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Application of Wave Equation Models to the Dynamics, Spread, and Control of Youth Drug Addiction in Nigeria

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ABSTRACT
Youth drug addiction is a rising concern in schools and communities. This work develops a mathematical model that combines a damped wave equation for the spread of peer influence with reaction–diffusion equations for student populations—susceptible, users, and rehabilitated. The model shows how influence waves can trigger drug surges even when baseline risk is low, and identifies key parameters such as wave speed, damping, and intervention strength. An optimal control framework is introduced to evaluate targeted awareness and rehabilitation strategies. Numerical results illustrate how interventions can suppress wave-driven outbreaks and reduce overall prevalence. The model provides a tool for understanding and mitigating drug abuse among youths.

KEYWORDS: youth drug addiction, wave equation, peer influence, reaction–diffusion, optimal control.

I. INTRODUCTION

Youth involvement in hard drugs has become a major public health, social, and educational challenge across many developing nations, particularly in Nigeria. Reports indicate that the prevalence of drug abuse among secondary and tertiary students is rising, with severe consequences on academic performance, mental health, and crime [1,2]. Peer pressure, availability of substances, and social media exposure contribute significantly to the initiation and persistence of drug use among students [3]. Consequently, mathematical models are increasingly being used to study the dynamics of drug addiction, evaluate policy strategies, and provide insights into effective control measures [4–6].

Classical epidemiological models, such as compartmental SIR-type frameworks, have been adapted to describe transitions between susceptible individuals, active drug users, and rehabilitated groups [7,8]. These models capture the temporal dynamics of addiction but often fail to incorporate spatial heterogeneity and the rapid spread of influence through peer networks and media. Yet, empirical studies suggest that the rise and fall of drug-use trends among youths resemble "waves" of social influence, propagating with finite speed across campuses and communities [9]. This motivates the integration of wave-based models with traditional reaction–diffusion equations.

In this study, we develop a spatial–temporal mathematical model that couples **a** damped wave (telegraph) equation for the spread of influence with reaction–diffusion dynamics for youth subpopulations. The model partitions the student population into three classes: susceptible or vulnerable (S), active users (U), and rehabilitated (R). The influence field ($P(x,t)$) represents peer pressure, social exposure, and availability signals that spread as waves, while intervention ($I(x,t)$) captures awareness campaigns, counseling, and enforcement efforts. This framework extends the work of previous researchers who applied wave equations to model nonlinear social and physical phenomena [10,11], and provides a new perspective for understanding addiction as a socio-dynamic "epidemic" with wave-like propagation.

Despite numerous campaigns and government interventions, youth involvement in hard drugs in Nigeria continues to rise. The challenge is compounded by the fact that addiction is not merely an individual problem but one that spreads across social networks in a community. Current mathematical models often fail to account for the wave-like propagation of influence, which can explain sudden surges in drug use across schools and campuses.

There is, therefore, a need for a more robust model that captures both the biological-like

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transitions (from susceptible to user to rehabilitate) and the wave-like social dynamics of addiction[12]. Without such a framework, interventions may be poorly timed, misallocated, or insufficiently targeted, allowing drug use to persist and escalate among students.

II. MODEL FORMULATION

We consider a bounded spatial domain **(space** $x \in \Omega \subset R^2$, $time\ t \geq 0$**)** with smooth boundary $\partial\Omega$ representing schools or communities. The population is divided into three compartments:

- $S(x,t)$: susceptible/vulnerable students,
- $U(x,t)$: active drug users,
- $R(x,t)$: rehabilitated individuals.

We introduce an influence field $P(x,t)$ governed by a damped wave equation, representing peer pressure, availability, and social influence. An intervention control $I(x,t)$ models awareness, counseling, and enforcement efforts. The coupled system is:

Wave (telegraph) equation for influence

$$P_{tt} + \delta P_t - v^2 \nabla P + kP = \eta U - \chi I(x,t)$$

(1)

υ : propagation speed of influence,

- δ>0: damping (loss of attention/fad decay),
- κ≥0: restorative term (baseline forgetting),
- η>0: how strongly current users excite/spread influence,
- I(x,t): intervention intensity (awareness campaigns, policing, counseling),
- χ>0: how strongly interventions suppress influence.

III. POPULATION DYNAMICS (REACTION–DIFFUSION)

$$S_t = D_s \Delta S + \Lambda - \mu S - \lambda(P,U)S + \omega R \qquad (2)$$

$$U_t = D_u \Delta U + \lambda(P,U)S - (\gamma + \mu)U - \xi I(x,t)U \qquad (3)$$

$$R_t = D_R \Delta R + \gamma U - (\omega + \mu)R \qquad (4)$$

- D: spatial mobility (school transfers, hangouts),
- Λ: entry/inflow of new students,
- μ: exit (graduation/relocation/background mortality),
- λ(P,U): initiation rate into use,
- γ: rehab/recovery rate,
- ω: relapse to susceptibility,
- ξ>0: direct effect of interventions on reducing active use.

3.1 Initiation (nonlinear "pressure" response)

Pick a saturating form that blends peer pressure and prevalence:

$$\lambda(P,U) = \lambda_0 + \frac{\beta_P P}{I + \alpha_P P} + \frac{\beta_U U}{K + U}$$

(5)

Where λ_0, β_P, β_U are sensitive; α_p, k set saturation

where the initiation rate is

$$P_{tt} + \delta P_t - v^2 \nabla P + kP = \eta U - \chi I(x,t)$$

(6)

$$\lambda(P,U) = \lambda_0 + \frac{\beta_P P}{I + \alpha_P P} + \frac{\beta_U U}{K + U}$$

(7)

We assume homogeneous Neumann boundary conditions:

$$\nabla S \cdot n = \nabla U \cdot n = \nabla R \cdot n = \nabla P \cdot n = 0$$

(8)

and nonnegative initial conditions:

- **Initial conditions:** measured baselines

$$S(x,0) = U(x,t),\ R(x,t)\ and\ P(x,t), P_t(x,t)$$

(9)

- **Boundary conditions:** no-flux on school/city boundary ∂Ω:
- $$\nabla S \cdot n = \nabla U \cdot n = \nabla R \cdot n = \nabla P \cdot n = 0$$ 10)

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3.2 Parameter Definitions

Symbol	Meaning	Units
Λ	Recruitment rate of students	persons/time
M	Natural exit rate (graduation/other)	1/time
Λ	Baseline initiation rate	1/time
Bp	Peer-influence initiation coefficient	1/(time × influence)
Ap	Saturation parameter for influence	–
Γ	Rehabilitation rate	1/time
Ω	Relapse rate	1/time
DS, DU, DR	Diffusion coefficients	persons/area/time
C	Propagation speed of influence waves	distance/time
Δ	Damping coefficient of influence	1/time
K	Natural decay of influence	1/time
H	Influence generation from users	influence/person/time
X	Suppression of influence by intervention	influence/unit effort
Ξ	Effectiveness of intervention on users	1/time

3.3 Positivity of Solutions

- **Lemma 1:** If $S(x,0), U(x,0), R(x,0)$ *and* $P(x,0) \geq 0$ then the solutions remain nonnegative for all $t > 0$.
- Reaction terms are quasi-monotone: at $S = 0, U = 0, R = 0,$ the PDEs yield nonnegative derivatives.
- The Neumann boundary condition preserves positivity under diffusion.
  Thus, solutions remain biologically feasible.

3.4 Drug-Free Equilibrium (DFE)

At steady state without users

$(U = 0, R = 0, P = 0)$:

$$(S^*, U^*, R^*, P^*) = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right) \tag{11}$$

3.5 Basic Reproduction Number

Linearizing around the DFE, with P≈0 we obtain

$$R_0 = \frac{\lambda_0\left(\frac{\Lambda}{\mu}\right)}{\gamma + \mu} R < 1 \tag{12}$$

- If $R_0 < 1$, the DFE is locally asymptotically stable → drug addiction cannot invade.
- If $R_0 > 1$, addiction persists and spreads. Note: Coupling with the wave field PP can generate transient surges even when $R_0 < 1$, reflecting real-world outbreaks of peer-influenced drug use.

IV. ANALYSIS OF THE MODEL

4.1 Positivity and Boundedness of Solutions

For a model to be epidemiologically (or socially) meaningful, its solutions must remain non-negative and bounded for all $t \geq 0$.From the system equations 2, 3 and 4

$$S_t = D_s \Delta S + \Lambda - \mu S - \lambda(P, U)S + \omega R$$
$$U_t = D_u \Delta U + \lambda(P, U)S - (\gamma + \mu)U - \xi I(x,t)U$$
$$R_t = D_R \Delta R + \gamma U - (\omega + \mu)R$$

it can be shown (using the theory of parabolic PDEs and maximum principle) that if initial conditions are non-negative, then $S(x,t), U(x,t), R(x,t) \geq 0$

Moreover, the total population

$$N(x,t) = S(x,t) + U(x,t) + R(x,t) \tag{13}$$

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Satisfies

$$N_t \le \Lambda - \mu N \qquad (14)$$

implying that

$$N(x,t) \le \frac{\lambda}{\mu} \quad t \ge 0 \qquad (15)$$

Thus, the solutions remain bounded within a feasible region.

4.2 Equilibrium of the Model

Two equilibrium states are of interest:

1. **Drug-Free Equilibrium (DFE):**
Occurs when no addiction is present $(U = 0, P = 0)$:

$$E_0 = (S^*, U^*, R^*, P^*) = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right) \qquad (16)$$

2. **Endemic Equilibrium (EE):**
Occurs when drug use persists $(U > 0, P > 0)$

$$E^* = (S^*, U^*, R^*, P^*) \qquad (17)$$

where S^*, U^*, R^* satisfy:

$$\lambda(P^*, U^*)S^* = (\gamma + \mu + \xi)U^* \quad R^* = \frac{\gamma U^*}{\omega + \mu} \qquad (18)$$

and

$$P^* = \frac{\eta U^*}{k + \chi I} \qquad (19)$$

This equilibrium exists if the initiation rate exceeds the removal rate of users.

4.3 Basic Threshold Parameter (R_0)

To derive the basic threshold condition, we linearize the system near the drug-free equilibrium. At E_0:

$$S \approx \frac{\Lambda}{\mu}, \; U \approx 0, \; P \approx 0 \qquad (20)$$

- The effective initiation rate is approximately λ_0

Thus, the **basic threshold parameter** is defined as:

$$R_0 = \frac{\lambda_0 \frac{\Lambda}{\mu}}{\gamma + \mu} \qquad (21)$$

- If R_0 <1, the DFE is stable, meaning drug addiction cannot invade.
- If R_0 >1, the DFE is unstable, and addiction can spread.

This result parallels classical epidemic thresholds, but here, wave reinforcement (η) and peer sensitivity (βP) can transiently elevate λ(P,U), pushing the system above threshold.

4.4 Local Stability Analysis

The Jacobian of the reduced (S,U,R)-subsystem at the DFE is:

$$J(E_0) = \begin{bmatrix} -(\mu + \lambda_0) & 0 & \omega \\ \lambda_0 & (-\gamma + \mu) & 0 \\ 0 & \gamma & -(\omega + \mu) \end{bmatrix} \qquad (22)$$

The eigenvalues are negative if R_0 <1. Thus, the DFE is **locally asymptotically stable** when $R_0 < 1$.

At the endemic equilibrium R^*, stability depends on nonlinear interactions between U and P. Numerical simulations (see Chapter Five) are required to confirm the existence of oscillations or wave-driven surges.

4.5 Influence Wave Dynamics

The influence equation

$$P_{tt} + \delta P_t - v^2 \nabla P + kP = \eta U - \chi I(x,t) \qquad (23)$$

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describes a **damped driven wave**.

- Without users (U=0), P decays exponentially.
- With persistent users, U > 0, the forcing term ηU sustains influence, generating wave-like surges.
- Interventions (I) increase damping and reduce amplitude.

Traveling wave solutions of the form $P(x,t)=\phi(x-ct)$ exist when the wave speed $c \geq \upsilon$. These solutions explain how addiction influence can "move" across schools and communities.

Optimal control (policy design)

Choose $I(x,t)$ minimize users while limiting costs $I \geq O$

$$\min_{I} J = \int_{0}^{T}\int_{\Omega}\left(U + \frac{c_1}{2}I^2 + \frac{c_2}{2}|\nabla I|^2\right) \qquad (24)$$

subject to the PDEs. Derive adjoint system and gradient for numerical optimization.

4.6 Summary of Analytical Results

1. Solutions remain non-negative and bounded.
2. The system admits a drug-free equilibrium (DFE) and an endemic equilibrium (EE).
3. The drug-free equilibrium is stable if $R_0 < 1$.
4. Peer influence and wave reinforcement can temporarily destabilize the DFE, leading to outbreaks.
5. Interventions act as dampng terms that suppress wave amplitudes and reduce spread.

V. NUMERICAL SIMULATIONS, RESULTS AND DISCUSSION

5.1 Numerical Method

To illustrate the model dynamics, we implemented a finite difference scheme in space and time for the wave equation governing peer influence, and an implicit–explicit (IMEX) scheme for the reaction–diffusion equations describing the student compartments. Simulation experiments were carried out in a one-dimensional spatial domain (representing a school corridor or campus neighborhood), with initial conditions set as:

$$S(x,0)=\frac{\Lambda}{\mu}-5,\quad U(x,0)=5e^{-(x-5)^2},\quad R(x,0)=0,\quad P(x,0)=0$$

Wave P, damped leapfrog new

$$P_j^{n+1}=(2-\delta\Delta t)P_j^n-\left(1-\frac{\delta\Delta t}{2}\right)P_j^{n-1}+\Delta t^2\left(\upsilon^2\Delta x P_j^n-kP_j^n+\eta U_j^n-\chi I_j^n\right)$$

$$S,U,R: IMEX$$

$$\chi^{n+1}-D_X\Delta t\Delta_x\chi^{n+1}=\chi^n+\Delta t R_X\left(X^n P^n I^n\right)$$

$$X\in\{S,U,R\}$$

Where R_X are the reaction terms

These conditions simulate a small cluster of drug users initially concentrated at the center of the domain. Parameter values were adapted from previous modeling studies [1, 4, 9, 13].

5.2 Baseline Dynamics (No Intervention)

- Without intervention, the cluster of users generates a **wave of peer influence** that travels outward with speed approximately equal to the parameter υ.
- The susceptible population declines in areas reached by the wave, while the user population rises.
- Over time, the wave damps out, but not before seeding new drug users across the domain. This confirms the role of wave propagation in explaining sudden surges of addiction in schools.

5.3 Impact of Peer-Pressure Sensitivity (βP)

When the sensitivity of students to peer influence is high:

- The wave amplitude of $P(x,t)$ is amplified.
- Even small initial clusters of users cause widespread outbreaks.
- The minimal wave speed increases, meaning addiction spreads faster through the population.

Conversely, reducing βP delays and attenuates wave fronts, showing that **peer resilience and awareness programs** are crucial.

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5.4 Effect of Interventions $(I(x,t))$

Three intervention strategies were tested:

1. **Uniform Awareness Campaign:**

A constant $I(x,t) = I_0$ across the domain increases damping χI and reduces initiation.

- Result: Wave amplitude decreases, but drug use persists at low levels.

2. **Targeted Intervention:**

$I(x,t)$ applied only in regions where $U(x,t)$ exceeds a threshold.

- Result: Surges are suppressed quickly at hotspots, preventing further spread.
- More cost-effective compared to uniform intervention.

3. **Pulsed Campaigns:**

Periodic interventions ($I(x,t)$ alternating between active and inactive phases).

- Result: Effective at reducing long-term prevalence but less efficient at quenching initial outbreaks.

5.5 Long-Term Outcomes

Numerical experiments confirm the analytical result:

- If $R_0 < 1$, drug use eventually dies out.
- If $R_0 > 1$, addiction persists, maintained by recurrent waves of peer influence.
- Effective intervention strategies act by either reducing R_0 below unity or increasing damping to suppress influence waves.

5.6 Policy Implications

1. Early Detection:Small clusters of drug users can trigger large outbreaks through influence waves. Early intervention is essential.
2. **Targeted Campaigns:** Concentrating resources at hotspots is more effective than uniform, unfocused interventions.
3. **Peer Influence Management:** Programs that strengthen resilience to peer pressure (mentorship, counseling, role models) reduce wave amplitude.
4. **Sustained Awareness:** Recurrent interventions are necessary to prevent resurgence after temporary declines.
5. **Integration with Security and Counseling Services:** Mathematical modeling highlights the need for collaboration between NDLEA, schools, religious groups, and community leaders.

5.7 Summary of Findings

- The wave equation captures **spatial surges** in drug addiction better than classical ODE models.
- Peer influence sensitivity and propagation speed are **critical control parameters**.
- Interventions act as damping mechanisms, effectively quenching addiction waves when optimally applied.
- The framework provides quantitative insights that can guide policy, outreach, and rehabilitation planning in Nigeria and beyond.

VI. CONCLUSION

This study developed and analyzed a mathematical model for youth drug addiction using a coupled wave–reaction–diffusion framework**.** The model integrated peer influence dynamics, represented by a damped wave equation, with population transitions between susceptible, user, and rehabilitated groups. Analytical results established that the system admits a drug-free equilibrium (DFE) and an endemic equilibrium (EE). The threshold parameter R_0 determined whether addiction persists or dies out.Simulations showed that drug addiction can spread in wave-like patterns, where small clusters of users generate peer influence that propagates across schools and communities. The results highlighted that:

- Drug use persists when $R_0 > 1$, driven by strong peer influence.
- Interventions such as awareness campaigns, rehabilitation, and peer resistance programs act as damping terms**,** suppressing wave propagation.
- Targeted interventions at hotspots are more effective than uniform strategies.

This study demonstrates that mathematical modeling provides powerful insights into the mechanisms of youth drug addiction and the design of effective policies.

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