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Bayesian Optimization Model in Multi-purpose/Multi-objective Projects in Anambra – Imo River Basin Resources Project Utilization, Nigeria.

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ABSTRACT

The work is aimed at investigating optimization of river basin resources utilization in Anambra – Imo river basin to mitigate climate variability using Bayesian theory model. The objective is to determine optimal benefits under various net benefits (objectives) is a multi-purpose/multi-objective capital projects to develop sustainability in the river basin. The methodology involves the use of Bayesian Decision Theory (BDT) model based on the data generated from the Bill of Engineering Measurement and Evaluation (BEME). The result shows that the optimal solution from the Bayesian model analysis of the Maximum Expected Monetary Value (EMV*) was N68.72b on third iteration. When the amount of N12.504 billion released to Anambra – Imo river basin for the period were appropriated to various purpose/objectivewas deducted from the revenue generated from Bayesian EMV* (N68.72billion), N56.22 billion emerged as profit margin from the investment. The world recommended that since there are much uncertainties in climate change projection which impacts on the environment, optimal strategies should incorporate delivery benefits irrespective of climate variability which Bayesian optimal strategies would mitigate.

Keywords: Optimization, multi-purpose/multi-objective, River basin, Bayesian theory.

I. INTRODUCTION

The multi-purpose/multi-objective river basin development project planning and management will help to determine levels of development to be apportioned to various purposes for water resources projects. The planning and management of these projects are multi-disciplinary and may involve a lot of complex situations. Barrow (1998) opined that River basin development and planning is the process of identifying the best way in which a river and its tributaries may be used to meet competing demands while maintaining river health. It includes the allocation of scarce water resources between different users and purposes, choosing between environmental objectives and competing human needs and choosing between competing food risk management requirements (Molle, 2006). The increasing complexity of many of the river basins occasioned by increasing development and population pressure, have resulted many serious crisis related to floods, degradation of water quality, acute water shortage and degradation of ecological health. The various approaches to river basin planning is ultimately playing significant roles to the adaptation of the local circumstances. The considerationof economic efficiency, federal economic redistribution, regional economic redistribution, state economic redistribution, local economic redistribution, social well-being, environmental quality improvement, youth employment, gender equality and security are becoming more relevant due to some political, ecological and health concern of the people. Ezenweani (2017) identified inability of management of river basin to control the whole basin and lack of baseline data with inadequate monitoring are some of the problems that hinders

River basin development planning and management. Klare (2001) also said that politics to determine who is to be employed, what is on the agenda and how river basin development planning and management proceeds also affects them. The required decisions will need to be made by concerned stakeholders in the government and river basin development authority for adequate benefits to be derived from the resources development and utilization.

The Bayesian Decision Theory (BDT) model is a Dynamic programming techniques concerned with the method of computing posterior probabilities from prior probabilities using Bayes' theorem. An initial probability statement to evaluate expected payoff is called a prior probability distribution. The one which has been revised in the light of new information is called posterior probability distribution. What is a posterior to one sequence of state of nature becomes the prior on others which are yet to happen. A further analysis of problems using these probabilities with respect to new expected payoffs with additional information is called prior-posterior analysis.

II. AIM AND OBJECTIVES

The aim is to investigate optimization of river basin resources utilization in Anambra – Imo river basin to mitigate climate variability using Bayesian theory model. The objective is to firstuse the net benefits generated from Bill of Engineering Measurement and Evaluation (BEME) to determine optimal benefits under the various objectives in a multi-purpose/multi-objective projects to champion the course of green revolution in the river basin management planning and development.

Secondly, to determine the magnitude of differences between alternative courses of action with the degree of association indicators available for decision making under the situation of certainty and uncertainty in the river basin.

III. METHODOLOGY

This involve the use of Bayesian Decision Theory (BDT) model from the data generated from the Bill of Engineering Measurement and Evaluation (BEME). The policy iteration algorithm were used to determine the optimal benefits under the various objectives in the river basin.

IV. ANALYSIS AND DISCUSSION OF RESULTS

4.1 Net Benefits Multi-Purpose under Various Multi-Objectives

Table 1: Summary of Net Benefits for all the Objectives against the Purposes in Billion Naira

S/N	Purpose	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10
(1)	Irrigated Agriculture	3.65	4.84	6.36	3.60	3.44	4.37	4.05	4.22	1.12	8.73
(2)	Hydro-electric power generation	13.38	7.55	9.60	9.68	9.29	5.46	6.05	6.39	1.37	10.95
(3)	Water supply	4.54	4.34	6.04	3.78	3.52	4.56	4.22	4.37	1.13	9.13
(4)	Navigation	8.30	5.83	10.46	8.19	8.24	11.39	10.96	12.20	3.33	25.77
(5)	Drainage/ Dredging	17.21	6.01	12.26	3.68	6.08	8.96	11.51	10.83	3.00	21.96
(6)	Flood control	19.43	5.58	10.20	3.39	1.55	8.68	10.32	11.35	2.90	22.12
(7)	Recreation / Tourism	16.93	3.94	10.36	3.42	3.33	10.57	11.33	12.25	3.33	25.94
(8)	Erosion control	13.91	3.01	10.27	3.15	3.26	9.56	7.13	8.72	2.21	16.78
(9)	Plantation / Forestry	14.01	6.83	8.08	6.40	6.59	8.96	7.66	8.40	2.26	18.08
(10)	Reservoir/	82.72	5.66	12.16	3.36	3.48	19.99	20.54	20.71	5.77	41.23

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	Gullies										

B_1= Economic efficiency,
B_2 = Federal Economic Redistribution,
B_3 = Regional Economic Redistribution,
B_4= State Economic Redistribution,

B_5 = Local Economic Redistribution,
B_6 = Social Well-being,
B_7 = Youth Empowerment,
B_8 = Environmental Quality Improvement,
B_9 = Gender Equality,
B_{10} = Security

Discussion of Results in Table 1:
The Table 1 explained the summary results calculation of Net benefits from Bill of Engineering Measurement and Evaluation (BEME) in billions of naira.

(i) Under Irrigation Agriculture the highest benefits of ₦8.73 billion from Security while the least amount of benefit was ₦1.12 billion from Gender Equality
(ii) On Hydro-electric Power Generation, Economic Efficiency has the highest value of ₦13.38 billion while lowest value of ₦1.37 billion was on Gender Equality
(iii) Under the purpose of Reservoir and Gullies, the highest benefit of ₦82.72 billion was from objective of Economic Efficiency and the lowest was ₦3.36 billion on State Economic Redistribution
(iv) In other purposes the Net benefis has the highest from objectives on Security with the following values; ₦9.13 billion from Water Supply; ₦25.77 billion from Navigation; ₦21.96 billion from Drainage/Dredging; ₦22.12 billion from Flood Control; ₦25.94 billion from Recreation/Tourism; ₦16.78 billion from Erosion Control and ₦18.08 billion from Plantation/Forestry.Except for Hydro-electric power generation and Reservoir/Gullies, other purpose have the highest net benefits under the objective of security improvement.
(v) Except for state economic redistribution under reservoir/gullies that has the lowest net benefits of ₦3.36billion, the rest of the lowest net benefits under all other purpose were under the objective of Gender Equality. These are ₦1.12 billion from irrigation agriculture, ₦1.37 billion from hydro-electric power generation, ₦1.13 billion from Water Supply; ₦3.33 billion from Navigation; ₦3.00 billion from Drainage/Dredging; ₦2.90 billion from Flood Control; ₦3.33 billion from Recreation/Tourism ; ₦2.21 billion from Erosion Control and ₦2.26 billion from Plantation/Forestry.

4.2 Bayesian Decision Model Simulation Based on Courses of Action

Using the Bayesian Decision Analysis, the prior probability was derived from the benefits and used in the analysis for previous prediction i.e. states of nature probabilities; N_1 = 0.02, N_2 = 0.07, N_3 = 0.03, N_4 = 0.04, N_5 = 0.09, N_6 = 0.10, N_7 = 0.09, N_8 = 0.07, N_9 = 0.08, N_{10} = 0.41.

4.2.1 Calculation of Likelihood Forecast of Probabilities

Table 2: The Likelihood Forecast of Probability Estimated from the Various Courses of Action for Net Benefits.

States of Nature	**Courses of Action**									
	B_1	**B_2**	**B_3**	**B_4**	**B_5**	**B_6**	**B_7**	**B_8**	**B_9**	**B_{10}**
N_1	0.08	0.11	0.14	0.08	0.08	0.10	0.09	0.10	0.02	0.20
N_2	0.17	0.09	0.12	0.12	0.12	0.07	0.07	0.08	0.02	0.14
N_3	0.10	0.10	0.13	0.08	0.08	0.10	0.09	0.10	0.02	0.20
N_4	0.08	0.05	0.10	0.08	0.08	0.11	0.10	0.12	0.03	0.25
N_5	0.17	0.06	0.12	0.03	0.06	0.09	0.11	0.11	0.03	0.22
N_6	0.20	0.06	0.11	0.03	0.02	0.09	0.11	0.12	0.03	0.23
N_7	0.17	0.04	0.10	0.04	0.03	0.10	0.11	0.12	0.03	0.26
N_8	0.18	0.04	0.13	0.04	0.04	0.12	0.09	0.11	0.03	0.22
N_9	0.16	0.08	0.09	0.07	0.07	0.10	0.09	0.10	0.03	0.21
N_{10}	0.38	0.02	0.06	0.01	0.02	0.09	0.10	0.10	0.03	0.19

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Where the courses of action are;
N_1 = Irrigation Agriculture,

N_2 = Hydro-electric Power Generation,
N_3 = Water Supply,

N_4 = Navigation/Water Transport,
N_5 = Drainage/Dredging,
N_6 = Flood Control,
N_7 = Recreation/Tourism,
N_8 = Erosion Control,
N_9 = Plantation/Forestry,
N_{10} = Reservoir/Gullies

Where the states of nature are;
B_1 = Economic efficiency,

B_2 = Federal Economic Redistribution,
B_3 = Regional Economic Redistribution,
B_4 = State Economic Redistribution,

B_5 = Local Economic Redistribution,
B_6 = Social Well-being,
B_7 = Youth Empowerment,
B_8 = Environmental Quality Improvement,
B_9 = Gender Equality,
B_{10} = Security

Discussion of Results in Table 2:

Table 2 shows the likelihood forecast probabilities from various courses of action for the purposes. These probabilities were used in calculating the Joint probability outcomes on first iteration in order to determine the Marginal probability outcomes.
The next step is to calculate the Expected Monetary Values (EMVs) using the Prior Probabilities for the States of Nature.

4.2.2 Determination of Expected Monetary Value (EMVs) at First Iteration

The Expected Monetary Values (EMVs) or Expected Utility explains criteria for various courses of action (alternatives) under risk. The EMV is the weighted sum of possible payoffs from each alternative. It is obtained by adding up the payoffs of each course of action multiplied by the probabilities associated with each state of nature. This was calculated and shown on Table 3 below as follows;

Table 3: Calculation of Expected Monetary Values (EMVs) at First Iteration

States of Nature	Prior Probability	Conditional Net Benefits Course of Action in Billions of Naira									
		S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
N_1	0.02	3.65	4.84	6.36	3.60	3.44	4.37	4.05	4.22	1.12	8.73
N_2	0.07	13.38	7.55	9.60	9.68	9.29	5.46	6.05	6.39	1.37	10.95
N_3	0.03	4.54	4.34	6.04	3.78	3.52	4.56	4.22	4.37	1.13	9.13
N_4	0.04	8.30	5.83	10.46	8.19	8.24	11.39	10.96	12.20	3.33	25.77
N_5	0.09	17.21	6.01	12.26	3.68	6.08	8.96	11.51	10.83	3.00	21.96
N_6	0.10	19.43	5.58	10.20	3.39	1.55	8.68	10.32	11.35	2.90	22.12
N_7	0.09	16.93	3.94	10.36	3.42	3.33	10.57	11.33	12.25	3.33	25.94
N_8	0.07	13.91	3.01	10.27	3.15	3.26	9.56	7.13	8.72	2.21	16.78
N_9	0.08	14.01	6.83	8.08	6.40	6.59	8.96	7.66	8.40	2.26	18.08
N_{10}	0.41	82.72	5.66	12.16	3.36	3.48	19.99	20.54	20.71	5.77	41.23

States of Nature	Expected Net Benefits in Billions of Naira Course of Action									
	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
N_1	0.073	0.0968	0.1272	0.072	0.0688	0.0874	0.081	0.0844	0.0224	0.1746
N_2	0.9366	0.5285	0.672	0.6776	0.6503	0.3822	0.4235	0.4473	0.0959	0.7665
N_3	0.1362	0.1302	0.1812	0.1134	0.1056	0.1368	0.1266	0.1311	0.0339	0.2739
N_4	0.332	0.2332	0.4184	0.3276	0.3296	0.4556	0.4384	0.488	0.1332	1.0308
N_5	1.5489	0.5409	1.1034	0.3312	0.5472	0.8064	1.0359	0.9747	0.27	1.9764
N_6	1.943	0.558	1.02	0.339	0.155	0.868	1.032	1.135	0.290	2.212
N_7	1.5237	0.3546	0.9324	0.3078	0.2997	0.9513	1.0197	1.1025	0.2997	2.3346
N_8	0.9737	0.2107	0.7189	0.2205	0.2282	0.6692	0.4991	0.6104	0.1547	1.1746
N_9	1.1208	0.5464	0.6464	0.512	0.5272	0.7168	0.6128	0.672	0.1808	1.4464
N_{10}	33.9972	2.3206	0.8856	1.3776	1.4268	8.1959	8.4214	8.4911	2.3657	16.9043
Expected monetary values (EMVs)	**42.5851**	**5.5199**	**6.7055**	**4.2787**	**4.3364**	**13.2496**	**13.6904**	**14.1365**	**3.8463**	**28.2941**

The Maximum Expected Monetary Value (EMV*) = ₦42.5851 Billionon Economic Efficiency is the optimal course of action with other optimal course of action with other objectives to be considered for maximum benefits.

Where the states of nature are;
S_1 = Economic efficiency,
S_2 = Federal Economic Redistribution,
S_3 = Regional Economic Redistribution,
S_4 = State Economic Redistribution,

S_5 = Local Economic Redistribution,
S_6 = Social Well-being,
S_7 = Youth Empowerment,
S_8 = Environmental Quality Improvement,
S_9 = Gender Equality,
S_{10} = Security

Discussion of Result in Table 3:

(i). The prior probabilities are multiplied by the Conditional Net benefits courses of action to get the Expected Monetary Values (EMVs). The Maximum Expected Monetary Value (EMV*) is S_1 = ₦42.59 billion, S_2 = ₦5.52 billion, S_3 = ₦6.71 billion, S_4 = ₦4.28 billion, S_5 = ₦4.34 billion, S_6 = ₦13.25 billion, S_7 = ₦13.69 billion, S_8 = ₦14.14 billion, S_9 = ₦3.85 billion, S_{10} = ₦28.29 billion.

(ii). The Maximum Expected Benefit for each states of nature was ₦42.5851 billion.

(iii). Expected Profit with Perfect Information (EPPI) = 0.02(8.73) + 0.07(13.38) + 0.03(9.13) + 0.04 (25.77) + 0.09 (21.96) + 0.10 (22.12) + 0.09 (25.94) + 0.07 (16.78) + 0.08 (18.08) + 0.41 (82.92) = ₦45.5571 billion.

(iv). The Expected Value of Perfect Information (EVPI) = EPPI – EMV* = ₦45.5571 - ₦42.5851 = ₦2.972 billion.

4.2.3 Determination of Forecast Likelihood of Probabilities

The Table 4 shows the Forecast Likelihood of Probabilities determined from the table of net benefits on Tables 1 and 2respectively.

Table 4: Forecast Likelihood of Probabilities

States of Nature	Forecast Likelihood									
	B_1/N	B_2/N	B_3/N	B_4/N	B_5/N	B_6/N	B_7/N	B_8/N	B_9/N	B_{10}/N
N_1	0.08	0.11	0.14	0.08	0.08	0.10	0.09	0.10	0.02	0.20
N_2	0.17	0.09	0.12	0.12	0.12	0.07	0.07	0.08	0.02	0.14
N_3	0.10	0.10	0.13	0.08	0.08	0.10	0.09	0.10	0.02	0.20
N_4	0.08	0.05	0.10	0.08	0.08	0.11	0.10	0.12	0.03	0.25
N_5	0.17	0.06	0.12	0.03	0.06	0.09	0.11	0.11	0.03	0.22
N_6	0.21	0.06	0.11	0.03	0.02	0.09	0.11	0.12	0.03	0.23
N_7	0.17	0.04	0.10	0.04	0.03	0.10	0.11	0.12	0.03	0.26
N_8	0.18	0.04	0.13	0.04	0.04	0.12	0.09	0.11	0.03	0.22
N_9	0.16	0.08	0.09	0.07	0.07	0.10	0.09	0.10	0.03	0.21
N_{10}	0.38	0.02	0.06	0.01	0.02	0.09	0.10	0.10	0.03	0.19

Discussion of Results in Table 4:

(i). The Forecast Likelihood Probabilities in Table 4 was calculated by dividing each value of the summary of Net benefits in Table 1 by the total value in each row of the table.

(ii). The Expected Profit with Perfect Information (EPPI) was obtained by multiplying the each prior probability by their respective highest net benefit on each row and adding up the values which is ₦45.5571 billion.

(iii). The Expected Value of Perfect Information (EVPI) was obtained by subtracting the value of Maximum Expected Monetary Value (EMV*) ₦42.5851 from the amount of Expected Profit with Perfect Information (EPPI) ₦45.5571 billion which gives a balance of ₦45.5571 billion.

(iv). For each of the forecast result, the prior and posterior probabilities are calculated in Tables 5 and 6 respectively.

4.2.4 Determination of Joint Probabilities Outcomes on First Iteration

The determination of Joint Probabilities Outcomes on First (1st) Iteration was obtained by multiplying the states of nature (prior) probabilities $P(N_i)$ with each of the conditional probabilities (B_i/N). These were calculated for each course of action outcomes (Bi) as shown in Table 5.

It should be noted that in these tables the prior probabilities of states of Nature $P(N_i)$ for i = 1, 2 3, 4, 5, 6, 7, 8, 9 and 10 are multiplied by each of the conditional probabilities outcomes $P(B_i/N_i)$ to get the joint values probabilities outcomes i.e. $P(B_i \cap N_i)$ = $P(N_i)$ $P(B_i/N_i)$ as shown below.i.e. $(P(N_I)$ for (I=1, 2, 3, 4, 5, 6, 7, 8, 9, 10) $\times P(B_i/N_i)$ for i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10). For Example, purpose B_i (irrigation agriculture) for states of nature N_i, Joint Probability = $P(N_i) \times P(B_i/N_i)$

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For $B_1 = 0.02 \times 0.08 = 0.0016$
For $B_2 = 0.02 \times 0.11 = 0.0022$
For $B_3 = 0.02 \times 0.14 = 0.0028$ etc.
The Marginal Probability for each benefits B_i for (i = 1,2,3,4,5,6,7,8,9,10)

$$\text{for } B_i = \sum\{P(N_i) \times P(B_i/N_i)\}$$

$$\text{for } B_1 = \sum\{N_1B_1 + N_2B_1 + N_3B_1 + N_4B_1 + N_5B_1 + N_6B_1 + N_7B_1 + N_8B_1 + N_9B_1 + N_{10}B_1\}$$

$$\text{for } B_2 = \sum\{N_1B_2 + N_2B_2 + N_3B_2 + N_4B_2 + N_5B_2 + N_6B_2 + N_7B_2 + N_8B_2 + N_9B_2 + N_{10}B_2\}$$

… etc to N_iB_{10} for i = …, 3, 4, 5, 6, 7, 8, 9, 10.

Table 5: Joint Probabilities Outcomes at First Iteration

States of Nature (N_i)	Prior Probability $P(N_i)$	Outcomes (B_i)	Conditional Probability $P(B_i/N_i)$	Joint Probability $P(B_i \cap N_i) = P(N_i)\,P(B_i/N_i)$									
				B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
N_1	0.02	B_1	0.08	0.0016									
		B_2	0.11		0.0022								
		B_3	0.14			0.0028							
		B_4	0.08				0.0016						
		B_5	0.08					0.0016					
		B_6	0.10						0.0020				
		B_7	0.09							0.0018			
		B_8	0.10								0.0020		
		B_9	0.02									0.0004	
		B_{10}	0.20										0.00040
N_2	0.07	B_1	0.17	0.00119									
		B_2	0.09		0.0063								
		B_3	0.12			0.0084							
		B_4	0.12				0.0084						
		B_5	0.12					0.0084					
		B_6	0.07						0.0049				
		B_7	0.07							0.0049			
		B_8	0.08								0.0056		
		B_9	0.02									0.0014	
		B_{10}	0.14										0.0098
N_3	0.03	B_1	0.10	0.003									
		B_2	0.10		0.003								
		B_3	0.13			0.0039							
		B_4	0.08				0.0024						
		B_5	0.08					0.0024					
		B_6	0.10						0.003				
		B_7	0.09							0.0027			
		B_8	0.10								0.003		
		B_9	0.02									0.0006	
		B_{10}	0.20										0.006

Note: This Table 5 continued on the next page from states of nature N_4

Table 5: Joint Probabilities Outcomes at First Iteration Continued

States of Nature (N_j)	Prior Probability $P(N_j)$	Outcomes (B_i)	Conditional Probability $P(B_i/N_j)$	Joint Probability $P(B_i \cap N_j) = P(N_j)\,P(B_i/N_j)$									
				B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
N_4	0.04	B_1	0.06	0.032									
		B_2	0.05		0.002								
		B_3	0.10			0.004							
		B_4	0.08				0.0032						
		B_5	0.08					0.0032					
		B_6	0.11						0.0044				
		B_7	0.10							0.0040			
		B_8	0.12								0.0012		
		B_9	0.03									0.0012	
		B_{10}	0.25										0.0100
N_5	0.09	B_1	0.17	0.0153									
		B_2	0.06		0.0054								
		B_3	0.12			0.0108							
		B_4	0.03				0.0027						
		B_5	0.06					0.0054					
		B_6	0.09						0.0081				
		B_7	0.11							0.0099			
		B_8	0.11								0.0099		
		B_9	0.03									0.0027	
		B_{10}	0.22										0.0198
N_6	0.10	B_1	0.20	0.0200									
		B_2	0.06		0.0060								
		B_3	0.11			0.0110							
		B_4	0.03				0.003						
		B_5	0.02					0.002					
		B_6	0.09						0.009				
		B_7	0.11							0.0110			
		B_8	0.12								0.012		
		B_9	0.03									0.003	
		B_{10}	0.23										0.023

Note: This Table 5 continued on the next page from states of nature N_7

Table 5: Joint Probabilities Outcomes at First Iteration Continued

States of Nature (N_j)	Prior Probability $P(N_j)$	Outcomes (B_i)	Conditional Probability $P(B_i/N_j)$	Joint Probability $P(B_i \cap N_j) = P(N_j)\,P(B_i/N_j)$									
				B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
N_7	0.09	B_1	0.17	0.0153									
		B_2	0.04		0.0036								
		B_3	0.10			0.009							
		B_4	0.04				0.0036						
		B_5	0.03					0.0027					
		B_6	0.10						0.009				
		B_7	0.11							0.0099			
		B_8	0.12								0.0108		
		B_9	0.03									0.0027	
		B_{10}	0.26										0.0234
N_8	0.07	B_1	0.18	0.0126									
		B_2	0.04		0.0028								
		B_3	0.13			0.0091							
		B_4	0.04				0.0028						
		B_5	0.04					0.0028					
		B_6	0.12						0.0084				
		B_7	0.09							0.0063			
		B_8	0.11								0.0077		
		B_9	0.03									0.0021	
		B_{10}	0.22										0.0154

Note: This Table 5 continued on the next page from states of nature N_9

Table 5: Joint Probabilities Outcomes at First Iteration Continued

States of Nature (N_i)	Prior Probability $P(N_i)$	Outcomes (B_i)	Conditional Probability $P(B_i/N_i)$	Joint Probability $P(B_i \cap N_i) = P(N_i) P(B_i/N_i)$									
				B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
N_9	0.08	B_1	0.16	0.0128									
		B_2	0.03		0.0064								
		B_3	0.09			0.0072							
		B_4	0.07				0.0056						
		B_5	0.07					0.0056					
		B_6	0.10						0.0088				
		B_7	0.09							0.0072			
		B_8	0.10								0.0080		
		B_9	0.03									0.0024	
		B_{10}	0.21										0.0168
N_{10}		B_1	0.38	0.1558									
		B_2	0.02		0.0082								
		B_3	0.06			0.0246							
		B_4	0.01				0.0041						
		B_5	0.02					0.0082					
		B_6	0.09						0.0369				
		B_7	0.10							0.041			
		B_8	0.10								0.041		
		B_9	0.03									0.0123	
		B_{10}	0.19										0.0779
Marginal Probability				**0.2515**	**0.0599**	**0.0908**	**0.0374**	**0.0423**	**0.0941**	**0.0987**	**0.1048**	**0.0288**	**0.2061**

Discussion of Results in Table 5:

(i). The joint values probabilities outcomes were calculated by multiplying prior probability of each states of nature by the conditional probability outcomes and adding of the result of each of them to obtain the marginal probability values as shown on Table 5.

(ii) The values of the marginal probabilities are 0.2515 for economic efficiency; 0.0599 for federal economic redistribution; 0.0908 for regional economic redistribution; 0.0374 for State economic redistribution; 0.0423 for local economic redistribution; 0.0941 for social well-being; 0.0987 for youth empowerment; 0.1048 for environmental quality improvement; 0.0288 for gender equality and 0.2061 for security.

4.2.5 Determination of Posterior Probability Outcomes at First Iteration

The Posterior Probability $P(N_i/B_i) = P(N_i \cap B_i)/P(B_i)$ where $P(B_i)$ is the values of the marginal probabilities which is the total sum of each values of the joint probabilities outcomes $P(N_i \cap B_i)$.

The Posterior Probability Outcomes at first iteration on Table 6 is computed by dividing each states of nature (N_i) for i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 by each values of marginal probability outcomes $P(B_i)$ for i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 under each group values. These results are shown on Table 6 below.

Table 6: Posterior Probability Outcomes at First Iteration

Outcomes B_i	**Probability** $P(B_i)$	**States of Nature** N_i	**Joint Probability** $P(B_i \cap N_i) = P(N_i) P(B_i/N_i)$	**Posterior Probability** $P(N_i/B_i) = P(N_i \cap B_i)/ P(B_i)$
B_1	0.2515	N_1	0.0016	0.0016/0.2515 = 0.0064
		N_2	0.0119	0.0119/0.2515 = 0.0473
		N_3	0.003	0.003/0.2515 = 0.0119
		N_4	0.0032	0.0032/0.2515 = 0.0127
		N_5	0.0153	0.0153/0.2515 = 0.0608
		N_6	0.020	0.020/0.2515 = 0.0795
		N_7	0.0153	0.0153/0.2515 = 0.0608
		N_8	0.0126	0.0126/0.2515 = 0.0501

		N_9	0.0128	0.0128/0.2515 = 0.0509
		N_{10}	0.1558	0.1558/0.2515 = 0.6195
B_2	0.0599	N_1	0.0022	0.0022/0.0599 = 0.0367
		N_2	0.0063	0.0063/0.0599 = 0.1052
		N_3	0.003	0.003/0.0599 = 0.0501
		N_4	0.002	0.002/0.0599 = 0.0334
		N_5	0.0054	0.0054/0.0599 = 0.0902
		N_6	0.006	0.006/0.0599 = 0.1002
		N_7	0.0036	0.0036/0.0599 = 0.0601
		N_8	0.0028	0.0028/0.0599 = 0.0467
		N_9	0.0064	0.0064/0.0599 = 0.1068
		N_{10}	0.0082	0.0082/0.0599 = 0.1369
B_3	0.0908	N_1	0.0028	0.0028/0.0908 = 0.0308
		N_2	0.0084	0.0084/0.0908 = 0.0925
		N_3	0.0039	0.0039/0.0908 = 0.0430
		N_4	0.0040	0.0040/0.0908 = 0.0441
		N_5	0.0108	0.0108/0.0908 = 0.1189
		N_6	0.0110	0.0110/0.0908 = 0.1211
		N_7	0.009	0.009/0.0908 = 0.0991
		N_8	0.0091	0.0091/0.0908 = 0.1002
		N_9	0.0072	0.0072/0.0908 = 0.0793
		N_{10}	0.0246	0.0246/0.0908 = 0.2709
B_4	0.0374	N_1	0.0016	0.0016/0.0374 = 0.0428
		N_2	0.0084	0.0084/0.0374 = 0.2246
		N_3	0.0024	0.0024/0.0374 = 0.0642
		N_4	0.0032	0.0032/0.0374 = 0.0856
		N_5	0.0027	0.0027/0.0374 = 0.0722
		N_6	0.003	0.003/0.0374 = 0.0802
		N_7	0.0036	0.0036/0.0374 = 0.0963
		N_8	0.0028	0.0028/0.0374 = 0.0749
		N_9	0.0056	0.0056/0.0374 = 0.1497
		N_{10}	0.0041	0.0041/0.0374 = 0.1096

Table 6: Posterior Probability Outcomes at First Iteration Continued

Outcomes B_i	**Probability P(** B_{ii} **)**	**States of Nature** N_i	**Joint Probability P(** $B_i \cap N_i$ **) = P(** N_i **) P(** B_i/N_i **)**	**Posterior Probability P(** N_i/B_i **) = P(** $N_i \cap B_i$ **)/ P(** B_i **)**
B_5	0.0423	N_1	0.0016	0.0016/0.0423 = 0.0378
		N_2	0.0084	0.0084/0.0423 = 0.1986
		N_3	0.0024	0.0024/0.0423 = 0.0567
		N_4	0.0032	0.0032/0.0423 = 0.0757
		N_5	0.0054	0.0054/0.0423 = 0.1277
		N_6	0.002	0.002/0.0423 = 0.0473
		N_7	0.0027	0.0027/0.0423 = 0.0638
		N_8	0.0028	0.0028/0.0423 = 0.0662
		N_9	0.0056	0.0056/0.0423 = 0.1324
		N_{10}	0.0082	0.0082/0.0423 = 0.1939
B_6	0.0941	N_1	0.0020	0.0020/0.0941 = 0.0213
		N_2	0.0049	0.0049/0.0941 = 0.0521
		N_3	0.003	0.003/0.0941 = 0.0319
		N_4	0.0044	0.0044/0.0941 = 0.0468
		N_5	0.0081	0.0081/0.0941 = 0.0861

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		N_6	0.009	0.009/0.0941 = 0.0956
		N_7	0.009	0.009/0.0941 = 0.0956
		N_8	0.0084	0.0084/0.0941 = 0.0893
		N_9	0.0088	0.0088/0.0941 = 0.0935
		N_{10}	0.00369	0.00369/0.0941 =0.3921
B_7	0.0987	N_1	0.0018	0.0018/0.0987 = 0.0182
		N_2	0.0049	0.0049/0.0987 = 0.0496
		N_3	0.0027	0.0027/0.0987 = 0.0274
		N_4	0.004	0.004/0.0987 = 0.0405
		N_5	0.0099	0.0099/0.0987 = 0.1003
		N_6	0.012	0.012/0.0987 = 0.1216
		N_7	0.0099	0.0099/0.0987 = 0.1003
		N_8	0.0063	0.0063/0.0987 = 0.0638
		N_9	0.0072	0.0072/0.0987 = 0.0729
		N_{10}	0.041	0.041/0.0987 = 0.1418
B_8	0.1048	N_1	0.0020	0.0020/0.1048 = 0.0191
		N_2	0.0056	0.0056/0.1048 = 0.0534
		N_3	0.003	0.003/0.1048 = 0.0286
		N_4	0.0048	0.0048/0.1048 = 0.0458
		N_5	0.0099	0.0099/0.1048 = 0.0859
		N_6	0.012	0.012/0.1048 = 0.1145
		N_7	0.0108	0.0108/0.1048 = 0.1031
		N_8	0.0077	0.0077/0.1048 = 0.0735
		N_9	0.008	0.008/0.1048 = 0.0763
		N_{10}	0.041	0.041/0.1048 = 0.3912
B_9	0.0288	N_1	0.0004	0.0004/0.0288 = 0.1389
		N_2	0.0014	0.0014/0.0288 = 0.0486
		N_3	0.0006	0.0006/0.0288 = 0.2083
		N_4	0.0012	0.0012/0.0288 = 0.0417
		N_5	0.0027	0.0027/0.0288 = 0.0938
		N_6	0.003	0.003/0.0288 = 0.1042
		N_7	0.0027	0.0027/0.0288 = 0.0938
		N_8	0.0021	0.0021/0.0288 = 0.0729
		N_9	0.0024	0.0024/0.0288 = 0.0833
		N_{10}	0.0123	0.0123/0.0288 = 0.4271

Table 6: Posterior Probability Outcomes at First Iteration Continued

Outcomes B_i	**Probability P(** B_i**)**	**States of Nature** N_i	**Joint Probability P(**$B_i \cap N_i$**) = P(**N_i**) P(**B_i/N_i**)**	**Posterior Probability P(**N_i/B_i**) = P(**$N_i \cap B_i$**)/ P(**B_i**)**
B_{10}	0.2061	N_1	0.004	0.004/0.2061 = 0.0194
		N_2	0.0098	0.0098/0.2061 = 0.0475
		N_3	0.006	0.006/0.2061 = 0.0291
		N_4	0.010	0.010/0.2061 = 0.0485
		N_5	0.0198	0.0198/0.2061 = 0.0961
		N_6	0.023	0.023/0.2061 = 0.1116
		N_7	0.0234	0.0234/0.2061 = 0.1135
		N_8	0.0154	0.0154/0.2061 = 0.0747
		N_9	0.0168	0.0168/0.2061 = 0.8151
		N_{10}	0.0779	0.0779/0.2061 = 0.3780

Discussion of Result in Table 6:
(i). The Posterior Probability was computed by dividing each states of Nature total joint probabilities (referred to as marginal probabilities) by probability values of each outcomes for each of the course of action (B_i) for B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9 and B_{10} as stated before for all N_1. N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9 and N_{10} for each set of B_i.

4.2.6 Determination of Forecast Outcomes for the Objectives/Benefits at First Iteration (Posterior Expected Opportunity Loss)

These are determined by calculating the forecast outcomes for the objectives/benefits which is the sum of the multiplication of each respective value of the posterior probability results with the conditional opportunity loss of each of the states of nature to get the Expected Opportunity Loss (EOL). The sum totals of each set of values are referred to as the Posterior Expected Opportunity Loss for each of the objectives/benefits. The Conditional Opportunity Loss is obtained for each states of nature by subtracting each objectives/benefits (B_i) from the highest benefits of each group. For example, B_1 (economic efficency); the COL for N_1 = 8.73 – 3.65 = 5.08; the COL for N_2 = 8.73 – 4.84 = 3.89; the COL for N_3 = 8.73 – 6.36 = 2.37; the COL for N_4 = 8.73 – 3.6 = 5.13; the COL for N_5 = 8.73 – 3.44 = 5.29; the COL for N_6 = 8.73 – 4.37 = 4.36 etc. the COL for N_7 = 8.73 – 4.05 = 4.68; the COL for N_8 = 8.73 – 4.22 = 4.51; the COL for N_9 = 8.73 – 1.12 = 7.61; etc.
These details are shown on Table 7 below.

Table 7: Forecast outcomes for objectives/Benefit at first iteration (Posterior Expected Opportunity Loss)

States of Nature	B_1 (Econic Efficency)			B_2 (Federal Economic Redistribution)			B_3 (Regional Economic Redistribution)			B_4 (State Economic Redistribution			B_5 (Local Economic Redistribution)		
	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL
N_1	0.0064	5.08	0.0325	0.0367	0	0	0.0308	4.59	0.1414	0.0428	17.49	0.7477	0.0378	4.95	0.1796
N_2	0.0473	3.89	0.1840	0.1052	5.83	0.6133	0.0925	4.79	0.4431	0.2246	19.94	4.4785	0.1986	15.95	3.1677
N_3	0.0119	2.37	0.0282	0.0501	3.78	0.1894	0.0430	3.09	0.1329	0.0642	15.31	0.9829	0.0567	9.70	0.5500
N_4	0.0127	5.13	0.0652	0.0334	3.70	0.1236	0.0441	5.35	0.2359	0.0856	17.58	1.5048	0.0757	18.28	1.3838
N_5	0.0608	5.29	0.3216	0.0902	4.09	0.3689	0.1189	5.61	0.6670	0.0722	17.53	0.2657	0.1277	15.88	2.0279
N_6	0.0795	4.36	0.3466	0.1002	7.92	0.7936	0.1211	4.57	0.5534	0.0802	14.38	0.1533	0.0473	13.00	0.6149
N_7	0.0608	4.68	0.2845	0.0601	7.33	0.4405	0.0991	4.91	0.4866	0.0963	14.81	1.4262	0.0638	10.45	0.6667
N_8	0.0501	4.51	0.2260	0.0467	6.99	0.3264	0.1002	4.76	0.4770	0.0749	13.57	1.0164	0.0662	11.13	0.7369
N_9	0.0509	7.61	0.3873	0.1068	12.01	1.2827	0.0793	8.0	0.6344	0.1497	22.44	3.3593	0.1324	18.96	2.5103
N_{10}	0.6195	0	0	0.1369	2.43	0.3327	0.2709	0	0	0.1096	0	0	0.1939	0	0
	Posterior EOL		1.8959			4.4711			3.7717			15.9348			11.8377
States of Nature	B_6 (Social Well-being			B_7 (Youth Empowerment)			B_8 (Environmental Quality Improvement)			B_9 (Gender Equality)			B_{10} (Security)		
	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL
N_1	0.0213	2.69	0.0573	0.0182	9.01	0.1640	0.0191	2.87	0.0548	0.1389	4.07	0.5653	0.0194	0	0
N_2	0.0521	16.54	0.8617	0.0496	22.00	1.0912	0.0534	13.77	0.7353	0.0486	11.25	0.5468	0.0475	77.26	3.6699
N_3	0.0319	11.92	0.3802	0.0274	15.58	0.4269	0.0286	6.51	0.1862	0.2083	10.00	2.0830	0.0291	80.76	2.3501
N_4	0.0468	18.73	0.8766	0.0405	22.52	0.9121	0.0458	13.63	0.6243	0.0417	11.68	0.4891	0.0485	79.56	3.8587
N_5	0.0861	20.57	1.7711	0.1003	22.61	2.2678	0.0859	13.52	1.1614	0.0938	11.49	1.778	0.0961	79.44	7.6342
N_6	0.0956	13.44	1.2849	0.1216	15.37	1.8690	0.1145	7.22	0.8267	0.1042	9.12	0.9503	0.1116	62.93	7.0230
N_7	0.0956	11.80	1.1281	0.1003	14.61	1.4654	0.1031	9.65	0.9949	0.0938	10.42	0.9774	0.1135	62.38	7.0801
N_8	0.0893	10.77	0.9618	0.0638	13.69	0.8734	0.0735	8.06	0.5924	0.0729	9.68	0.9057	0.0747	62.21	4.6471
N_9	0.0935	19.22	1.7971	0.0729	22.61	1.6483	0.0763	5.7	1.1117	0.0833	15.82	1.3178	0.8151	77.15	62.8850
N_{10}	0.3921	0	0	0.1418	0	0	0.3912	0	0	0.4271	0	0	0.3780	41.69	15.7588
	Posterior EOL		9.1188			10.7181			6.2877			8.7112			114.9069

Discussion of Results in Table 7: (i). The forecast outcomes for the objectives/benefits (Posterior Expected Opportunity Loss) was obtained by multiplying each of the posterior probabilities for each state of nature by the Conditional Opportunity Loss (COL) and adding up the results. (ii) The total Posterior Expected Opportunity Loss (EOL) for the objectives are; ₦1.8959 billion for economic efficiency; ₦4.4711 billion for federal economic redistribution; ₦3.7717 billion for regional economic redistribution; ₦15.9348 billion for state economic redistribution; ₦11.8377 billion for local economic redistribution; ₦9.1188 billion for social well-being; ₦10.81 billion for youth empowerment; ₦6.2877 billion for environmental quality improvement; ₦8.7112 billion for gender equality; and ₦114.9069 billion for security.

4.2.7 Determination of the Expected Value of Sample Information at First Iteration

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These are calculated based on the information from Marginal probabilities multiplied by values of Expected Opportunity Loss (E.O.L.). The expected value of sample information was obtained by multiplying Posterior Expected Opportunity Loss (EOL) with the Marginal probabilities of various outcomes as shown on Table 8.

Table 8: The Expected Value of Sample Information at First Iteration

Outcomes B_i	**Marginal probability** $P(B_i)$	**Expected Opportunity Loss (E.O.L.)**	**Expected Value of Sample Information**
B_1	0.2515	1.8759	0.4718
B_2	0.0599	4.4711	0.2678
B_3	0.0908	3.7717	0.3425
B_4	0.0374	15.9348	0.5960
B_5	0.0423	11.8377	0.5001
B_6	0.0941	9.1188	0.8581
B_7	0.0987	10.7181	1.0579
B_8	0.1048	6.2877	0.6590
B_9	0.0288	8.7112	0.2509
B_{10}	0.2061	114.9069	23.6823
EVSI	**TOTAL**		**28.6864**

Discussion of Results in Table 8

(i) The Expected Value of Sample Information (EVSI) for each of the objectives/benefits is obtained by multiplying the marginal probabilities of each objectives by the Expected Opportunity Loss of each objectives. The values are ₦0.4718 billion for economic efficiency; ₦0.278 billion for federal economic redistribution₦0.3425 billion for regional economic redistribution; ₦0.5960 billion for state economic redistribution; ₦0.5001 billion for local economic redistribution; ₦0.8581 billion for social well-being; ₦1.0579 billion for youth empowerment; ₦0.659 billion for environmental quality improvement; ₦0.2509 billion for gender equality; and ₦23.6823 billion for security.

(ii) The total Expected Value of Sample Information (EVSI) of ₦28, 6564 billion indicates the money which can be paid for hiring the services of consultants for the River Basin operation yield for all purposes which include; Irrigation Agriculture, Hydroelectric Power Generation, Water Supply, Navigation, Drainage/Dredging, Flood Control, Recreation/Tourism, Erosion Control, Plantation / Forestry, Reservoir/Gullies if all the objectives as stated are to be achieved for optimization of resources utilization in Anambra-Imo River Basin Development Authority covering the five (5) Eastern states of Nigeria.

4.3 Second Bayesian Decision Model Iteration Process

The Bayesian theory can be subjective but its subjectivity can be employed as a powerful attribute which considers experts' unbiased opinion as input into the policy iteration algorithm to produce an optimum solution or decision. It describes the magnitude of difference between the alternative actions and provides a variety of estimates for consideration. The decision problem involving prior probabilities are called "data problems" which the second and third iterations tend to achieve.

It should be noted that the Bayesian Decision Model or Payoff Matrix involves the policy algorithm which can handle number of "state of nature" and alternative course of action infinitely. This has justified the need for the second iteration process of the Bayesian Decision Model to improve on the results on the first iteration process.

4.3.1 Determination of Expected Monetary Values (EMVs) on Second (2nd) Iteration of Bayesian analysis with the Expected Profit with Perfect Information (EPPI) and Expected Value of Sample Information.

The optimal Bayes strategy is generally referred to as one which maximizes the expected monetary value. The expected (or mean) value is the long run average value that would result if the decision were repeated a large number of times.

The Posterior probability of the course of action having the maximum Expected Monetary Value (EMV*) in the first iteration is used in the second iteration process as prior probability. The revised probabilities will be used to recalculate the Expected Monetary Value (EMV) which was generated based on perfect information. This can be referred to as value with data and are stated on Table 9 as shown.

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In this case, the benefits that has the maximum Expected Monetary Value (EMV*) is on B_1 with the values in the Posterior probabilities as in (B_1) which are; $N_1 = 0.0064$, $N_2 = 0.0473$, $N_3 = 0.0119$, $N_4 = 0.0127$, $N_5 = 0.0608$, $N_6 = 0.0795$, $N_7 = 0.0608$, $N_8 = 0.0501$, $N_9 = 0.0509$, $N_{10} = 0.6195$.

Table 9: Calculation of Expected Monetary Values (EMVs) at Second (2nd) Iteration

States of Nature	Prior Probability $P(N_i)$	Conditional Net Benefits Course of Action in Billions of Naira										Expected Net Benefits in Billions of Naira Course of Action									
		S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
N_1	0.0064	3.65	4.84	6.36	3.60	3.44	4.37	4.05	4.22	1.12	8.73	0.0234	0.0310	0.0407	0.0230	0.0220	0.0280	0.0259	0.0270	0.0072	0.0559
N_2	0.0473	13.38	7.55	9.60	9.68	9.29	5.46	6.05	6.39	1.37	10.95	0.6329	0.3571	0.4541	0.4579	0.4394	0.2583	0.2862	0.3022	0.0648	0.5179
N_3	0.0119	4.54	4.34	6.04	3.78	3.52	4.56	4.22	4.37	1.13	9.13	0.0540	0.5165	0.0719	0.4500	0.0419	0.0543	0.0502	0.0520	0.0134	0.1086
N_4	0.0127	8.30	5.83	10.46	8.19	8.24	11.39	10.96	12.20	3.33	25.77	0.1054	0.0740	0.1328	0.1040	0.1046	0.1447	0.1392	0.1549	0.0423	1.3283
N_5	0.0608	17.21	6.01	12.26	3.68	6.08	8.96	11.51	10.83	3.00	21.96	1.0464	0.3654	0.7454	0.2237	0.3697	0.5448	0.6998	0.6585	0.1824	1.3352
N_6	0.0795	19.43	5.58	10.20	3.39	1.55	8.68	10.32	11.35	2.90	22.12	1.5447	0.4436	0.8109	0.2695	0.1232	0.6901	0.8204	0.9023	0.2301	1.7585
N_7	0.0608	16.93	3.94	10.36	3.42	3.33	10.57	11.33	12.25	3.33	25.94	1.0293	0.2396	0.6299	0.2079	0.2025	0.6427	0.6889	0.7448	0.2025	1.5772
N_8	0.0501	13.91	3.01	10.27	3.15	3.26	9.56	7.13	8.72	2.21	16.78	0.6969	0.1508	0.5145	0.1578	0.1633	0.4790	0.3572	0.4369	0.1107	0.8407
N_9	0.0509	14.01	6.83	8.08	6.40	6.59	8.96	7.66	8.40	2.26	13.88	0.7131	0.3476	0.4113	0.3258	0.3354	0.4561	0.3899	0.4276	0.1150	0.9203
N_{10}	0.6195	82.72	5.66	2.16	3.36	3.48	19.99	20.54	20.71	5.77	41.23	51.3689	3.5064	1.3381	2.0815	2.1559	12.3838	12.7245	12.8298	3.5745	25.5420
		Expected Monetary Values (EMVs) =										**57.215**	**6.032**	**5.1496**	**4.3011**	**3.9579**	**15.6818**	**16.1822**	**16.536**	**4.5429**	**32.9836**

Discussion of Results in Table 9

The information on Table 4.38 shows that the expected monetary values of each of the objectives for the second iteration are: ₦57.215 billion for economic efficency; ₦6.032 billion on federal economic redistribution; ₦5.1496 billion for regional economic redistribution; ₦4.3011 billion for state economic redistribution; ₦3.95 billion for local economic redistribution; ₦15.6818 billion for fsocial well-being;—₦16.1822 billion youth empowerment; ₦16.563 billion for environmental quality improvement; ₦4.5429 billion for gender equality; and ₦32.9836 billion for security. The policy algorithm of Bayesian Model at 2nd iteration of EMVs is an improvement from the first iteration. The maximum Expected Monetary Value (EMV*) = ₦57.215 billion on economic efficiency. This shows that with information provided by expert or consultant the maximum Expected Monetary Value (EMV*) increased. Referring to the data on Table 4.38, the maximum benefit for each state of nature is used to calculate the Expected Profit with Perfect Information (EPPI) i.e. EPPI = 0.0064 (8.73) + 0.0473(13.38) + 0.0119 (9.13) + 0.0127(25.77) + 0.0608 (21.96) + 0.0795(22.12) + 0.0608 (25.94) + 0.0501(16.78) + 0.0509 (18.08) + 0.6195 (82.92) = ₦58.9254 billion. The Expected Value of Perfect Information (EVPI) = EPPI – EMV* = ₦58.9254 - ₦57.215 billion = ₦1.7104 billion. For each of the forecast result the prior and posterior probabilities are calculated in Tables 4.39 and 4.40 respectively.

4.3.2 Determination of Joint Probabilities Outcomes at Second (2nd) Iteration

The determination of Joint Probabilities Outcomes at Second (2nd) Iteration was obtained by multiplying the revised State of nature (Prior) probabilities P(Ni) from posterior probability outcomes of the first iteration with the conditional probability outcomes P(Bi/N). These are calculated for each course of action outcomes (Bi) as shown on Table 10 below.

The Joint Probabilities are calculated by multiplying value of prior probability by conditional probability which will be totaled to get the marginal probability.

Table .10: Joint Probabilities Outcomes at Second (2nd) Iteration

States of Nature (N_i)	Prior Probability $P(N_i)$	Outcomes (B_i)	Conditional Probability $P(B_i/N_i)$	Joint Probability $P(B_i \cap N_i) = P(N_i) P(B_i/N_i)$									
				B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
N_1	0.0064	B_1	0.08	0.0005									
		B_2	0.11		0.0007								
		B_3	0.14			0.0009							
		B_4	0.08				0.0005						
		B_5	0.08					0.0005					
		B_6	0.10						0.0006				
		B_7	0.09							0.0006			
		B_8	0.10								0.0006		
		B_9	0.02									0.0001	
		B_{10}	0.20										0.0013
N_2	0.0473	B_1	0.17	0.0080									
		B_2	0.09		0.0043								
		B_3	0.12			0.0057							
		B_4	0.12				0.0057						
		B_5	0.12					0.0057					
		B_6	0.07						0.0033				
		B_7	0.07							0.0033			
		B_8	0.08								0.0038		
		B_9	0.02									0.0009	
		B_{10}	0.14										0.0066
N_3	0.0119	B_1	0.10	0.0012									
		B_2	0.10		0.0012								
		B_3	0.13			0.0015							
		B_4	0.08				0.0010						
		B_5	0.08					0.001					
		B_6	0.10						0.0012				
		B_7	0.09							0.0011			
		B_8	0.10								0.0012		
		B_9	0.02									0.0002	
		$_{10}$	0.20										0.0024

Table 10: Joint Probabilities Outcomes at Second (2nd) Iteration Continued

States of Nature (N_i)	Prior Probability $P(N_i)$	Outcomes (B_i)	Conditional Probability $P(B_i/N_i)$	Joint Probability $P(B_i \cap N_i) = P(N_i) P(B_i/N_i)$									
				B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
N_4	0.0127	B_1	0.08	0.0010									
		B_2	0.05		0.0006								
		B_3	0.10			0.0013							
		B_4	0.08				0.001						
		B_5	0.08					0.001					
		B_6	0.11						0.0014				
		B_7	0.10							0.0013			
		B_8	0.10								0.0013		
		B_9	0.03									0.0004	
		B_{10}	0.25										0.0032
N_5	0.0608	B_1	0.17	0.0003									
		B_2	0.06		0.0036								
		B_3	0.12			0.0073							
		B_4	0.03				0.0018						
		B_5	0.06					0.0036					
		B_6	0.09						0.0055				
		B_7	0.11							0.0067			
		B_8	0.11								0.0067		
		B_9	0.03									0.0018	
		B_{10}	0.22										0.0134
N_6	0.0795	B_1	0.20	0.0159									
		B_2	0.06		0.0048								
		B_3	0.11			0.0087							
		B_4	0.03				0.0024						
		B_5	0.02					0.0016					
		B_6	0.09						0.0072				
		B_7	0.11							0.0087			
		B_8	0.12								0.0095		
		B_9	0.03									0.0024	
		B_{10}	0.23										0.00183

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Table 10: Joint Probabilities Outcomes at Second (2nd) Iteration Continued

States of Nature (N_i)	Prior Probability $P(N_i)$	Outcomes (B_i)	Conditional Probability $P(B_i/N_i)$	Joint Probability $P(B_i \cap N_i) = P(N_i)\, P(B_i/N_i)$									
				B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
N_7	0.0608	B_1	0.17	0.0103									
		B_2	0.04		0.0024								
		B_3	0.10			0.0061							
		B_4	0.04				0.0024						
		B_5	0.03					0.0018					
		B_6	0.10						0.0061				
		B_7	0.11							0.0067			
		B_8	0.12								0.0073		
		B_9	0.03									0.0018	
		B_{10}	0.26										0.0158
N_8	0.0501	B_1	0.18	0.009									
		B_2	0.04		0.002								
		B_3	0.13			0.0065							
		B_4	0.04				0.002						
		B_5	0.04					0.002					
		B_6	0.12						0.006				
		B_7	0.09							0.0045			
		B_8	0.11								0.0055		
		B_9	0.03									0.0015	
		B_{10}	0.22										0.011
N_9	0.0509	B_1	0.16	0.0081									
		B_2	0.08		0.0041								
		B_3	0.09			0.0046							
		B_4	0.07				0.0036						
		B_5	0.07					0.0036					
		B_6	0.10						0.0051				
		B_7	0.09							0.0046			
		B_8	0.10								0.0051		
		B_9	0.03									0.0015	

Table 10: Joint Probabilities Outcomes at Second (2nd) Iteration Continued

States of Nature (N_i)	Prior Probability $P(N_i)$	Outcomes (B_i)	Conditional Probability $P(B_i/N_i)$	Joint Probability $P(B_i \cap N_i) = P(N_i)\, P(B_i/N_i)$									
				B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
N_{10}	0.6195	B_1	0.38	0.2354									
		B_2	0.02		0.0124								
		B_3	0.06			0.0372							
		B_4	0.01				0.0062						
		B_5	0.02					0.0124					
		B_6	0.09						0.0558				
		B_7	0.10							0.0620			
		B_8	0.10								0.0620		
		B_9	0.03									0.0186	
		B_{10}	0.19										0.1177
		Marginal Probability =		**0.2987**	**0.0361**	**0.0798**	**0.0266**	**0.026**	**0.0922**	**0.0955**	**0.1030**	**0.0292**	**0.2004**

Discussion of Results in Table 10:

(i). The joint probabilities outcomes were calculated by multiplying prior probability of each states of nature by the conditional probability outcomes and adding of the result of each of them to obtain the marginal probability values as shown on Table 10.

(ii) The marginal probabilities values are: 0.2987 for economic efficiency; 0.0361 for federal economic redistribution; 0.0798 for regional economic redistribution; 0.0266 for state economic redistribution; 0.026 for local economic redistribution; 0.0922 forsocial well-being; 0.0955 for youth empowerment; 0.1030 for environmental quality improvement; 0.0292 for gender equality and 0.2004 for security.

(iii) Comparing the second iteration results with the results obtained from first iteration, there was an increase in joint probability for B_1 (economic efficiency) = 0.2987 and B_9 (gender equality) = 0.0292 while other show a reduction of the values.

4.3.3 Determination of Posterior Probability Outcomes at Second Iteration

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The Posterior Probability $P(N_i/B_i) = P(N_i \cap B_i)/P(B_i)$ where $P(B_i)$ is the values of the marginal probabilities which is the total sum of each values of the joint probabilities outcomes $P(N_i \cap B_i)$.

The Posterior Probability Outcomes at first iteration is computed by dividing each states of nature (N_i) for i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 by each values of marginal probability outcomes $P(B_i)$ for i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 under each group values. These results are shown on Table 11 below.

Table 11: Posterior Probability Outcomes at Second Iteration

Outcomes B_i	**Probability P(** B_{ii}**)**	**States of Nature (**N_i**)**	**Joint Probability** $P(B_i \cap N_i)=P(N_i)\ P(B_i/N_i)$	**Posterior Probability** $P(N_i/B_i) = P(N_i \cap B_i)/P(B_i)$
B_1	0.2987	N_1	0.0005	0.0005/0.2987= 0.0017
		N_2	0.008	0.0080/0.2987 = 0.0024
		N_3	0.0012	0.0012/0.2987 = 0.0040
		N_4	0.001	0.0010/0.2987 = 0.0043
		N_5	0.0103	0.0103/0.2987 = 0.0345
		N_6	0.0159	0.0159/0.2987 = 0.0532
		N_7	0.0103	0.0103/0.2987 = 0.0345
		N_8	0.009	0.0090/0.2987 = 0.0276
		N_9	0.0081	0.0081/0.2987 = 0.0281
		N_{10}	0.2354	0.2354/0.2987 = 0.7881
B_2	0.0361	N_1	0.0007	0.0007/0.0361 = 0.0194
		N_2	0.0043	0.0043/0.0361 = 0.1191
		N_3	0.0012	0.0012/0.0361 = 0.0332
		N_4	0.0006	0.0006/0.0361 = 0.0166
		N_5	0.0036	0.0036/0.0361 = 0.0997
		N_6	0.0048	0.0048/0.0361 = 0.1330
		N_7	0.0024	0.0024/0.0361 = 0.0665
		N_8	0.002	0.002-/0.0361 = 0.0554
		N_9	0.0081	0.0081/0.0361 = 0.2244
		N_{10}	0.0124	0.0124/0.0361 = 0.3435
B_3	0.0798	N_1	0.0009	0.0009/0.0798 = 0.0113
		N_2	0.0057	0.0057/0.0798 = 0.0714
		N_3	0.0015	0.0015/0.0798 = 0.0188
		N_4	0.0013	0.0013/0.0798 = 0.0163
		N_5	0.0073	0.0073/0.0798 = 0.0915
		N_6	0.0087	0.0087/0.0798 = 0.1090
		N_7	0.0061	0.0061/0.0798 = 0.0764
		N_8	0.0065	0.0065/0.0798 = 0.0815
		N_9	0.0046	0.0046/0.0798 = 0.0576
		N_{10}	0.0372	0.0372/0.0798 = 0.4662
B_4	0.0266	N_1	0.0005	0.0005/0.0266 = 0.0188
		N_2	0.0057	0.0057/0.0266 = 0.2143
		N_3	0.001	0.0010/0.0266 = 0.0376
		N_4	0.001	0.0010/0.0266 = 0.0376
		N_5	0.0018	0.0018/0.0266 = 0.0677
		N_6	0.0024	0.0024/0.0266 = 0.0902
		N_7	0.0024	0.0024/0.0266 = 0.0902
		N_8	0.002	0.002/0.0266 = 0.0752
		N_9	0.0036	0.0036/0.0266 = 0.1353
		N_{10}	0.0062	0.0062/0.0266 = 0.2331

Table 11: Posterior Probability Outcomes at Second Iteration continued

Outcomes B_i	**Probability P(** B_{ii}**)**	**States of Nature (**N_i	**Joint Probability P(**$B_i \cap N_i$**)=P(**N_i**) P(** B_i/N_i**)**	**Posterior Probability P(**N_i/B_i**) = P(**$N_i \cap B_i$**)/ P(**B_i**)**
B_5	0.026	N_1	0.0005	0.0005/0.026 = 0.0192
		N_2	0.0057	0.0057/0.026 = 0.2192
		N_3	0.001	0.0010/0.026 = 0.0385
		N_4	0.001	0.0010/0.026 = 0.0385
		N_5	0.0036	0.0036/0.026= 0.1385
		N_6	0.0016	0.0016/0.026 = 0.0615
		N_7	0.0018	0.0018/0.026 = 0.0692
		N_8	0.002	0.0020/0.026 = 0.0769
		N_9	0.0036	0.0036/0.026 = 0.1385
		N_{10}	0.0124	0.0124/0.026= 0.4769
B_6	0.0922	N_1	0.0006	0.0006/0.0922= 0.0065
		N_2	0.0033	0.0033/0.0922 = 0.0358
		N_3	0.0012	0.0012/0.0922 = 0.0130
		N_4	0.0014	0.0014/0.0922 = 0.0152
		N_5	0.0055	0.0055/0.0922 = 0.0542
		N_6	0.0072	0.0072/0.0922 = 0.0781
		N_7	0.0061	0.0061/0.0922 = 0.0662
		N_8	0.006	0.006/0.0922 = 0.0651
		N_9	0.0051	0.0051/0.0922 = 0.0553
		N_{10}	0.0558	0.0558/0.0922 = 0.6052
B_7	0.0995	N_1	0.0006	0.0006/0.0995 = 0.0060
		N_2	0.0033	0.0033/0.0995 = 0.0332
		N_3	0.0011	0.0011/0.0995 = 0.0111
		N_4	0.0013	0.0013/0.0995 = 0.0131
		N_5	0.0067	0.0067/0.0995 = 0.0673
		N_6	0.0087	0.0087/0.0995 = 0.0874
		N_7	0.0067	0.0067/0.0995 = 0.0673
		N_8	0.0045	0.0045/0.0995 = 0.0452
		N_9	0.0046	0.0046/0.0995 = 0.0462
		N_{10}	0.0620	0.0620/0.0995 = 0.0623
B_8	0.1030	N_1	0.0006	0.0006/0.1030 = 0.0058
		N_2	0.0038	0.0038/0.1030 = 0.0369
		N_3	0.0012	0.0012/0.1030 = 0.0117
		N_4	0.0013	0.0013/0.1030 = 0.0126
		N_5	0.0067	0.0067/0.1030 = 0.0650
		N_6	0.0095	0.0095/0.1030 = 0.0922
		N_7	0.0073	0.0073/0.1030 = 0.0709
		N_8	0.0055	0.0055/0.1030 = 0.0534
		N_9	0.0051	0.0051/0.1030 = 0.0495
		N_{10}	0.0620	0.0620/0.1030 = 0.6019
B_9	0.0292	N_1	0.0001	0.0001/0.0292 = 0.0034
		N_2	0.0009	0.0009/0.0292 = 0.0308
		N_3	0.0002	0.0002/0.0292 = 0.0685
		N_4	0.0004	0.0004/0.0292 = 0.0137
		N_5	0.0018	0.0018/0.0292 = 0.0616
		N_6	0.0024	0.0024/0.0292 = 0.0822
		N_7	0.0018	0.0018/0.0292 = 0.0616
		N_8	0.0015	0.0015/0.0292 = 0.0514

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		N_9	0.0015	0.0015/0.0292 = 0.0514
		N_{10}	0.0186	0.0186/0.0292 = 0.6370

Table 11: Posterior Probability Outcomes at Second Iteration continued

Outcomes B_i	**Probability P(** B_{ii}**)**	**States of Nature** (N_i)	**Joint Probability P(** $B_i \cap N_i$**)=P(** N_i**) P(** B_i/N_i**)**	**Posterior Probability P(** N_i/B_i**) = P(** $N_i \cap B_i$**)/ P(** B_i**)**
B_{10}	0.2004	N_1	0.0013	0.0013/0.2004 = 0.0065
		N_2	0.0066	0.0066/0.2004 = 0.0329
		N_3	0.0024	0.0024/0.2004 = 0.0120
		N_4	0.0032	0.0032/0.2004 = 0.0160
		N_5	0.0134	0.0134/0.2004 = 0.0669
		N_6	0.0183	0.0183/0.2004 = 0.0913
		N_7	0.0158	0.0158/0.2004 = 0.0788
		N_8	0.011	0.011/0.2004 = 0.0549
		N_9	0.0107	0.0107/0.2004 = 0.0534
		N_{10}	0.1177	0.1177/0.2004 = 0.5873

Discussion of Result in Table 11

(i). The Posterior Probability was computed by dividing each states of Nature total joint probabilities (referred to as marginal probabilities) by probability values of each outcomes for each of the objectives (B_i) for B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9 and B_{10} as stated before for all N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9 and N_{10} for each set of B_i.

(ii) For example, the benefits B_1 (economic efficiency and the values of posterior probabilities under it were: N_1 (irrigated agriculture) = 0.0017, N_2 = 0.0024, N_3 = 0.0040, N_4 = 0.0043, N_5 = 0.0345, N_6 = 0.0532, N_7 = 0.0345, N_8 = 0.0276, N_9 = 0.0281 and N_{10} = 0.7881. It follows the same pattern for the benefits B_2 (Federal economic redistribution), B_3 (Regional economic redistribution), B_4(Stateeconomic redistribution), B_5 (Local economic redistribution), B_6 (Social well-being), B_7 (Youth empowerment), B_8(Enviromental quality improvement), B_9 (Gender equality) and B_{10} (Security). These are shown on Table 11.

(iii) Comparing the results from the first iterations, there are reductions in the posterior probability outcomes while only on purpose of reservoir and gullies, the value increased from 0.6915 to 0.7881.

4.3.4 Determination of Forecast Outcomes for Benefits at Second (2nd) Iteration (Posterior Expected Opportunity Loss)

This is determined by calculating the forecast outcomes for the benefits which is the sum of the multiplication of each respective value of the posterior probability results with the cconditional opportunity loss of each of the states of nature to get the Expected Opportunity Loss (EOL). The sum totals of each set of values are referred to as the Posterior Expected Opportunity Loss for each of the objectives/benefits. The Conditional Opportunity Loss is obtained for each states of nature by subtracting each net benefit (B_i) from the highest benefits of each group. For example, B_1 (Economic efficiency); the COL for N_1 = 8.73 – 3.65 = 5.08; the COL for N_2 = 8.73 – 4.84 = 3.89; the COL for N_3 = 8.73 – 6.36 = 2.37; the COL for N_4 = 8.73 – 3.6 = 5.13; the COL for N_5 = 8.73 – 3.44 = 5.29; the COL for N_6 = 8.73 – 4.37 = 4.36 etc. the COL for N_7 = 8.73 – 4.05 = 4.68; the COL for N_8 = 8.73 – 4.22 = 4.51; the COL for N_9 = 8.73 – 1.12 = 7.61; etc. This is calculated by multiplying the individual posterior probabilities with the conditional Opportunity Loss as shown on Table 12.

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Table 12 Forecast Outcomes for Objectives/Benefits at Second (2nd) Iteration (Posterior Expected Opportunity Loss)

States of Nature	B_1 (Economic Efficiency)			B_2 (Federal Economic Redistribution)			B_3 (Regional Economic Redistribution)			B_4 (State Economic Redistribution)			B_5 (Local Economic Redistribution)		
	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL
N_1	0.0017	5.08	0.0086	0.0194	0	0	0.0113	4.59	0.0519	0.0188	17.49	0.3284	0.0192	4.95	0.0912
N_2	0.0024	3.89	0.0934	0.1191	5.83	0.6944	0.0714	4.79	0.3420	0.2143	19.94	4.2731	0.2192	15.95	3.4962
N_3	0.0040	2.37	0.0095	0.0332	3.78	0.1255	0.0188	3.09	0.0581	0.0376	15.31	0.5757	0.0385	9.70	0.3735
N_4	0.0043	5.13	0.0221	0.0166	3.70	0.0614	0.0163	5.35	0.0872	0.0376	17.58	0.6610	0.0385	18.28	0.7038
N_5	0.0345	5.29	0.1825	0.0997	4.09	0.4078	0.0915	5.61	0.5133	0.0677	17.53	1.1868	0.1385	15.88	2.1994
N_6	0.0532	4.36	0.2320	0.1330	7.92	1.0534	0.1090	4.57	0.4981	0.0902	14.38	1.2971	0.0615	13.00	0.7995
N_7	0.0345	4.68	0.1615	0.0665	7.33	0.4874	0.0764	4.91	0.3751	0.0902	14.81	1.3359	0.0692	10.45	0.7231
N_8	0.0276	4.51	0.1245	0.0554	6.99	0.3872	0.0815	4.76	0.3879	0.0752	13.57	1.0205	0.0769	11.13	0.8559
N_9	0.0281	7.61	0.2138	0.2244	12.01	2.6950	0.0576	8.0	0.4608	0.1353	22.44	3.0361	0.1385	18.96	2.6260
N_{10}	0.7881	0	0	0.3435	2.43	0.8396	0.4662	0	0	0.2331	0	0	0.4769	0	0
Posterior EOL			1.0479			6.7517			2.7744			13.7146			11.0983
States of Nature	B_6 (Social Well-being)			B_7 (Youth Empowerment)			B_8 (Environmental Quality Improvement)			B_9 (Gender Equality)			B_{10} (Security)		
	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL
N_1	0.0065	2.69	0.0175	0.0060	9.01	0.0541	0.0058	2.87	0.0166	0.0034	4.07	0.0138	0.0065	0	0
N_2	0.0358	16.54	0.5921	0.0332	22.00	0.7304	0.0369	13.77	0.5081	0.0308	11.25	0.3465	0.0329	77.26	2.5419
N_3	0.0130	11.92	0.1550	0.0111	15.58	0.1729	0.0117	6.51	0.0762	0.0685	10.00	0.6850	0.0120	80.76	0.9691
N_4	0.0152	18.73	0.2847	0.0131	22.52	0.2950	0.0126	13.63	0.1717	0.0137	11.68	0.1600	0.0160	79.56	1.2730
N_5	0.0542	20.57	1.1149	0.0673	22.61	1.5217	0.0650	13.52	0.8788	0.0616	11.49	0.7078	0.0669	79.44	5.3145
N_6	0.0781	13.44	1.0497	0.0874	15.37	1.3433	0.0922	7.22	0.6657	0.0822	9.12	0.7497	0.0913	62.93	5.7455
N_7	0.0662	11.80	0.7812	0.0673	14.61	0.9833	0.0709	9.65	0.6842	0.0616	10.42	0.6419	0.0788	62.38	4.9155
N_8	0.0651	10.77	0.7011	0.0452	13.69	0.6188	0.0534	8.06	0.4304	0.0514	9.68	0.4976	0.0549	62.21	3.4153
N_9	0.0553	19.22	1.0629	0.0462	22.61	1.0446	0.0495	14.	0.7212	0.0514	15.82	1.8131	0.0534	77.15	4.1198
N_{10}	0.6052	0	0	0.0623	0	0	0.6019	0	0	0.6370	0	0	0.5873	41.69	24.4845
Posterior EOL			5.7591			6.7641			4.1529			4.6154			52.7791

Discussion of Results in Table 12: (i) The forecast outcomes for the objectives/benefits (Posterior Expected Opportunity Loss) was obtained by multiplying each of the posterior probabilities for each state of nature by the Conditional Opportunity Loss (COL) and adding up the results. (ii) The total Posterior Expected Opportunity Loss (EOL) for the objectives are; ₦1.0479 billion for economic efficiency; ₦6.7517 billion for federal economic redistribution; ₦2.7744 billion for regional economic redistribution; ₦13.7146 billion for state economic redistribution; ₦ 11.0983 billion for local economic redistribution; ₦ 5.7591 billion for social well-being; ₦ 6.7641 billion for youth empowerment; ₦ 4.1529 billion for environmental quality improvement; ₦4.6154 billion for gender equality; and ₦52.7791 billion for security.

3.5 Determination of Expected Value of Sample Information Outcomes at Second (2nd) Iteration

The Expected Value of Sample Information (EVSI) is calculated by multiplying each value of the Marginal probabilities with the Expected Opportunity Loss Values (EOL) as shown on Table 13.

Table 13: The Expected Value of Sample Information Outcomes at Second (2nd) Iteration

Outcomes B_i	**Marginal probability** $P(B_i)$	**Expected Opportunity Loss (EOL)**	**Expected Value of Sample Information**
B_1	0.2987	1.0479	0.3130
B_2	0.0361	6.7517	0.2437
B_3	0.0798	2.7744	0.2214
B_4	0.0266	13.7146	0.3648
B_5	0.0260	11.0983	0.2886
B_6	0.0922	5.7591	0.5310
B_7	0.0995	6.7641	0.6730
B_8	0.1030	4.1529	0.4277
B_9	0.0292	4.6154	0.1348
B_{10}	0.2004	52.7791	10.5769
TOTAL (EVSI) =			**13.7749 billion**

Discussion of Results in Table 13:

(i) The Expected Value of Sample Information (EVSI) for each of the objectives are obtained by multiplying the marginal probabilities of each objectives by the Expected Opportunity Loss of each the objective. The values are: ₦0.3130 billion for economic efficiency; ₦0.2437 billion for federal economic redistribution; ₦0.2214 billion for regional economic redistribution; ₦0.3648 billion for state economic redistribution; ₦0.2886 billion for local economic redistribution; ₦0.5310 billion for social well-being₦0.6730 billion for youth empowerment; ₦0.4277 billion for environmental quality improvement; ₦0.1348 billion for gender equality; and ₦10.5769 billion for security.

(ii) The total expected Value of Sample Information (EVSI) as calculated in Table 13 is ₦13.7749 billion which indicates the money which can be paid for hiring the services of consultants for the River Basin operation yield for all the ten (10) purposes of irrigation agriculture, hydroelectric power generation, water supply, navigation, drainage/dredging, flood control, recreation/tourism, erosion control, plantation/ forestry, reservoir/gullies etc. respectively.

4.4 Third (3rd) Bayesian Decision Model Iteration Process

The Posterior productivity of the course of action having the maximum Expected Monetary Value (EMV*) in the second iteration process is used in the third iteration process. The revised probabilities will be used to recalculate the Expected Monetary Value (EMV) which was generated based on perfect information. This can be referred to as when more data were provided based on the performance of the previous data.

It should be noted that the Bayesian Decision Model or Payoff Matrix involves the policy algorithm which can handle number of "state of nature" and alternative course of action infinitely. This has justified the need for the third iteration process of the Bayesian Decision Model to improve on the results on the second iteration process.

4.4.1 Determination of Expected Monetary Values (EMVs) on Third Iteration

The optimal Bayes strategy is generally referred to as one which maximizes the expected monetary value. The expected (or mean) value is the long run average value that would result if the decision were repeated a large number of times.

The Posterior probability of the course of action having the maximum Expected Monetary Value (EMV*) in the second iteration is used in the third iteration process as prior probability. The revised probabilities will be used to recalculate the Expected Monetary Value (EMV) which was generated based on perfect information. This can be referred to as value with additional data as shown.

In this case, the objective that has the maximum Expected Monetary Value (EMV*) is on B_1 which is economic efficiency with the values in the Posterior probabilities as in (B_1) which are; $N_1 = 0.0017$, $N_2 = 0.0024$, $N_3 = 0.0040$, $N_4 = 0.0043$, $N_5 = 0.0345$, $N_6 = 0.0532$, $N_7 = 0.0345$, $N_8 = 0.0276$, $N_9 = 0.0281$, $N_{10} = 0.7881$.

The Expected Monetary Values (EMVs) are calculated by multiplying each values of the conditional net benefits course of action by each value of the corresponding value of probability. Subsequently, the expected profit with perfect information is calculated as the sum of each value of the total maximum expected monetary values of each expected net benefit course of action as shown on Table 14 and the Expected Value of Perfect Information (EVPI).

Table 14: Calculation of Expected Monetary Values (EMVs) at Third (3rd) Iteration

States of Nature	Prior Probability $P(N_i)$	Conditional Net Benefits Course of Action in Billions of Naira										Expected Net Benefits in Billions of Naira Course of Action									
		S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
N_1	0.0017	3.65	4.84	6.36	3.60	3.44	4.37	4.05	4.22	1.12	8.73	0.0062	0.0082	0.0108	0.0061	0.0058	0.0074	0.0069	0.0072	0.0019	0.0148
N_2	0.0024	13.38	7.55	9.60	9.68	9.29	5.46	6.05	6.39	1.37	10.95	0.3211	0.1812	0.2304	0.2323	0.2230	0.1310	0.1452	0.1534	0.0329	0.2628
N_3	0.0040	4.54	4.34	6.04	3.78	3.52	4.56	4.22	4.37	1.13	9.13	0.0182	0.0174	0.0242	0.0151	0.0141	0.0182	0.0502	0.0175	0.0045	0.0365
N_4	0.0043	8.30	5.83	10.46	8.19	8.24	11.39	10.96	12.20	3.33	25.77	0.0357	0.0251	0.0450	0.0352	0.0354	0.0490	0.0471	0.0525	0.0143	0.1108
N_5	0.0345	17.21	6.01	12.26	3.68	6.08	8.96	11.51	10.83	3.00	21.96	0.5937	0.2073	0.4230	0.1270	0.2098	0.3091	0.3971	0.03736	0.1035	0.7576
N_6	0.0532	19.43	5.58	10.20	3.39	1.55	8.68	10.32	11.35	2.90	22.12	1.0337	0.2969	0.5426	0.1803	0.0825	0.4618	0.5490	0.6038	0.1543	1.1768
N_7	0.0345	16.93	3.94	10.36	3.42	3.33	10.57	11.33	12.25	3.33	25.94	0.5841	0.1359	0.3574	0.1180	0.1149	0.3647	0.3909	0.4226	0.1149	0.8949
N_8	0.0276	13.91	3.01	10.27	3.15	3.26	9.56	7.13	8.72	2.21	16.78	0.3839	0.0831	0.2835	0.0869	0.0900	0.2639	0.1968	0.2407	0.0610	0.4631
N_9	0.0281	14.01	6.83	8.08	6.40	6.59	8.96	7.66	8.40	2.26	18.08	0.3937	0.1919	0.2270	0.1798	0.1852	0.2518	0.2152	0.2360	0.0635	0.5080
N_{10}	0.7881	82.72	5.66	2.16	3.36	3.48	19.99	20.54	20.71	5.77	41.23	63.3493	4.4606	1.7023	2.6480	2.7426	15.7541	16.1876	16.3216	4.5473	32.4934
		Expected Monetary Values (EMVs) =										**68.7196**	**5.6076**	**3.8462**	**3.6287**	**3.7033**	**17.611**	**18.1527**	**18.4289**	**5.0981**	**36.7187**

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Discussion of Results in Table 14:
(i) The information on Table 14 shows that the expected monetary values of each of the objectives for the third iteration are: ₦68.7196 billion for economic efficiency; ₦5.6076 billion on federal economic redistribution; ₦3.8462 billion for regional economic redistribution; ₦3.6287 billion for state economic redistribution; ₦3.7033 billion for local economic redistribution; ₦17.611 billion for social well-being; ₦18.1527 billion for youth empowerment; ₦18.4289 billion for environmental quality improvement; ₦5.0981 billion for gender equality; and ₦36.7187 billion for security. (ii) The policy algorithm of Bayesian Model at third iteration of EMVs is an improvement from the second iteration. (iii). The maximum Expected Monetary Value (EMV*) = ₦68.7196 billion for economic efficiency. (iv) This shows that with information provided by expert or consultant the maximum Expected Monetary Value (EMV*) increased. Referring to the data on Table 14, the maximum benefit for each states of nature is used to calculate the Expected Profit with Perfect Information (EPPI)= 0.0017(8.73) + 0.024(13.38) + 0.004 (9.13) + 0.0043 (25.77) + 0.0345 (21.96) + 0.0532 (22.12) + 0.0345(25.94) + 0.0276 (16.78) + 0.0281(18.08) + 0.7881 (82.92) = ₦69.633
The Expected Value of Perfect Information (EVPI) = EPPI – EMV = ₦69.633 - ₦68.7196 billion= ₦0.9134 billion
For each of the forecast result, the Prior and Posterior probabilities are calculated in Tables 15 and 16 respectivly.

4.4.2 Determination of Joint Probabilities Outcomes on Third (3rd) Iteration

The determination of Joint Probabilities Outcomes at third (3rd) Iteration was obtained by multiplying the revised State of nature (Prior) probabilities $P(N_i)$ from posterior probability outcomes of the second iteration with the conditional probability outcomes $P(B_i/N)$. These are calculated for each courses of action outcomes (B_i) as shown on Table 15 below.The Joint Probabilities are calculated by multiplying value of prior probability by conditional probability which will be totaled to get the marginal probability

Table 15: Joint Probabilities Outcomes at Third (3rd) Iteration

States of Nature (N_i)	Prior Probability $P(N_i)$	Outcomes (B_i)	Conditional Probability $P(B_i/N_i)$	Joint Probability $P(B_i \cap N_i) = P(N_i)\,P(B_i/N_i)$									
				B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
N_1	0.0017	B_1	0.08	0.0001									
		B_2	0.11		0.0002								
		B_3	0.14			0.0002							
		B_4	0.08				0.0001						
		B_5	0.08					0.0001					
		B_6	0.10						0.0002				
		B_7	0.09							0.0002			
		B_8	0.10								0.0002		
		B_9	0.02									0.00003	
		B_{10}	0.20										0.0003
N_2	0.024	B_1	0.17	0.0041									
		B_2	0.09		0.0022								
		B_3	0.12			0.0029							
		B_4	0.12				0.0029						
		B_5	0.12					0.0029					
		B_6	0.07						0.0017				
		B_7	0.07							0.0017			
		B_8	0.08								0.0019		
		B_9	0.02									0.0005	
		B_{10}	0.14										0.0034

Table 15: Joint Probabilities Outcomes at Third (3rd) Iteration Continued

States of Nature (N_i)	Prior Probability $P(N_i)$	Outcomes (B_j)	Conditional Probability $P(B_j/N_i)$	Joint Probability $P(B_j \cap N_i) = P(N_i)\,P(B_j/N_i)$									
				B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
N_3	0.004	B_1	0.10	0.0004									
		B_2	0.10		0.0004								
		B_3	0.13			0.0005							
		B_4	0.08				0.0003						
		B_5	0.08					0.0003					
		B_6	0.10						0.0004				
		B_7	0.09							0.0004			
		B_8	0.10								0.0004		
		B_9	0.02									0.00008	
		B_{10}	0.20										0.0008
N_4	0.0043	B_1	0.08	0.0003									
		B_2	0.05		0.0002								
		B_3	0.10			0.0004							
		B_4	0.08				0.0003						
		B_5	0.08					0.0003					
		B_6	0.11						0.0005				
		B_7	0.10							0.0004			
		B_8	0.12								0.0005		
		B_9	0.03									0.0001	
		B_{10}	0.25										0.0011
N_5	0.0345	B_1	0.17	0.0059									
		B_2	0.06		0.0021								
		B_3	0.12			0.0041							
		B_4	0.03				0.0010						
		B_5	0.06					0.0021					
		B_6	0.09						0.0031				
		B_7	0.11							0.0038			
		B_8	0.11								0.0038		
		B_9	0.03									0.0010	
		B_{10}	0.22										0.0076

Table 15: Joint Probabilities Outcomes at Third (3rd) Iteration Continued

States of Nature (N_i)	Prior Probability $P(N_i)$	Outcomes (B_j)	Conditional Probability $P(B_j/N_i)$	Joint Probability $P(B_j \cap N_i) = P(N_i)\,P(B_j/N_i)$									
				B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
N_6	0.0532	B_1	0.20	0.0106									
		B_2	0.06		0.0032								
		B_3	0.11			0.0059							
		B_4	0.03				0.0016						
		B_5	0.02					0.0011					
		B_6	0.09						0.0048				
		B_7	0.11							0.0059			
		B_8	0.12								0.0064		
		B_9	0.03									0.0016	
		B_{10}	0.23										0.0122
N_7	0.0345	B_1	0.17	0.0059									
		B_2	0.04		0.0014								
		B_3	0.10			0.0035							
		B_4	0.04				0.0014						
		B_5	0.03					0.0010					
		B_6	0.10						0.0035				
		B_7	0.11							0.0038			
		B_8	0.12								0.0041		
		B_9	0.03									0.0010	
		B_{10}	0.26										0.0090
N_8	0.0276	B_1	0.18	0.0050									
		B_2	0.04		0.0011								
		B_3	0.13			0.0036							
		B_4	0.04				0.0011						
		B_5	0.04					0.0011					
		B_6	0.12						0.0033				
		B_7	0.09							0.0025			
		B_8	0.11								0.0030		
		B_9	0.03									0.0008	
		B_{10}	0.22										0.0061

Table 15: Joint Probabilities Outcomes at Third (3rd) Iteration Continued

States of Nature (N_i)	Prior Probability $P(N_i)$	Outcomes (B_i)	Conditional Probability $P(B_i/N_i)$	Joint Probability $P(B_i \cap N_i) = P(N_i)\,P(B_i/N_i)$									
				B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
N_9	0.0281	B_1	0.16	0.0045									
		B_2	0.08		0.0022								
		B_3	0.09			0.0025							
		B_4	0.07				0.002						
		B_5	0.07					0.002					
		B_6	0.10						0.0028				
		B_7	0.09							0.0025			
		B_8	0.10								0.0028		
		B_9	0.03									0.0008	
		B_{10}	0.21										0.0059
N_{10}	0.7881	B_1	0.38	0.2995									
		B_2	0.02		0.0158								
		B_3	0.06			0.0473							
		B_4	0.01				0.0079						
		B_5	0.02					0.0158					
		B_6	0.09						0.0709				
		B_7	0.10							0.0788			
		B_8	0.10								0.0788		
		B_9	0.03									0.0236	
		B_{10}	0.19										0.1497
Marginal Probability =				**0.3363**	**0.0288**	**0.0709**	**0.0188**	**0.0267**	**0.0912**	**0.1000**	**0.1019**	**0.0291**	**0.1961**

Discussion of Results in Table 15:

(i). The joint probabilities outcomes were calculated by multiplying prior probability of each states of nature by the conditional probability outcomes and adding of the result of each of them to obtain the marginal probability values as shown on Table 15.

(ii) The marginal probabilities values are: 0.3363 for economic efficiency; 0.0288 for federal economic redistribution; 0.0709 for regional economic redistribution; 0.0188 for stateeconomic redistribution; 0.0267 for local economic redistribution; 0.0912 for social well-being; 0.1000 for youth empowerment; 0.1019 for environmental quality improvement; 0.0291 for gender equality and 0.1961 for security.

(iii) Comparing this third iteration with the results obtained from second iteration, B_1 (economic efficiency) increased from 0.2987 to 0.3363; B_5 (local economic redistribution) increased from 0.026 to 0.0267; B_7 (youth empowerment) increased from 0.0955 to 0.1000.

4.4.3 Determination of Posterior Probability Outcomes on Third Iteration

The Posterior Probability $P(N_i/B_i) = P(N_i \cap B_i)/P(B_i)$ where $P(B_i)$ is the values of the marginal probabilities which is the total sum of each values of the joint probabilities outcomes $P(N_i \cap B_i)$.

The Posterior Probability Outcomes at second iteration on Table 12 is computed by dividing each states of nature (N_i) for i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 by each values of marginal probability outcomes $P(B_i)$ for i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 under each group values.

The Posterior Probability is obtained by dividing each Joint Probability Outcomes with the total of each marginal Probability Outcomes as shown in Table 16.

Table 16: Posterior Probability Outcomes at Third Iteration

Outcomes B_i	**Probability** $P(B_{ii})$	**States of Nature** (N_i)	**Joint Probability** $P(B_i \cap N_i) = P(N_i)\,P(B_i/N_i)$	**Posterior Probability** $P(N_i/B_i) = P(N_i \cap B_i)/P(B_i)$
B_1	0.3363	N_1	0.0001	0.0001/0.3363 = 0.0003
		N_2	0.0041	0.0041/0.3363 = 0.0122

		N_3	0.0004	0.0004/0.3363 = 0.0012
		N_4	0.0003	0.0003/ 0.3363 = 0.0009
		N_5	0.0059	0.0059/0.3363= 0.0175
		N_6	0.0106	0.0106/0.3363 = 0.0315
		N_7	0.0059	0.0059/0.3363 = 0.0175
		N_8	0.0050	0.0050/0.3363 = 0.0149
		N_9	0.0043	0.0043/0.3363 = 0.0134
		N_{10}	0.2995	0.2995/0.3363 = 0.8906
B_2	0.0288	N_1	0.0002	0.0002/0.0288 = 0.0069
		N_2	0.0022	0.0022/0.0288 = 0.0764
		N_3	0.0004	0.0004/0.0288 = 0.0139
		N_4	0.0002	0.0002/0.0288 = 0.0069
		N_5	0.0021	0.0021/0.0288 = 0.0729
		N_6	0.0032	0.0032/0.0288 = 0.1111
		N_7	0.0014	0.0014/0.0288 = 0.0486
		N_8	0.0011	0.0011/0.0288 = 0.0417
		N_9	0.0022	0.0022/0.0288 = 0.0764
		N_{10}	0.0158	0.0158/ 0.0288 = 0.5486

Table 16 Posterior Probability Outcomes at Third Iteration Continued

Outcomes B_i	**Probability** **P(** B_{ii}**)**	**States of Nature** (N_i)	**Joint Probability** **P(**$B_i \cap N_i$**)** **=P(**N_i**) P(** B_i/N_i**)**	**Posterior Probability** **P(**N_i/B_i**) = P(**$N_i \cap B_i$**)/ P(**B_i**)**
B_3	0.0709	N_1	0.0002	0.0002/0.0709 = 0.0028
		N_2	0.0029	0.0029/0.0709 = 0.0409
		N_3	0.0005	0.0005/0.0709 = 0.0071
		N_4	0.0004	0.0004/0.0709 = 0.0056
		N_5	0.0041	0.0041/0.0709 = 0.0578
		N_6	0.0059	0.0059/0.0709 = 0.0832
		N_7	0.0035	0.0035/0.0709 = 0.0494
		N_8	0.0036	0.0036/0.0709 = 0.0508
		N_9	0.0025	0.0025/0.0709 = 0.0353
		N_{10}	0.0473	0.0473/0.0709 = 0.6671
B_4	0.0188	N_1	0.0001	0.0001/0.0188= 0.0053
		N_2	0.0029	0.0029/0.0188= 0.1543
		N_3	0.0003	0.0003/0.0188= 0.0160
		N_4	0.0003	0.0003/0.0188= 0.0160
		N_5	0.0010	0.0010/0.0188= 0.0532
		N_6	0.0016	0.0016/0.0188= 0.0811
		N_7	0.0014	0.0014/0.0188= 0.0213
		N_8	0.0011	0.0011/0.0188= 0.0585
		N_9	0.0020	0.0020/0.0188= 0.1064
		N_{10}	0.0079	0.0079/0.0188= 0.4202
B_5	0.0267	N_1	0.0001	0.0001/0.0267= 0.0037
		N_2	0.0029	0.0029/0.0267= 0.1086
		N_3	0.0003	0.0003/0.0267= 0.0112
		N_4	0.0003	0.0003/0.0267= 0.0112
		N_5	0.0021	0.0021/0.0267= 0.0787
		N_6	0.0011	0.0011/0.0267= 0.0412
		N_7	0.0010	0.0010/0.0267= 0.0375
		N_8	0.0011	0.0011/0.0267= 0.0412
		N_9	0.0020	0.0020/0.0267= 0.0749

		N_{10}	0.0158	0.0158/0.0267= 0.5918
B_6	0.0912	N_1	0.0002	0.0002/0.0912 = 0.0022
		N_2	0.0017	0.0017/0.0912 = 0.0186
		N_3	0.0004	0.0004/0.0912 = 0.0044
		N_4	0.0005	0.0005/0.0912 = 0.0055
		N_5	0.0031	0.0031/0.0912 = 0.0340
		N_6	0.0048	0.0048/0.0912 = 0.0526
		N_7	0.0035	0.0035/0.0912 = 0.0384
		N_8	0.0033	0.0033/0.0912 = 0.0362
		N_9	0.0028	0.0028/0.0912 = 0.0307
		N_{10}	0.0709	0.0709/0.0912 = 0.7774
B_7	0.1	N_1	0.0002	0.0002/0.10 = 0.0020
		N_2	0.0017	0.0017/0.10 = 0.017
		N_3	0.0004	0.0004/0.10 = 0.004
		N_4	0.0004	0.0004/0.10 = 0.004
		N_5	0.0038	0.0038/0.10= 0.038
		N_6	0.0059	0.0059/0.10 = 0.059
		N_7	0.0038	0.0038/0.10= 0.038
		N_8	0.0025	0.0025/0.10 = 0.025
		N_9	0.0025	0.0025/0.10 = 0.025
		N_{10}	0.0788	0.0788/0.10 = 0.788

Table 16: Posterior Probability Outcomes at Third Iteration Continued

Outcomes B_i	**Probability** **P(** B_{ii}**)**	**States of Nature** N_i	**Joint Probability** $P(B_i \cap N_i)=P(N_i)$ $P(B_i/N_i)$	**Posterior Probability** $P(N_i/B_i) = P(N_i \cap B_i)/ P(B_i)$
B_8	0.1019	N_1	0.0002	0.0002/0.1019 = 0.0020
		N_2	0.0017	0.0017/0.1019 = 0.0167
		N_3	0.0004	0.0004/0.1019 = 0.0039
		N_4	0.0005	0.0005/0.1019 = 0.0049
		N_5	0.0038	0.0038/0.1019 = 0.0373
		N_6	0.0064	0.0064/0.1019 = 0.0628
		N_7	0.0041	0.0041/0.1019 = 0.0402
		N_8	0.0030	0.0030/0.1019 = 0.0294
		N_9	0.0028	0.0028/0.1019 = 0.0275
		N_{10}	0.0788	0.0788/0.1019 = 0.7733
B_9	0.0291	N_1	0.00003	0.00003/0.0291 = 0.0103
		N_2	0.0005	0.0005/0.0291 = 0.0172
		N_3	0.00008	0.00008/0.0291 = 0.0027
		N_4	0.0001	0.0001/0.0291 = 0.0034
		N_5	0.0010	0.0010/0.0291 = 0.0344
		N_6	0.0016	0.0016/0.0291 = 0.0550
		N_7	0.0010	0.0010/0.0291 = 0.0344
		N_8	0.0008	0.0008/0.0291 = 0.0275
		N_9	0.0008	0.0008/0.0291 = 0.0275
		N_{10}	0.0236	0.0236/0.0291 = 0.8110
B_{10}	0.1961	N_1	0.0003	0.0003/0.1961 = 0.0015
		N_2	0.0034	0.0034/0.1961 = 0.0173
		N_3	0.0008	0.0008/ 0.1961 = 0.0041
		N_4	0.0011	0.0011/0.1961 = 0.0056
		N_5	0.0076	0.0076/0.1961 = 0.0388
		N_6	0.0122	0.0122/0.1961 = 0.0622

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		N_7	0.0090	0.0090/ 0.1961 = 0.0459
		N_8	0.0061	0.0061/0.1961 = 0.0311
		N_9	0.0059	0.0059/ 0.1961 = 0.0301
		N_{10}	0.1497	0.1497/0.1961 = 0.7634

Discussion of Result in Table 16:

(i). The Posterior Probability was computed by dividing each states of nature total joint probabilities (referred to as marginal probabilities) by probability values of each outcomes for each of the objectives (B_i) for B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8, B_9 and B_{10} as stated before for all N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9 and N_{10} for each set of B_i.

(ii) For example, the objective B_1 (Economic efficiency) and the values of posterior probabilities under it were: N_1 (states of nature) = 0.0003, N_2 = 0.0122, N_3 = 0.0012, N_4 = 0.0009, N_5 = 0.0175, N_6 = 0.0315, N_7 = 0.0175, N_8 = 0.0149, N_9 = 0.0134 and N_{10} = 0.8906. It follows the same pattern for B_2 (Federal economic redistribution), B_3 (Regional economic redistribution), B_4 (Stateeconomic redistribution), B_5 (Local economic redistribution), B_6 (Social well-being), B_7 (Youth empowerment), B_8 (Enviromental quality improvement), B_9 (Gender equality) and B_{10} (Security). These are shown on Table 16.

(iii) Comparing the results from the second iterations, there are reductions in the posterior probability outcomes while on the purpose of reservoir/gullies, the value increased from 0.7881 to 0.8906.

4.4.4 Determination of Forecast Outcomes for Objectives/Benefits at Third (3rd) Iteration (Posterior Expected Opportunity Loss)

This is determined by calculating the forecast outcomes for the objectives/benefits which is the sum of the multiplication of each respective value of the posterior probability results with the conditional opportunity loss of each of the states of nature to get the Expected Opportunity Loss (EOL). The sum totals of each set of values are referred to as the Posterior Expected Opportunity Loss for each of the benefits. The Conditional Opportunity Loss is obtained for each states of nature by subtracting each net benefit (B_i) from the highest benefits of each group. For example, B_1 (economic efficiency); the COL for N_1 = 8.73 – 3.65 = 5.08; the COL for N_2 = 8.73 – 4.84 = 3.89; the COL for N_3 = 8.73 – 6.36 = 2.37; the COL for N_4 = 8.73 – 3.6 = 5.13; the COL for N_5 = 8.73 – 3.44 = 5.29; the COL for N_6 = 8.73 – 4.37 = 4.36 etc. the COL for N_7 = 8.73 – 4.05 = 4.68; the COL for N_8 = 8.73 – 4.22 = 4.51; the COL for N_9 = 8.73 – 1.12 = 7.61; etc.The Forecast Outcomes for the Benefits (Posterior Expected Opportunity Loss) are obtained as the sum of the multiple of each Posterior Probabilities with the Conditional Opportunity Loss (COL) as shown in Table 17

Table 17: Forecast Outcomes for Objectives/Benefits at Third (3rd) Iteration (Posterior Expected Opportunity Loss)

States of Nature	B_1 (Economic Efficiency)			B_2 (Federal Economic Redistribution)			B_3 (Regional Economic Redistribution)			B_4 (State Economic Redistribution)			B_5 (Local Economic Redistribution)		
	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL
N_1	0.0003	5.08	0.0015	0.0069	0	0	0.0028	4.59	0.0129	0.0053	17.47	0.0926	0.0037	4.75	0.0176
N_2	0.0122	3.89	0.0475	0.0764	5.83	0.4454	0.0409	4.79	0.1959	0.1543	19.94	3.0767	0.1086	15.95	1.7322
N_3	0.0012	2.37	0.0028	0.0139	3.78	0.0525	0.0071	3.09	0.0219	0.0160	15.31	0.2450	0.0112	9.70	0.1086
N_4	0.0009	5.13	0.0046	0.0069	3.70	0.0255	0.0056	5.35	0.0300	0.0160	17.58	0.2813	0.0112	18.28	0.2047
N_5	0.0175	5.29	0.0926	0.0729	4.09	0.2982	0.0578	5.61	0.3243	0.0532	17.53	1.9326	0.0787	15.88	1.2498
N_6	0.0315	4.36	0.1373	0.1111	7.92	0.8799	0.0832	4.57	0.3802	0.0811	14.38	1.1662	0.0412	13.00	0.5356
N_7	0.0175	4.68	0.0819	0.0486	7.33	0.3562	0.0494	4.91	0.2426	0.0213	14.81	0.3155	0.0375	10.45	0.3919
N_8	0.0149	4.51	0.0672	0.0417	6.99	0.2915	0.0508	4.76	0.2418	0.0585	13.57	0.7938	0.0412	11.13	0.4586
N_9	0.0134	7.61	0.1020	0.0764	12.01	0.9176	0.0353	8.0	0.2824	0.1064	22.44	2.3876	0.0749	18.96	1.4201
N_{10}	0.89475	0	0	0.5486	2.43	1.3331	0.6671	0	0	0.4202	0	0	0.5918	0	0
Posterior EOL			0.5374			4.5999			1.7328			9.2913			6.1191

States of Nature	B_6 (Social Well-being)			B_7 (Youth Empowerment)			B_8 (Environmental Quality Improvement)			B_9 (Gender Equality)			B_{10} (Security)		
	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL	Prob.	COL	EOL
N_1	0.0022	2.69	0.0059	0.002	9.01	0.0180	0.0020	2.87	0.0057	0.0103	4.07	0.0419	0.0015	0	0
N_2	0.0186	16.54	0.3076	0.017	22.00	0.374	0.0167	13.77	0.2300	0.0172	11.25	0.1935	0.0173	77.26	1.3383
N_3	0.0044	11.92	0.0524	0.004	15.58	0.0623	0.0039	6.51	0.0254	0.0027	10.00	0.027	0.0041	80.76	0.3311
N_4	0.0055	18.73	0.1030	0.004	22.52	0.0901	0.0049	13.63	0.0668	0.0034	11.68	0.0397	0.0056	79.56	0.4455
N_5	0.0340	20.57	0.6994	0.038	22.61	0.8592	0.0373	13.52	0.5043	0.0344	11.49	0.3953	0.0388	79.44	3.0823
N_6	0.0526	13.44	0.7069	0.059	15.37	0.9068	0.0628	7.22	0.4534	0.0550	9.12	0.5016	0.0622	62.93	3.9142
N_7	0.0384	11.80	0.4531	0.038	14.61	0.5552	0.0402	9.65	0.3879	0.0344	10.42	0.3584	0.0459	62.38	2.8632
N_8	0.0362	10.77	0.3901	0.025	13.69	0.3423	0.0294	8.06	0.2370	0.0275	9.68	0.2662	0.0311	62.21	1.9347
N_9	0.0307	19.22	0.5901	0.025	22.61	0.5653	0.0275	14.57	0.4007	0.0275	15.82	0.4351	0.0301	77.15	2.3222
N_{10}	0.7774	0	0	0.788	0	0	0.7733	0	0	0.8110	0	0	0.7634	41.69	31.8261
Posterior EOL			3.3085			3.7732			2.3112			2.6187			48.0576

Discussion of Results on Table 17:

(i) The total Posterior Expected Opportunity Loss (EOL) for the objectives are; ₦0.5374 billion for economic efficiency; ₦4.5999 billion for federal economic redistribution;₦1.7328 billion for regional economic redistribution; ₦9.2913 billion for stateeconomic redistribution; ₦6.1191 billion for local economic redistribution; ₦3.3085 billion for social well-being; ₦3.7732 billion for youth empowerment; ₦2.3112 billion for environmental quality improvement; ₦2.6187 billion for gender equality; and ₦48.0576 billion for security.

4.4.5 Determination of Expected Value of Sample Information (EVSI) Outcomes at Third (3rd) Iteration.

The Expected Value of Sample Information (EVSI) is calculated by multiplying Posterior Expected Opportunity Loss (EOLs) values with the marginal probabilities as shown on Table 18.

Table 18: Expected Value of Sample Information (EVSI) Outcomes at Third (3rd) Iteration

Outcomes B_i	**Marginal probability** P(B_i)	**Expected Opportunity Loss (EOL)**	**Expected Value of Sample Information**
B_1	0.3363	0.5374	0.1807
B_2	0.0288	4.5999	0.1325
B_3	0.0709	1.7320	0.1228
B_4	0.0188	9.2913	0.1747
B_5	0.0267	6.1191	0.1634
B_6	0.0912	3.3085	0.3017
B_7	0.100	3.7732	0.3773
B_8	0.1019	2.3112	0.2355
B_9	0.0291	2.6187	0.0762
B_{10}	0.1961	48.0576	9.4241
		TOTAL (EVSI) =	**₦11.1889 billion**

Discussion of Results in Table 18:

(i) The highest Expected opportunity loss of ₦48.0576 billion multiply by the marginal probability of 0.1961 results to ₦9.4241 billion of Expected Value of Sample Information under Reservoir /Gullies while the least is on Plantation/Forestry with the EVSI of ₦0.0762 billion.

(ii) The Expected Value of Sample Information (EVSI) is ₦11.1889 billion which indicates the money which can be paid for hiring the services of consultants for the River Basin operation yield for all the ten (10) purposes of Irrigation, Hydroelectric Power Generation, Water Supply, Navigation, Drainage/Dredging, Flood Control, Recreation/Tourism, Erosion Control, Plantation / Forestry, Reservoir/Gullies etc. respectively.

V. CONCLUSION AND RECOMMENDATIONS

The optimal utilization of river basin resources entails employment of all the purposes of Irrigation Agriculture, Hydro-electric power generation, Water supply, Navigation or Water transport, Drainage/Dredging, Flood control, Recreation/Tourism, Erosion control, Plantation/ Forestry and Reservoir /Gullies for the optimum benefits based on the objectives of Economic Efficiency, Federal Economic Redistribution, Regional Economic Redistribution, State Economic Redistribution, Local Economic Redistribution, Social Well-being, Youth Empowerment, Environmental quality improvement, Gender Equality and Security.

a) The Bayesian Decision Model analysis reveal that with a total of ₦12.50 billion released to Anambra-Imo River Basin for capital projects development from 2015 to 2020 for the multi-purpose/multi-objective projects will yield maximum Expected Monetary Value of ₦68.72 billion. This implies that with investment of ₦12.50 billion the river basin is expected to generate profit of ₦56.22 billion within the period. This is expected when there is perfect information or with data and the money appropriated for the purpose and objectives respectively.
b) The expected profit with perfect information also increased from ₦45.56 billion without data at first iteration to ₦69.63 billion on third iteration.
c) The Expected Value of Perfect Information reduced from ₦2.97 billion on first iteration to ₦0.9134 billion on third iteration while the expected value of Sample Information reduced from ₦28.69 billion on first iteration without data to ₦11.19 billion on third iteration. The expected value of Sample Information (EVSI)

of ₦11.19 billion is the maximum amount the river basin will pay for additional information for full utilization of the purpose and optimization of all the benefits.

d) The Posterior Expected Opportunity Loss reduced drastically for the objectives. For example the Expected Opportunity Loss for benefit of security decreased from ₦114.91 billion on first iteration to ₦48.06 billion on third iteration.

e) The River basin managers should use the Bayesian analysis to estimate expected monetary benefits for proper apportioning of available funds to various purposes and objectives in-order to realize optimal benefits from their investment in the light of the global climate change scenario and projections.

f) There should be measures to encourage the use of green and clean energy while implementing the purpose/objectives in a multi-purpose/multi-objective Anambra-Imo River basin to reduce the impact of soil erosion, flood disaster, failure of reservoirs and dams, improve hydro-electric power generation, improve water supply, and check insecurity etc. that ravage the living environment.

g) The implementation of these recommendations will be a fertile ground for the management of the river basin to generate revenue and financial benefits to the government, the community and social well-being of the inhabitants in the area.

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