 61.47584 & 17.30279 & 17.30279 \\ 17.30279 & 7.10101 & 4.86984 \\ 17.30279 & 4.86984 & 297.78343 \end{bmatrix}^{-1} \begin{bmatrix} 4.0000 \times 10^{-2} \\ 1.1905 \times 10^{-2} \\ 1.1905 \times 10^{-2} \end{bmatrix} \frac{q}{D} a^4 = \begin{bmatrix} 5.685 \times 10^{-4} \\ 2.898 \times 10^{-4} \\ 2.207 \times 10^{-6} \end{bmatrix} \frac{q}{D} a^4$$

The degree of freedom coefficients, k_i (k_o, k_1 and k_2) for other aspect ratios of the SSSS plate were obtained by similar approach and their results are shown in Table 3.2.

Table 3.2: Typical deflection coefficients of SSSS plate on elastic foundation at varying aspect ratio

Aspect ratio, P	k_o	k_1	k_2
1.0	5.685E-04	2.898E-04	2.207E-06
1.1	5.696E-04	2.918E-04	2.141E-06
1.2	5.703E-04	2.934E-04	2.099E-06
1.3	5.709E-04	2.945E-04	2.072E-06
1.4	5.713E-04	2.955E-04	2.053E-06
1.5	5.715E-04	2.962E-04	2.040E-06
1.6	5.718E-04	2.968E-04	2.030E-06
1.7	5.719E-04	2.973E-04	2.023E-06
1.8	5.721E-04	2.977E-04	2.018E-06
1.9	5.722E-04	2.980E-04	2.014E-06
2.0	5.723E-04	2.983E-04	2.011E-06

3.1.2 ALL EDGES CLAMPED SUPPORTED THIN RECTANGULAR PLATE (CCCC)

The stiffness coefficients are given after due substitutions on equation 2.48

$$\Pi_{0,0} = \left[1.2698 \times 10^{-3} + 7.2562 \times 10^{-4}\frac{1}{P^2} + 1.2698 \times 10^{-3}\frac{1}{P^4} + 2.5195 \times 10^{-6} ka^4\right]$$

$$\Pi_{0,1} = \left[3.6281 \times 10^{-4} + 1.8141 \times 10^{-4}\frac{1}{P^2} + 3.4632 \times 10^{-4}\frac{1}{P^4} + 6.8714 \times 10^{-7} ka^4\right]$$

$$\Pi_{0,2} = \left[3.4632 \times 10^{-4} + 1.8141 \times 10^{-4}\frac{1}{P^2} + 3.6281 \times 10^{-4}\frac{1}{P^4} + 6.8714 \times 10^{-7} ka^4\right]$$

$$\Pi_{1,0} = \left[3.6281 \times 10^{-4} + 1.8141 \times 10^{-4}\frac{1}{P^2} + 3.4632 \times 10^{-4}\frac{1}{P^4} + 6.8714 \times 10^{-7} ka^4\right]$$

$$\Pi_{1,1} = \left[3.6281 \times 10^{-4} + 1.0994 \times 10^{-4}\frac{1}{P^2} + 1.2432 \times 10^{-4}\frac{1}{P^4} + 2.4667 \times 10^{-7} ka^4\right]$$

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$$\Pi_{1,2} = \left[9.8949 \times 10^{-5} + 4.5351 \times 10^{-5}\frac{1}{P^2} + 9.8949 \times 10^{-5}\frac{1}{P^4} + 1.8740 \times 10^{-7}ka^4\right]$$

$$\Pi_{2,0} = \left[3.4632 \times 10^{-4} + 1.8141 \times 10^{-4}\frac{1}{P^2} + 3.6281 \times 10^{-4}\frac{1}{P^4} + 6.8714 \times 10^{-7}ka^4\right]$$

$$\Pi_{2,1} = \left[9.8949 \times 10^{-5} + 4.5351 \times 10^{-5}\frac{1}{P^2} + 9.8949 \times 10^{-5}\frac{1}{P^4} + 1.8740 \times 10^{-7}ka^4\right]$$

$$\Pi_{2,2} = \left[1.2432 \times 10^{-4} + 1.0994 \times 10^{-4}\frac{1}{P^2} + 3.6281 \times 10^{-4}\frac{1}{P^4} + 3.1746 \times 10^{-4}ka^4\right]$$

$\Pi_{i,i}$ is a variable dependent on the aspect ratio,
p of the plate, modulus of subgrade reaction, k and the transverse section of the plate, a., we obtained values
for stiffness coefficients using Short span length, a = 5
m; thickness of plate; h = 0.15m, modulus of elasticity
of plate; E = 205 GPa, modulus of subgrade reaction;
k = 40 N/m^3 (5700 psi) for clayey medium dense sand
soil taken from reference Timoshenko, (1959).

Table 3.3: Stiffness Coefficients Values for CCCC Plate at varying aspect ratio

Aspect ratio, P	$\boldsymbol{\Pi_{i,i}}$	**0**	**1**	**2**
1	0	0.06625	0.01807	0.01807
	1	0.01807	0.00676	0.00493
	2	0.01807	0.00493	7.93711
1.1	0	0.06573	0.01793	0.01792
	1	0.01793	0.00671	0.00489
	2	0.01792	0.00489	7.93697
1.2	0	0.06537	0.01783	0.01783
	1	0.01783	0.00667	0.00486
	2	0.01783	0.00486	7.93688
1.3	0	0.06513	0.01777	0.01776
	1	0.01777	0.00664	0.00485
	2	0.01776	0.00485	7.93682
1.4	0	0.06496	0.01772	0.01771
	1	0.01772	0.00662	0.00483
	2	0.01771	0.00483	7.93678
1.5	0	0.06483	0.01769	0.01768
	1	0.01769	0.00660	0.00482
	2	0.01768	0.00482	7.93675
1.6	0	0.06474	0.01767	0.01765
	1	0.01767	0.00659	0.00482
	2	0.01765	0.00482	7.93673
1.7	0	0.06466	0.01765	0.01763
	1	0.01765	0.00658	0.00481
	2	0.01763	0.00481	7.93671
1.8	0	0.06460	0.01763	0.01762
	1	0.01763	0.00658	0.00481
	2	0.01762	0.00481	7.93670
1.9	0	0.06456	0.01762	0.01762
	1	0.01762	0.00657	0.00480
	2	0.01760	0.00480	7.93669
2.0	0	0.06452	0.01761	0.01759
	1	0.01761	0.00656	0.00480
	2	0.01759	0.00480	7.93668

The degree of freedom coefficients, Ki (K_o, K_1 and K_2) for any aspect ratio of the SSSS plate were then obtained by substituting these stiffness coefficients, $\Pi_{i,i}$ for that aspect ratio into Equation 2.49, For example, at aspect ratio,
P = 1.0; the stiffness coefficients $\Omega_{i,i}$ were given (see Table 3.3).
Substituting these stiffness coefficients into Equation 2.48 and subsequently solving the resulting canonical equation, gave;

$$\begin{bmatrix} k_o \\ k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} \Pi_{0,0} & \Pi_{0,1} & \Pi_{0,2} \\ \Pi_{1,0} & \Pi_{1,1} & \Pi_{1,2} \\ \Pi_{2,0} & \Pi_{2,1} & \Pi_{2,2} \end{bmatrix}^{-1} \begin{bmatrix} 1.1111 \times 10^{-3} \\ 3.1746 \times 10^{-4} \\ 3.1746 \times 10^{-4} \end{bmatrix} \frac{q}{D} a^4$$

$$\begin{bmatrix} k_o \\ k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 0.06625 & 0.01807 & 0.01807 \\ 0.01807 & 0.00676 & 0.00493 \\ 0.01807 & 0.00493 & 7.93711 \end{bmatrix}^{-1} \begin{bmatrix} 1.1111 \times 10^{-3} \\ 3.1746 \times 10^{-4} \\ 3.1746 \times 10^{-4} \end{bmatrix} \frac{q}{D} a^4$$

$$= \begin{bmatrix} 1.463 \times 10^{-2} \\ 7.860 \times 10^{-3} \\ 1.819 \times 10^{-6} \end{bmatrix} \frac{q}{D} a^4 3.2$$

The degree of freedom coefficients, k_j (k_o, k_1 and k_2) for other aspect ratios of the CCCC plate were obtained by similar approach and their results are shown in Table 3.4
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Table 3.4: **Typical deflection coefficients of CCCC plate on elastic foundation at varying aspect ratio**

Aspect ratio, P	$\Pi_{i,i}$	**0**	**1**	**2**
1	0	0.06625	0.01807	0.01807
	1	0.01807	0.00676	0.00493
	2	0.01807	0.00493	7.93711
1.1	0	0.06573	0.01793	0.01792
	1	0.01793	0.00671	0.00489
	2	0.01792	0.00489	7.93697
1.2	0	0.06537	0.01783	0.01783
	1	0.01783	0.00667	0.00486
	2	0.01783	0.00486	7.93688
1.3	0	0.06513	0.01777	0.01776
	1	0.01777	0.00664	0.00485
	2	0.01776	0.00485	7.93682
1.4	0	0.06496	0.01772	0.01771
	1	0.01772	0.00662	0.00483
	2	0.01771	0.00483	7.93678
1.5	0	0.06483	0.01769	0.01768
	1	0.01769	0.00660	0.00482
	2	0.01768	0.00482	7.93675
1.6	0	0.06474	0.01767	0.01765
	1	0.01767	0.00659	0.00482
	2	0.01765	0.00482	7.93673
1.7	0	0.06466	0.01765	0.01763
	1	0.01765	0.00658	0.00481
	2	0.01763	0.00481	7.93671
1.8	0	0.06460	0.01763	0.01762
	1	0.01763	0.00658	0.00481
	2	0.01762	0.00481	7.93670
1.9	0	0.06456	0.01762	0.01762
	1	0.01762	0.00657	0.00480
	2	0.01760	0.00480	7.93669
2.0	0	0.06452	0.01761	0.01759
	1	0.01761	0.00656	0.00480
	2	0.01759	0.00480	7.93668

Where, the complete expression of the third degree of freedom deflection expression for plate is given in Equation 3.35 as:

$W(R,Q) = k_0(R^2 - 2R^3 + R^4)(Q^2 - 2Q^3 +$
$Q4 + k1R4-2R5+R6Q2-2Q3+Q4+$

$k_2(R^2 - 2R^3 + R^4)(Q^4 - 2Q^5 + Q^6)$.

3.1.3 OPPOSITE EDGES SIMPLY SUPPORTED AND CLAMPED THINRECTANGULAR PLATE (CSCS)

The stiffness coefficients were given in Equations 3.121 to 3.1106 as:

$$\Pi_{0,0} = \left[7.6190\times10^{-3} + 1.8503\times10^{-2}\frac{1}{P^2} + 3.9365\times10^{-2}\frac{1}{P^4} + 4.2958\times10^{-4}ka^4\right]$$

$$\Pi_{0,1} = \left[-9.0703\times10^{-4} + 5.2608\times10^{-3}\frac{1}{P^2} + 1.1140\times10^{-2}\frac{1}{P^4} + 1.2157\times10^{-4}ka^4\right]$$

$$\Pi_{0,2} = \left[2.0779\times10^{-3} + 4.6259\times10^{-3}\frac{1}{P^2} + 1.1247\times10^{-2}\frac{1}{P^4} + 1.1893\times10^{-4}ka^4\right]$$

$$\Pi_{1,0} = \left[-9.0703\times10^{-4} + 5.2608\times10^{-3}\frac{1}{P^2} + 1.1140\times10^{-2}\frac{1}{P^4} + 1.2157\times10^{-4}ka^4\right]$$

$$\Pi_{1,1} = \left[9.0703\times10^{-4} + 4.2823\times10^{-3}\frac{1}{P^2} + 4.5110\times10^{-3}\frac{1}{P^4} + 4.9228\times10^{-5}ka^4\right]$$

$$\Pi_{1,2} = \left[-2.4737\times10^{-4} + 1.3152\times10^{-3}\frac{1}{P^2} + 3.1828\times10^{-3}\frac{1}{P^4} + 3.3657\times10^{-5}ka^4\right]$$

$$\Pi_{2,0} = \left[2.0779\times1 + 4.6259\times10^{-3}\frac{1}{P^2} + 1.1247\times10^{-2}\frac{1}{P^4} + 1.1893\times10^{-4}ka^4\right]$$

$$\Pi_{2,1} = \left[-2.4737\times10^{-4} + 1.3152\times10^{-3}\frac{1}{P^2} + 3.1828\times10^{-3}\frac{1}{P^4} + 3.3657\times10^{-5}ka^4\right]$$

$$\Pi_{2,2} = \left[7.4592\times10^{-4} + 2.8035\times10^{-3}\frac{1}{P^2} + 1.1247\times10^{-2}\frac{1}{P^4} + 1.1905\times10^{-2}ka^4\right]$$

$\Pi_{i,i}$ is a variable dependent on the aspect ratio, p of the plate, modulus of subgrade reaction, k and the transverse section of the plate, a., we obtained v alues forstiffness coefficients using Short span

length, a = 5 m; thickness of plate; h = 0.15m, modulus of elasticity of plate;
E = 205 GPa, modulus of subgrade reaction;
k = 40 N/m^3 (5700 psi) for clayey medium dense sand soil taken from reference Timoshenko, (1959).

Table 3.5: Stiffness Coefficients Values for CSCS Plate at varying aspect ratio

Aspect ratio, P	$\Pi_{i,i}$	**0**	**1**	**2**
1	0	10.8050	3.0547	2.9913
	1	3.0547	1.2404	0.8457
	2	2.9913	0.8457	297.6338
1.1	0	10.7893	3.0502	2.9869
	1	3.0502	1.2382	0.8444
	2	2.9869	0.8444	297.6298
1.2	0	10.7789	3.0473	2.9840
	1	3.0473	1.2367	0.8436
	2	2.9840	0.8436	297.6272
1.3	0	10.7718	3.0453	2.9821
	1	3.0453	1.2357	0.8431
	2	2.9821	0.8431	297.6254
1.4	0	10.7668	3.0439	2.9807
	1	3.0439	1.2350	0.8427
	2	2.9807	0.8427	297.6242
1.5	0	10.7631	3.0428	2.9797
	1	3.0428	1.2344	0.8424
	2	2.9797	0.8424	297.6233
1.6	0	10.7603	3.0420	2.9789
	1	3.0420	1.2340	0.8422
	2	2.9789	0.8422	297.6226
1.7	0	10.7582	3.0414	2.9784
	1	3.0414	1.2336	0.8420
	2	2.9784	0.8420	297.6221
1.8	0	10.7566	3.0410	2.9779
	1	3.0410	1.2334	0.8419
	2	2.9779	0.8419	297.6217
1.9	0	10.7552	3.0406	2.9747
	1	3.0406	1.2331	0.8418
	2	2.9775	0.8418	297.6214
2.0	0	10.7542	3.0403	2.9773
	1	3.0403	1.2330	0.8417
	2	2.9773	0.8417	297.6212

they were then obtained by substituting these stiffness coefficients, $\Pi_{i,i}$ for that aspect ratio into Equation 2.49

For example, at aspect ratio, P = 1.0; the stiffness coefficients $\Omega_{i,i}$ were given (see Table 4.5). Substituting these stiffness coefficients into Equation 2.48 and subsequently solving the resulting canonical equation, gave;

$$\begin{bmatrix} k_o \\ k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} \Pi_{0,0} & \Pi_{0,1} & \Pi_{0,2} \\ \Pi_{1,0} & \Pi_{1,1} & \Pi_{1,2} \\ \Pi_{2,0} & \Pi_{2,1} & \Pi_{2,2} \end{bmatrix}^{-1} \begin{bmatrix} 6.6667 \times 10^{-3} \\ 1.9841 \times 10^{-3} \\ 1.9048 \times 10^{-3} \end{bmatrix} \frac{q}{D} a^4$$

$$\begin{bmatrix} k_o \\ k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 10.8050 & 3.0547 & 2.9913 \\ 3.0547 & 1.2404 & 0.8457 \\ 2.9913 & 0.8457 & 297.6338 \end{bmatrix}^{-1} \begin{bmatrix} 6.6667 \times 10^{-3} \\ 1.9841 \times 10^{-3} \\ 1.9048 \times 10^{-3} \end{bmatrix} \frac{q}{D} a^4$$

$$= \begin{bmatrix} 5.424 \times 10^{-4} \\ 2.638 \times 10^{-4} \\ 1.993 \times 10^{-7} \end{bmatrix} \frac{q}{D} a^4 3.2$$

The degree of freedom coefficients, K_i (k_o, k_1 and k_2) for other aspect ratios of the CCCC plate were obtained by similar approach and their results are shown in Table 3.6.

Table 3.6: **Typical deflection coefficients of CSCS plate on elastic foundation at varying aspect ratio**

Aspect ratio, P	k_o	k_1	k_2
1.0	5.424E-04	2.638E-04	1.993E-07
1.1	5.431E-04	2.644E-04	1.993E-07
1.2	5.435E-04	2.649E-04	1.993E-07
1.3	5.439E-04	2.652E-04	1.993E-07
1.4	5.441E-04	2.655E-04	1.993E-07
1.5	5.442E-04	2.657E-04	1.993E-07
1.6	5.444E-04	2.658E-04	1.992E-07
1.7	5.444E-04	2.660E-04	1.992E-07
1.8	5.445E-04	2.661E-04	1.992E-07
1.9	5.446E-04	2.661E-04	1.992E-07
2.0	5.446E-04	2.662E-04	1.992E-07

Where, the complete expression of the third degree of freedom deflection expression for plate were given in as:

$$W(R,Q) = k_0(R - 2R^3 + R^4)(Q^2 - 2Q^3 + Q^4) + k_1(R^3 - 2R^5 + R^6)(Q^2 - 2Q^3 + Q^4) + k_2\,(R - 2R^3 + R^4\,)(Q^4 - 2Q^5 + Q^6)$$

3.2 EXPRESSIONS OF DEFLECTIONS AND MOMENTS

In the deflection expressions that follow, the symbol, W_i shall be used to represent the functions of R, Q, and P. That is:

$$W(R,Q) = W_i \frac{q}{D} a^4 = \frac{q}{D} a^4 \sum k_i\,(R,Q) \quad for\ i = 0, 1, 2 \qquad 3.3$$

$Where,$
$W_i = \sum k_i\,(R, Q)\ for\ i = 0, 1, 2$ 3.4

3.2.1 ALL EDGES SIMPLY SUPPORTED PLATE (TYPE SSSS)

The second-degree v deflection function of the plate obtained in Equation 3.39 is given as:

$$W(R,Q) = k_0(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4) + k_1(R^3 - 2R^5 + R^6)(Q - 2Q^3 + Q^4) + k_2(R - 2R^3 + R^4)(Q^3 - 2Q^5 + Q^6)$$

But at mid span of plate for aspect ratio, P = 1,$i = 3$; $W_i = W_3$; $R = Q = \frac{1}{2}$; $K_O = 0.000568$; $K_1 = 0.000290$; $and\ K_2 = 0.000002$

$$W_3 = k_o(R - 2R^3 + R^4)(Q - 2Q^3 + Q^4) + k_1(R^3 - 2R^5 + R^6)(Q - 2Q^3 + Q^4) + k_2(R - 2R^3 + R^4)(Q^3 - 2Q^5 + Q^6)$$

$$W_3 = 0.00056\left(\frac{1}{2} - 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4\right)\left(\frac{1}{2} - 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4\right) +$$
$$0.000290\left(\left(\left(\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^6\right)\left(\frac{1}{2} - 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4\right)\right) +$$
$$0.000002\left(\left(\frac{1}{2} - 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4\right)\left(\left(\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right)^5 + \right.$$

126=6.264×10−5**3.3**

Using the values of k_i for different aspect ratios in Table 3.2, the midspan deflection coefficients, $W_i$for SSSS plate is shown in Table 3.7 for different degrees of deflections and aspect ratio.

According to Timoshenko and Woinowsky - Kriger (1959), the deflection for SSSS plate on elastic foundation is given as:

$$w = \frac{16q}{\pi^2}\sum_{m=1,3,5,\dots}^{\infty}\ \sum_{n=1,3,5,\dots}^{\infty}\frac{\sin\frac{m\pi x}{a}\sin\frac{n\pi x}{b}}{mn\left[\pi^4 D\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2 + k\right]} \quad \mathbf{3.4}$$

For the first term, m = n = 1, the Equation reduces as follows:

$$w = \frac{16q}{\pi^2}\frac{\sin\frac{\pi x}{a}\sin\frac{\pi x}{b}}{\left[\pi^4 D\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2 + k\right]} = \frac{16q}{\pi^2}\frac{\sin\frac{\pi x}{a}\sin\frac{\pi x}{b}}{\left[\pi^4 D\left(\frac{1}{a^2}+\frac{1}{b^2}\right)^2 + k\right]} \quad \mathbf{3.5}$$

Introducing the aspect ratio, p, and simplifying and expressing in non – dimensional form, we have:

$x = Ra$; y = Qb

we have;

$$p = \frac{b}{a};\quad ap = b$$

$$w = A_{11}\sin\frac{\pi Ra}{a}\sin\frac{\pi Qb}{b} = A_{11}\sin\pi R\sin\pi Q \quad \mathbf{3.6}$$

Figure 4.1 shows the plot of these coefficients of deflections for different degrees of freedom with that of Timoshenko and Woinowsky - Kriger (1959) for different aspect ratios.

Tables 3.7, 3.8, 3.9 and Figures 3.1, 3.2, and 3.3 are these coefficients of deflections for different degrees of freedom at aspect ratio of unity along the non – dimensional short span of the slab, $0 \le R \le 1$.

Table 3.7: Midspan deflection Coefficients Values for SSSS Plate at varying aspect ratio

Aspect ratio, P	Deflection coefficient, W_i		
	1st degree deflection coefficients, $W_1 \times 10^{-5}$	2nd degree deflection coefficients, $W_2 \times 10^{-5}$	Timoshenko deflection coefficients, $W_T \times 10^{-5}$
1	5.5516	6.2590	6.38504
1.1	5.5624	6.2748	6.39376
1.2	5.5697	6.2859	6.40073
1.3	5.5749	6.2939	6.40642
1.4	5.5786	6.3000	6.41114
1.5	5.5815	6.3046	6.41512
1.6	5.5836	6.3082	6.41852
1.7	5.5853	6.3111	6.42144
1.8	5.5867	6.3135	6.42399
1.9	5.5879	6.3154	6.42622
2.0	5.5887	6.3219	6.42800

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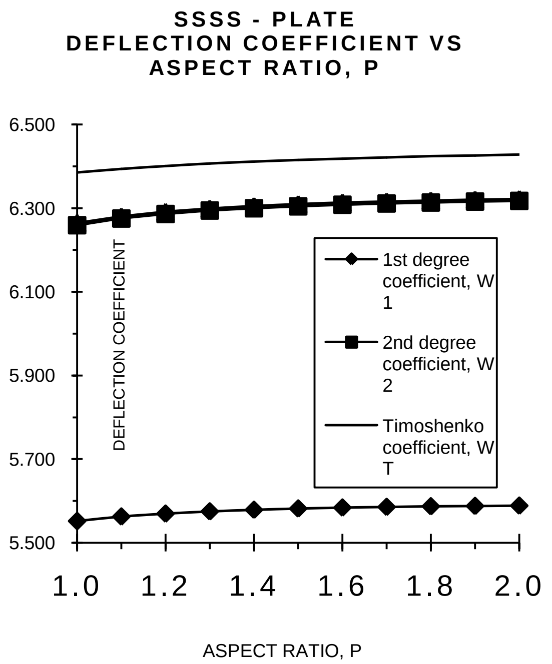

Figure 3.1 Plot of Midspan deflection Coefficients Values for SSSS Plate at varying aspect ratio

Similarly, the deflection for other type of plate is solved and gotten as shown over leaf

Table 3.8: Midspan deflection Coefficients Values for CCCC Plate at varying aspect ratio

Aspect ratio, P	1st degree coefficient, W_1 $\times 10^{-5}$	2nd degree coefficient, W_2 $\times 10^{-5}$
1	5.7134	6.4810
1.1	5.7592	6.5330
1.2	5.7899	6.5680
1.3	5.8113	6.5923
1.4	5.8265	6.6098
1.5	5.8378	6.6227
1.6	5.8463	6.6325
1.7	5.8528	6.6401
1.8	5.8579	6.6460
1.9	5.8620	6.6508
2	5.8653	6.6546

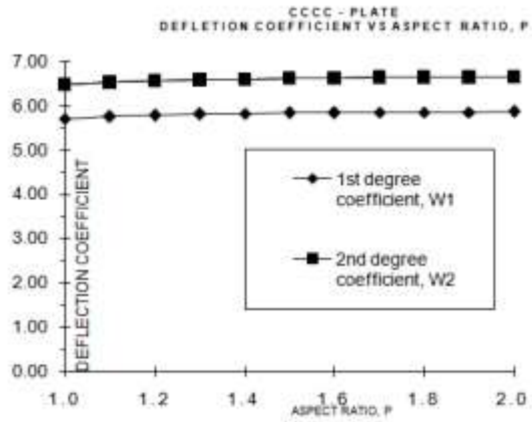

Figure 3.2: Plot of Midspan deflection Coefficients Values for CCCC Plate at varying aspect ratio

Table 3.9: Midspan deflection Coefficients Values for CSCS Plate at varying aspect ratio

Aspect ratio, P	**Deflection coefficient, W_i**	
	1st degree coefficient, W_1 $\times 10^{-3}$	2nd degree coefficient, W_2 $\times 10^{-3}$
1	1.0593	1.1881
1.1	1.0607	1.1898
1.2	1.0616	1.1910
1.3	1.0622	1.1917
1.4	1.0627	1.1923
1.5	1.0630	1.1927
1.6	1.0632	1.1930
1.7	1.0634	1.1932
1.8	1.0635	1.1934
1.9	1.0636	1.1935
2.0	1.0637	1.1937

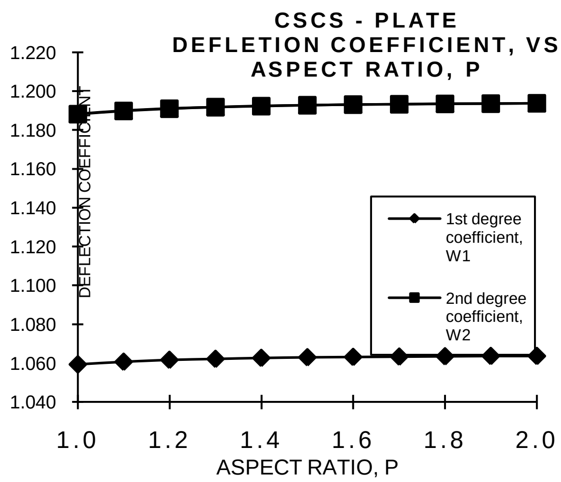

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Figure 3.3: Midspan deflection Coefficients Values for CSCS Plate at varying aspect ratio

3.2.2 EXPRESSIONS OF MOMENTS

The values of deflection coefficients K_i gotten are then substituted into the non – dimensional bending moment coefficients, ψ_{xi} and ψ_{yi} expressions in Equations 3.53, and 3.55. This gave the bending moment coefficients ψ_{xi} and ψ_{yi} as represented graphically for the various support conditions as follows:

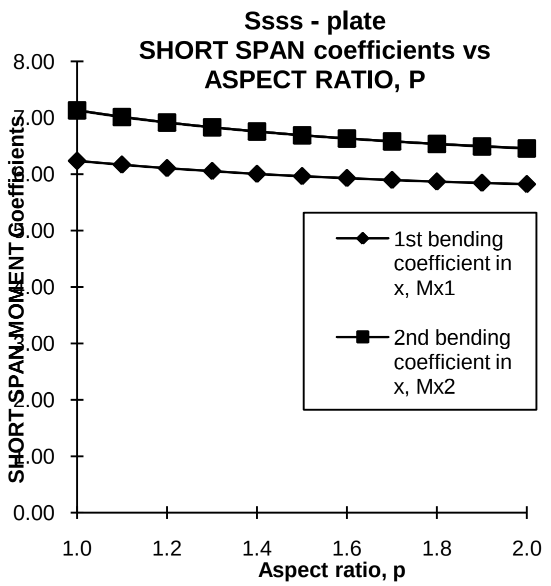

Figure 3.4: Plot of bending moment Coefficients Values of the degrees of freedom for SSSS Plate against the aspect ratio, p

This procedure is repeated for CCCC and CSCS., and their respective bending moment coefficients, ψ_{xi} and ψ_{yi} at unit aspect ratio along the short span are graphically shown in figure 3.5 and 3.6.

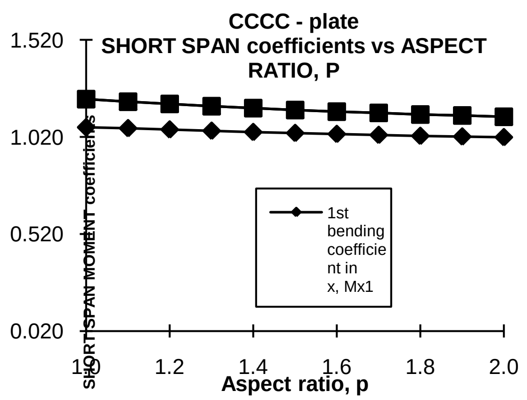

Figure 3.5: Plot of bending moment Coefficients Values in x – direction of the degrees of freedom for CCCC Plate against the aspect ratio, p

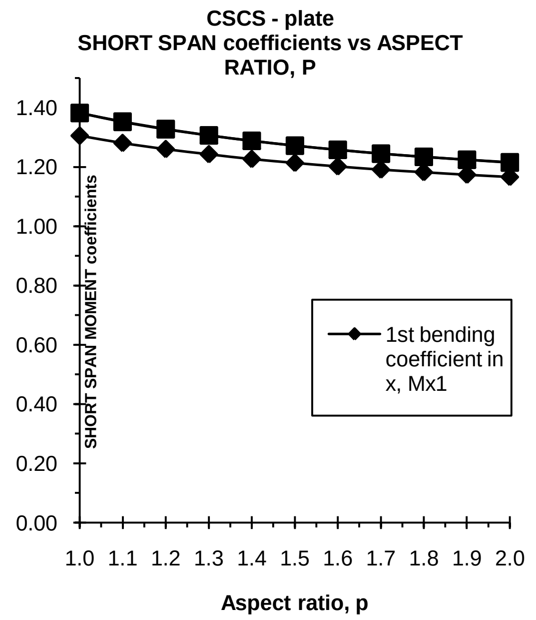

Figure 3.6: Plot of bending moment Coefficients Values of the degrees of freedom for CSCS Plate at unit aspect ratio along the short span,

3.3. DISCUSSIONS ON SSSS, CCC, AND CSCS PLATE RESULTS

Tables 3.1, 3.2, and 3.3 shows the results for the coefficients of maximum deflections at varying aspect ratios for SSSS, CCCC and CSCS thin rectangular plates. and figures 3.4, 3.5, and 3.6 shows diagrammatically bending moments obtained for the first and second degrees of freedom at varying aspect ratios for SSSS, CCCC and CSCS thin rectangular plates. From our gotten results, the higher degrees of freedom functionals would result to upper bound solutions whereas first or single degree of freedom deflection functional gave lower bound results in relation to Timoshenko and Woinowsky - Kriger (1959). Therefore, there is closeness of values gotten as the aspect ratio increases at 0.1 range margin, when compared with the Timoshenko and Woinowsky - Kriger (1959). This suggests that higher degrees of freedom deflection functions would lead to a better approximation of the deflection functions.

IV. CONCLUSION

This Research has analyzed thin rectangular plates of various support conditions at the edges using work principle method. This was obtained by applying the characteristic orthogonal polynomials after the shape functionals of various support conditions of thin rectangular plates has been derived. The satisfaction of the boundary conditions by the shape functions is an indication that there is improvement in approximation and suitability. Due to the inability of other variational methods such as Galerkin's and Ritz to analysis higher degrees of freedom of deflection functionals, Work principles method was adopted to solve the governing differential equation the plate. The resulting work error equation was minimized to enable the complete solutions of the three coefficients of third degree of freedom deflection functional. Ritz and Galerkin methods, though they are called variational method are not adequate for solution elastic plates' problem with multi – degree of free freedom deflection functional. Comparative analysis of this was also carried out with famous results of previous work as yardstick. The results showed better and faster approximation compared with Timoshenko et al (1959) and Oguejiofor (2017).

In conclusion;

a) Utilizing characteristic orthogonal polynomials, we develop shape functions for rectangular plates with increased degrees of freedom. These functions effectively approximate the deformed shape of thin rectangular plates under various boundary conditions.

b) The findings of this investigation align closely with existing literature, exhibiting an upper bound trend. Despite this, they offer a dependable basis for design, prioritizing safety and simplicity.

c) Ritz and Galerkin method are not suitable for solving thin rectangular plates' problem of multi – degrees of freedom. Ritz method could make reasonable advance in a multi – degree of free problem but would be rigorous and lengthy, while Galerkin's method is completely impossible.

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