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Another Representation of Riemann Zeta Function and Its Non-Trivial Zero

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Let
$S(s)= 1^{-s}$ (a_0)
Multiplying equation (a_0) by 2^{-s}both side we get

$2^{-s} S(s) = 2^{-s}$ (a_1)
Adding (a_0) and (a_1) we get

$S(s)(1+2^{-s})= 1+2^{-s}$ (a_2)

Multiplying eq.(a_2) by 3^{-s} we get

$S(s)(1+2^{-s})3^{-s} = 3^{-s}+ 6^{-s}$ (a_3)

Adding (a_2) and (a_3) we get
$S(s)(1+2^{-s})(1+3^{-s}) = 1+2^{-s}+3^{-s}+ 6^{-s}$ (a_4)

Multiplying eq.(a_4) by 5^{-s} we get

$S(s)(1+2^{-s})(1+3^{-s})5^{-s} = 5^{-s}+10^{-s}+15^{-s} +30^{-s}$ (a_5)

Adding (a_4) and (a_5) we get
$S(s)(1+2^{-s})(1+3^{-s})(1+5^{-s}) = 1+2^{-s}+3^{-s}+5^{-s}+6^{-s} +10^{-s}+15^{-s} +30^{-s}$ (a_6)

And by this way we can write for all primes we get following expression

$S(s)(1+2^{-s})(1+3^{-s})(1+5^{-s})\ldots\ldots(1+p^{-s})= 1+2^{-s}+3^{-s}+5^{-s}+\ldots\ldots.(2.3.5\ldots.p)^{-s}$ (a_n)

Multiplying eq.(a_n) by 2^{-2s} we get

$S(s)(1+2^{-s})(1+3^{-s})(1+5^{-s})\ldots\ldots(1+p^{-s})2^{-2s} = [1+2^{-s}+3^{-s}+5^{-s}+\ldots\ldots.(2.3.5\ldots.p)^{-s}]2^{-2s}$ (b_1)

Adding (a_n) and (b_1) we get

$S(s)(1+2^{-s})(1+3^{-s})(1+5^{-s})\ldots\ldots(1+p^{-s})(1+ 2^{-2s}) = [1+2^{-s}+3^{-s}+ 4^{-s}+5^{-s}+\ldots\ldots.2^{-2s}(2.3.5\ldots.p)^{-s}]$ (b_2)

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And by this way we can write for all primes we get following expression

$S(s)(1+2^{-s})(1+3^{-s})(1+5^{-s})\ldots\ldots(1+p^{-s})(1+ 2^{-2s})(1+3^{-2s})\ldots..(1+p^{-2s}) = [1+2^{-s}+3^{-s} + 4^{-s} +5^{-s}+\ldots\ldots.2^{-2s}(2.3.5\ldots.p)^{-s}\ldots\ldots]$ (b_n)

Multiplying eq.(b_n) by 2^{-4s} we get

$S(s)(1+2^{-s})(1+3^{-s})(1+5^{-s})\ldots\ldots(1+p^{-s})(1+ 2^{-2s})(1+3^{-2s})\ldots..(1+p^{-2s})2^{-4s} =[1+2^{-s}+3^{-s} + 4^{-s} +5^{-s}+\ldots\ldots.2^{-2s}(2.3.5\ldots.p)^{-s}\ldots\ldots] 2^{-4s}$ (c_1)

Adding (b_n) and (c_1) we get

$S(s)(1+2^{-s})(1+3^{-s})(1+5^{-s})\ldots\ldots(1+p^{-s})(1+ 2^{-2s})(1+3^{-2s})\ldots..(1+p^{-2s})(1+2^{-4s}) = [1+2^{-s}+3^{-s} + 4^{-s} +5^{-s}+\ldots\ldots.]$ (c_2)

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And by this way we can write for all primes we get following expression

$S(s)(1+2^{-s})(1+3^{-s})(1+5^{-s})\ldots\ldots(1+p^{-s})(1+ 2^{-2s})(1+3^{-2s})\ldots..(1+p^{-2s})(1+2^{-4s})\ldots(1+p^{-4s}) = 1+2^{-s}+3^{-s} + 4^{-s} +5^{-s} +\ldots\ldots.$ (c_n)

n^{th}term of this sequence is given by following expression

$S(s\ )(1+3^{-s})\ldots(1+p^{-s})\ (1+2^{-2s})\ldots(1+p^{-2s})\ (1+2^{-4s})\ldots(1+p^{-4s})\ldots(1+2^{-s})\ldots(_{1+[p]}^{-}(2)^n s) = 1+2^{-s}+3^{-s}+4^{-s}\ldots n^{-s}$ (n^{th})

Taking limit $[n\rightarrow\infty]$ both side of equation $[n^{th}]$ and taking all infinity primes we will get.

R.H.S.= $\sum_{n=1}^{\infty}(\boldsymbol{n}^{-s})$

L.H.S.=$\prod_{n=0}^{\infty} \prod_{P} [\ \mathbf{1} + P^{-2^n s}\ ]$

Now

R.H.S.=L.H.S.

$\sum_{n=1}^{\infty} (\boldsymbol{n^{-s}}) = \prod_{n=0}^{\infty} \prod_{P} [\ \mathbf{1} + P^{-2^n s}\ ]$

As we know that Riemann zeta function having Following expression.
Riemann zeta function = $\sum_{n=1}^{\infty} n^{-s}$

Or
Riemann zeta function = ζ(s) =$\sum_{n=1}^{\infty} n^{-s}$
Hence

ζ(s) =$\sum_{n=1}^{\infty} n^{-s} = \prod_{n=0}^{\infty} \prod_{P} [\ \mathbf{1} + P^{-2^n s}\ ]$

or

ζ(s) =$\prod_{n=0}^{\infty} \prod_{P} [\ \mathbf{1} + P^{-2^n s}\ ]$
now

Generating function of Riemann zeta function.

ψ(s,q) =$\prod_{n=0}^{\infty} \prod_{P} [\sum_{q=1}^{q} (\ \mathbf{1} + P^{-(m)^n sq})]$

Were m=q+1 q= 1,2,3,4,5,6…….

m is a fixed quantity for each value of q were q= 1,2,3,4,5,6…….

Example for q=1 we get following representation of Riemann zeta function.

ψ(s,q=1) = ζ(s) =$\prod_{n=0}^{\infty} \prod_{P} [\ \mathbf{1} + P^{-2^n s}\ ]$

zero of Riemann zeta function ζ(s)

ζ(s)=0

or$\prod_{n=0}^{\infty} \prod_{P} [\ \mathbf{1} + P^{-2^n s}\ ] = \mathbf{0}$

$[\ 1 + P^{-2^n s}\ ]=0$

$P^{-2^n s} = -1$

As we know that limit x→0 xln(x)=0

$P^{-2^n s} = e^{\pm i(2k-1)\pi}$ were k=1,2,3,4,5……

We get

S=[$\pm i(2k-1)\pi$]/[(2^n) ln(P)]

This is non-trivial roots of Riemann zeta function whose real part is zero.

If we take n=0 and p=2, k=1 we will get

S=[$\pm i\pi$]/[ln(P)] were p=prime numbers

S=[$\pm i\pi$]/[ln(2)] =$\pm i$(4.532…)

If we take n=0 and p=2 ,k=2 we will get

S=[$\pm i3\pi$]/[ln(2)] =$\pm i$(13.597…)
If we take n=0 and p=3, k=3 we will get

S=[±i5π]/[ln(3)] =±i(14.298…)

If we take n=0 and p=3,k=4 we will get

S=[±i7π]/[ln(3)] =±i(20.017…)

If we take n=4 and p=3,k=18 we will get

S=[±i35π]/[(4)ln(3)] =±i(25.0215…)

If we take n=0 and p=17,k=10 we will get

S=[±i19π]/[ln(17)] =±i (21.0680…)

Note [1] :Here we can see that real part of roots are equal to zero and imaginary part are very close to imaginary part (t) of those roots which are in form of [(1/2) + it] which are approximated by using Jensen polynomials of Riemann zeta function.
[2]:As we can see that there are infinite many non –trivial roots of Riemann zeta function whose real part may or may not be equal to zero
According to value of [q].