International Journal of Advances in Engineering and Management (IJAEM)
Volume 7, Issue 05 May 2025, pp: 122-125 www.ijaem.net ISSN: 2395-5252

Applying Matlab to Find Solutions of Nonlinear Transition Circuit Problems by Linearization Method for Small Amount of Nonlinearity

Ha Le Thi Thu [1] Nguyet Le Thi Minh [2]
(1) Faculty of Electrical Engineering, Thai Nguyen University of Technology, Vietnam
(2) Faculty of Electrical Engineering, College of economics and techniques Thai Nguyen, Vietnam

Date of Submission: 01-05-2025 Date of Acceptance: 10-05-2025

SUMMARY
The transition process in an electric circuit is the process of the circuit changing from the old steady state to the new steady state.
The article introduces the use of Matlab software to support finding solutions of nonlinear transition problems by linearization method for small nonlinear quantities more accurately.
Keywords: Transition; linearization; RL circuit; small nonlinear quantity.

I. CONCEPT OF TRANSIENT PROCESS IN NONLINEAR CIRCUITS

The transient process in a nonlinear circuit is described by a system of nonlinear differential equations written according to Kirchhoff's laws. The study of the transient process is reduced to solving a system of nonlinear differential equations. For example, with the circuit shown in Figure 1, the voltage balance equation of the circuit is:

$$e(t) = Ri + \frac{\partial \psi}{\partial i}.\frac{di}{dt} = Ri + L(i)\frac{di}{dt} \qquad (1)$$

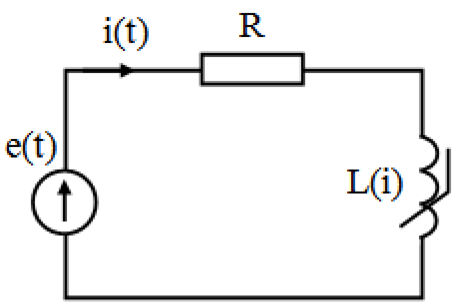

Figure 1: RL nonlinear circuit

The nonlinearity of equation (1) is determined by the nonlinearity of the relation $\psi(i)$.

For nonlinear circuits, there is usually no direct relationship between current and voltage, or in other words, there is no direct relationship between the two response and excitation quantities.

Nonlinear elements are often found in devices such as: electronic lamps, semiconductors; iron cores in electric machines, in relays... therefore, studying the transition process of nonlinear circuits has practical significance.

Calculating the transient process in nonlinear circuits is much more difficult than in linear circuits. Moreover, it is almost impossible to directly calculate the integral to find the solution of nonlinear equations, but must use approximate solution methods such as: linearization method for small nonlinear quantities, difference method, perturbation method, slow-varying amplitude and phase method, model method. Depending on each specific problem, we apply the appropriate solution method.

II. LINEARIZATION METHOD FOR SMALL NONLINEAR QUANTITIES

Consider the transient process when closing the voltage source u(t) to the iron core coil (nonlinear RL circuit) as shown in Figure 2. Given the characteristic

$\begin{cases} i = i(\psi) \\ \psi = \psi(i) \end{cases}$ shown in Figure 3.

Equation written according to Kirchhoff's law 2 for the circuit in figure 2: $u(t) = Ri(t) + \frac{d\psi}{dt}$ (2)

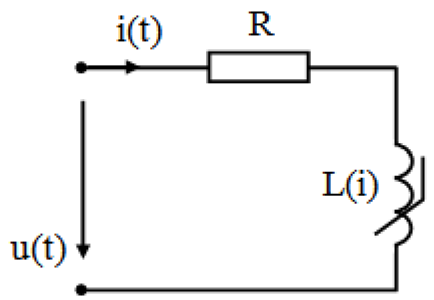

Figure 2: Closing the nonlinear RL circuit to the voltage source u(t)

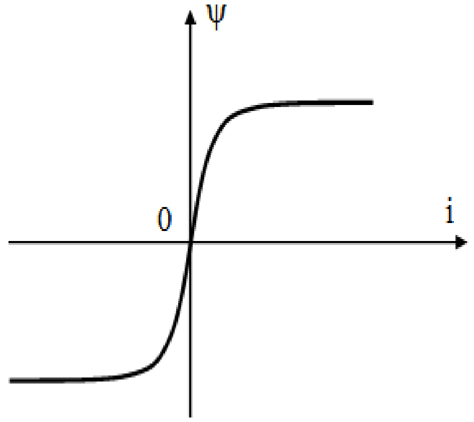

Figure 3: ψ(i) characteristic of nonlinear inductor

From equation (2) we have the following observations:

- If the coil has low dissipation (low loss) it means that: $Ri(t) << \frac{d\psi}{dt}$. From that we can deduce that:

$$u(t) = \underbrace{Ri(t)}_{\text{small amount}} + \frac{d\psi}{dt}$$

Since Ri(t) is a small quantity, we can consider the relationship i(ψ) as linear or nonlinear, which has little effect on the result of the problem. Therefore, to simplify, consider the relationship of i(ψ) as linear, meaning we have the relationship:

$$L = \frac{\psi}{i} \quad \text{hay} \quad i = \frac{\psi}{L} \qquad (3)$$

Substitute (3) into (2) to get:

$$u(t) = R\frac{\psi}{L} + \frac{d\psi}{dt} \Leftrightarrow u(t) = \frac{R}{L}.\psi + \psi' \qquad (4)$$

Equation (4) is a linear differential equation with respect to ψ, so when solving (4) we will find ψ(t) and when we have found ψ(t), based on the relationship ψ(i) - Wb-A characteristic we can find i(t) by drawing a graph.

- If the coil dissipates large (high loss), it means: $Ri(t) >> \frac{d\psi}{dt}$. From that we can deduce:

$$u(t) = Ri(t) + \underbrace{\frac{d\psi}{dt}}_{\text{small amount}}$$

From this we can deduce that the argument is similar to the case where Ri(t) is a small quantity, considering the relationship ψ(i) as linear or nonlinear has little effect on the result of the problem. Therefore, for simplicity, we consider the relationship of ψ(i) as linear, meaning we have the relationship:

$$\psi = L.i \qquad (5)$$

Similarly, substituting (5) into (2) we get:

$$u(t) = R.i(t) + \frac{d\psi}{dt} \Leftrightarrow u(t) = R.i(t) + L.i' \qquad (6)$$

Equation (6) is a linear equation for i(t), solving equation (6) we find i(t) and then based on i(ψ) - Wb-A characteristic we find ψ(t) by drawing a graph.

III. STEPS OF PROBLEM ANALYSIS BY LINEARIZATION METHOD FOR SMALL NONLINEAR QUANTITIES

Through the above analysis, the steps to analyze the nonlinear transition problem by the linearization method for small nonlinear quantities:

Step 1: Set up the equation

Step 2: Find the small term and linearize its relationship, deduce an approximate linear differential equation for the unknown.

Step 3: Solve the linear differential equation to find that unknown. Then use the nonlinear relationship to find the remaining unknown.

In linearization equations, the problem is how to choose the inductance value L. We see that when i varies from 0 to i_{max}, ψ also varies from 0 to ψ_{max}, so we can choose according to the formula:

$$L = \frac{\psi_{max}}{i_{max}}$$

According to the above method, we need to determine the small term of the equation (the small term may or may not be given in advance), we can consider a certain element as a small quantity and solve the circuit according to one of the two cases above from the viewpoint:

- If we need to find i(t) with full nonlinearity, we consider i(ψ) as linear, then we can derive a linear equation for the variable $\psi \rightarrow \psi(t)$ and then based on ψ(i) we can find i(t).

- If we need to find ψ(t) with full nonlinearity, we consider ψ(i) to be linear, then we can derive a linear

International Journal of Advances in Engineering and Management (IJAEM)
Volume 7, Issue 05 May 2025, pp: 122-125 www.ijaem.net ISSN: 2395-5252

equation for the variable i → i(t) and then based on i(ψ) we can find ψ(t).

IV. ILLUSTRATIVE EXAMPLE

Given the circuit as shown in Figure 4. Calculate the transient current i(t) when closing a DC voltage source U = 34 V to the iron core coil (nonlinear RL circuit), knowing R = 50 Ω, characteristic $\psi(i) = ai - bi^3 = 3i - 0,5i^3$.

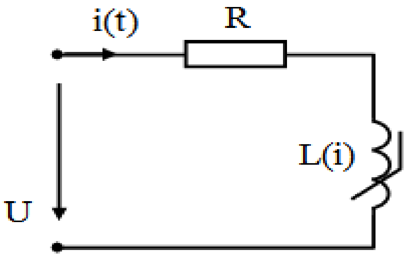

Figure 4: Closing the nonlinear RL circuit into a DC voltage source U

Linear differential equation describing the circuit in figure 4:

$$U = Ri(t) + \frac{d\psi}{dt} \qquad (7)$$

Consider i(ψ) to be linear with: $L = \frac{\psi}{i}$ hay $i = \frac{\psi}{L}$.

Substitute into equation (7):

$$U = \frac{d\psi}{dt} + \frac{R}{L}\psi \qquad (8)$$

Determine the coefficient:

$$I_{max} = \frac{U}{R} = \frac{34}{50} = 0,68\ (A)$$

$$\psi_{max} = L(2i_{max} - 0,5i_{max}^3) = 3.0,68 - 0,5.(0,68)^3 = 1,882\ (Wb)$$

$$\text{So } L = \frac{\psi_{max}}{i_{max}} = \frac{1,882}{0,68} = 2,768\ (H)$$

Substitute the numbers into equation (8):

$$\frac{d\psi}{dt} + \frac{50}{2,768}\psi = 34 \rightarrow \psi' + 18,063\psi = 34 \qquad (9)$$

Convert equation (9) to Laplace operator equation:

$$s\psi(s) + 18,063\psi(s) = \frac{34}{s}$$

$$\rightarrow \psi(s) = \frac{34}{s(s+18,063)} = \frac{A}{s} + \frac{B}{s+18,063}$$

$$\rightarrow \psi(s) = \frac{1,882}{s} - \frac{1,882}{s+18,063}$$

Look up image table - original:

$$\psi(t) = 1,882(1 - e^{-18,063t})\ (Wb)$$

Once the relation ψ(t) is found, to find i(t), we do the following: At a time t_1, close the characteristic curve ψ(t) to get the value ψ_1, then close it to the relation ψ(i) to get the value i_1. Doing this with many points will find the solution i(t) (Figure 5).

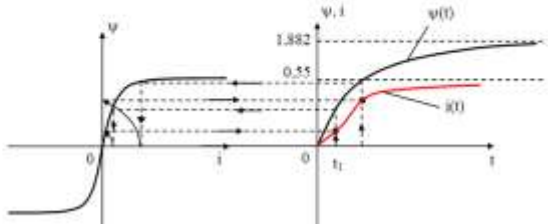

Figure 5: Solution i(t) of Figure 4 by graphing method

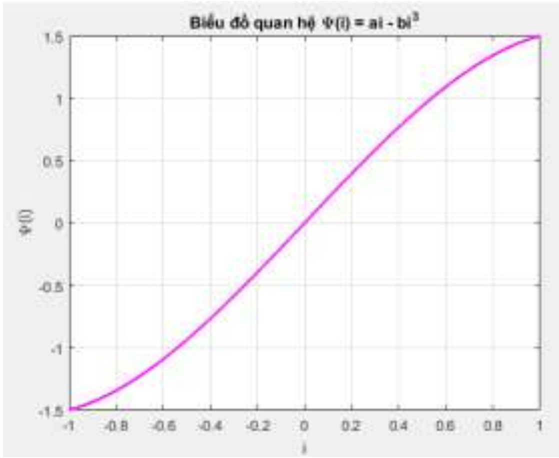

Figure 6: Characteristics $\psi(i) = ai - bi^3 = 3i - 0,5i^3$

To find the solution i(t) by finding each point will take a lot of time and requires meticulousness and precision. To find the solution i(t) of the above nonlinear problem, we can use Matlab software with the calculation program written as below. With the program written on Matlab, if we change the parameters a and b in the relationship ψ(i), we can also easily find the solution i(t).

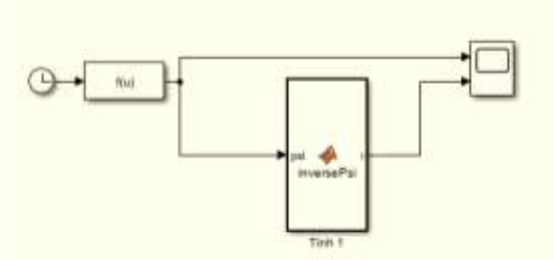

Figure 7: Example simulation block diagram circuit figure 4

International Journal of Advances in Engineering and Management (IJAEM)
Volume 7, Issue 05 May 2025, pp: 122-125 www.ijaem.net ISSN: 2395-5252

Using Matlab to find the solution i(t), we have the simulation diagram as shown in Figure 6.
Code program written for Calculation block 1:

```
function i = inversePsi(psi)
% Giải phương trình: 3i - .5i^3 = psi
coeffs = [-.5, 0, 3, -psi];
r = roots(coeffs);
realRoots = r(imag(r) == 0);
posRoots = realRoots(realRoots >= -2 & realRoots <= 2);
if ~isempty(posRoots)
    i = posRoots(1);
else
    i = NaN;
end
```

Running the simulation, we see that the solution i(t) also has the same form as the graphing method in Figure 5.

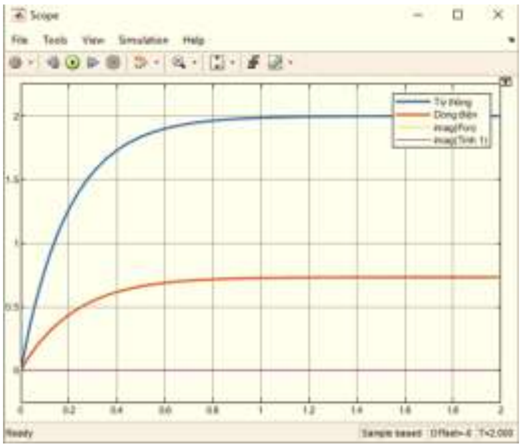

Figure 7: Solution i(t) corresponding to $\psi(i) = 3i - 0{,}5i^3$

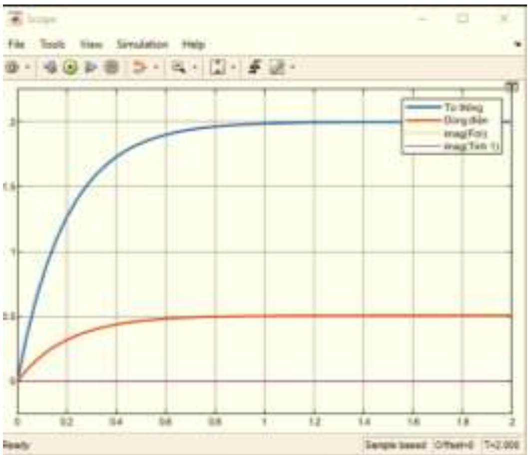

Figure 7: Solution i(t) corresponding to $\psi(i) = 4i - 0{,}2i^3$

V. CONCLUDE:

The paper introduces the method of analyzing nonlinear transient problems by linearization method for small nonlinear quantities. By using Matlab software, the solution of nonlinear transient problems is found more accurately and faster than by graphing method.

REFERENCES

[1] Lai Khac Lai; Theoretical basis of circuits - volume 2; Science and Technology Publishing House; 2009.
[2] Lecture of Electrical Engineering 2024.