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Analysis of the Steady State of Ergodicity of Embedded Morkov Chain- Queuing Models

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ABSTRACT:
Queuing system with repeated request has many useful applications in communication and computer system modeling. The main characteristics of the single server queue with repeated request is that, a customer who finds the server busy upon arrival return to the system after a random amount of time. Most queuing system with repeated orders are characterized by the following features, if the server is free then the arriving customer enter service, if the server is occupied the customer must leave the service area and enter a pool of unsatisfied customers. An arbitrary customer in the pool generates a system of repeated requests that is independent of the customer in the pool.

I. INTRODUCTION:

A computer network in which a host computer wishes to send a message to another computer. If the transition medium is available, the hot computer immediately sends the message; otherwise the message will be stored in a buffer and the host has to try again sometime later.

1.1.1 Mathematical Model:

We considered queuing system in which arrival is according to Poisson stream with rate $\lambda > 0$. Any customer who finds the server busy upon arrival leaves the service area and joins a pool of unsatisfied customer.

The control policy to access from this pool to the server is governed by an exponential law with linear intensity $\vartheta_n = \alpha(1-\delta_{n0}) + n\vartheta$, when the pool size is $n\epsilon N$. the service times are general with probability distribution function B (x) and mean $\beta_1 < \infty$. The input stream of arrival service times and internals between successive repeat requests are assumed to be mutually independent. Also, let $\beta(s)$ be the corresponding Laplace-Stieltjes transform.

The state of the system can be described by the process $X(t) = \{C(t), N(t), \xi(t)\}$, where $C(t)$ denotes the state of the server as 0 and 1 according to whether the server is free or busy, $N(t)$ the number of unsatisfied customers at time t and if $C(t) = 1$ then, $\xi(t)$ represents the elapsed time of the customer being served. We neglect $\xi(t)$ and consider only the pair $\{C(t), N(t),\}$ whose state space is $S = \{0,1\} * N$.

We study the necessary and sufficient condition for the system to be stable. To see this, we investigate the ergodicity of the embedded markov chain at the departure epochs. Let N_n be the number of the customers in the pool at the time of the n^{th} departure

The fundamental equation, $N_n = N_{n-1} - B_n + V_n$, where V_n is the number of customers arriving the n^{th} service time, and

$$V_n = \begin{cases} 1, \text{if the customer from the pool gets } n^{th} \text{serve} \\ 0, \ \text{otherwise} \end{cases}$$

Now $\{N_n, n\epsilon N\}$ is irreducible and aperiodic. To prove Ergodicity 'i' we derive Foster's condition.

1.1.2Ergodicity of Embedded Markov Chain Foster's condition

An irreducible and aperiodic Markov chain is ergodic if there exist a non-negative function ,$s\epsilon N$ and $\varepsilon > 0$ such that the mean drift f(s)

$$\varphi_s = E[f(N_{n+1}) - f(N_n) \setminus N_n = s$$

is finite for all $s\epsilon N$ and $\varphi_s \leq -\epsilon, \forall\, s\epsilon N$ except a finite number. First we consider the functionf(s) = s, then obtain,

$$\varphi_j = \begin{cases} \gamma, & j = 0 \\ \gamma - \frac{\alpha + j\nu\alpha}{\lambda + \alpha + j\nu}, & j = 1,2 \ldots \end{cases}$$

Where $\gamma = \alpha\beta_1$, clearly if γ satisfies the inequality

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$$\gamma < 1 - \frac{\lambda}{\lambda+\alpha}\delta_{0v}$$

$\lim_{j\to\infty}\varphi_j < 0$. Therefore the embedded Markov chain $\{N_n, n\epsilon N\}$ is ergodic. The same condition is also necessary for ergodicity. The proof follows by Sennot et al[58]., which states that it can quarantee non-ergodicity if $\{N_n, n\epsilon N\}$ satisfies Kaplan's condition,

$\varphi_j < +\infty$ $(j \geq 0)$ and there is a j_0 such that $\varphi_j \geq 0$ $(j \geq j_0)$. $P_{ij}=0$ $(j < i - k, i < 0)$ where $P = P_{ij}$ $i, j = 0,1,2 \ldots$ is the transition matrix associated to $\{N_n, n\epsilon N\}$. Since the arrival stream is poisson process, we use Burke's theorem given by cooper[21].

1.1.3 Analysis of the Steady state

Let $P_i(z)$ be the generating function of the sequence P_{ij}, $j \in N$, for $i \in (0,1)$. we also consider the number of customer $K(t)$ in the system in steady state whose generating function is given by

$$k_j = P_r[Y = j]$$
$$k_0 = P_{00}$$
$$k_1 = P_{01} + P_{10}$$
$$k_2 = P_{02} + P_{ii}$$
$$\ldots\ldots\ldots\ldots$$
$$k_j = P_{ij} + P_{ij-1\ldots\ldots}(j \geq 1).$$

The generating function of the customers in system in steady state is given by

$k(z) = \sum_{j=1}^{\infty}(P_{0j} + P_{1j-1})z^j = P_0(z) + zP_1(z)$, for $|z| \leq 1$.

We denote $Q(z)$ the generating function of the number of customers present in the system in the standard M/G/1 queue. This is given by

$$Q(z) = (1-\gamma)\frac{(1-z)\beta(\lambda-\lambda z)}{\beta(\lambda-\lambda z)-z}, \text{ for } |z| \leq 1.$$

Where $\beta(z)$ is the Laplace –Stieljes transform defined by

$$\beta(z) = \int_0^{\infty} e^{-st}\, d\beta(t), S > 0$$

II. CONCLUSION:

In this model, we analyzed the simplifying assumption that the queue for the station I and II are inexhaustible. After completion of the service I and II, they exit the common gate G, the gate will be occupied by the customers those who have completed the II and I service. After the service completion, the customer go to orbit. The service gate will be free if number of retrial customers go to exit gate.

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