International Journal of Advances in Engineering and Management (IJAEM)
Volume 5, Issue 3 March 2023, pp: 1259-1262 www.ijaem.net ISSN: 2395-5252

Bipolar Fuzzy Graphs

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Date of Submission: 12-03-2023 Date of Acceptance: 22-03-2023

ABSTRACT

In this paper, we studied three new operations on bipolar fuzzy graph; namely direct product, semi strong product and strong product. Also, we give sufficient condition for each one of them to be complete. Finally, we studied product bipolar fuzzy graphs. Bipolar fuzzy graph as solved in Direct product, Semi strong product, Strong product and also solved the sufficient condition for each one of them to be complete. We also discuss some Propositions of self complementary and self weak complementary strong bipolar fuzzy graphs.

Key words

Fuzzy Relation, Crisp Graph, Direct sum, Strong bipolar fuzzy Graph, Strong product of two bipolar fuzzy Graph, Complement.

PRELIMINARIES OF BIPOLAR FUZZY GRAPHS

In this section, we first review some definitions of undirected graphs that are necessary for this paper

Definition-1:

By graph, we mean a pair G* = (V, E), where V is the set and E is a relation on V. The elements of V are vertices of G* and the elements of E are edges of G*. We write x y ∈ E to mean {x y} ∈ E; and if e = x y ∈ E; we say x and y are adjacent. Formally, given a graph G* = (V, E), two vertices x, y ∈ V are said to be neighbors, or adjacent nodes, if x y ∈ E: The neighborhood of a vertex v in a graph G* is the induced sub graph of G* consisting of all vertices adjacent to v and all edges connecting two such vertices. The neighborhood is often denoted N(v).

Definition-2:

The degree deg (v) of vertex v is the number of edges incident on v or equivalently, deg (v) = |N(v)|. The set of neighbors, called a (open) neighborhood N(v) for a vertex v in a graph G*, consists of all vertices adjacent to v but not including v, that is N(v) = {u ∈ V | vu ∈ E}: When v is also included, it is called a closed neighbourhood N[v], that is, N[v] = N(v) ∪ {v}. A regular graph is a graph where each vertex has the same number of neighbors, i.e., all the vertices have the same open neighborhood degree.

Definition -3:

A fuzzy set A on a set X is characterized by a mapping m : X →[0,1], called the membership function. A fuzzy set is denoted as A = (X, m) . A fuzzy graph ξ = (V, σ, μ) is a non-empty set V together with a pair of functions σ :V →[0,1] and μ :V ×V →[0,1] such that for all u, v ∈ V, μ (u, v) ≤ σ (u) ∧ σ (v) (here x ∧ y denotes the minimum of x and y). Partial fuzzy sub graph ξ ′ = (V, τ , ν) of ξ is such that τ (v) ≤ σ (v) for all v ∈ V and μ (u ,v) ≤ ν (u ,v) for all u, v ∈ V. Fuzzy sub graph ξ ′′ = (P, σ ′, μ′) of ξ is such that P ⊆ V , σ(u) = σ (u) for all u ∈ P, μ ′(u, v) = μ (u, v) for all u, v ∈ P .

Definition -4:

A fuzzy graph is complete if μ (u, v) =σ (u) ∧ σ (v) for all u, v ∈ V. The degree of vertex u is d (u) = $\sum_{(u,v)\in\xi}\mu(u,v)$. The minimum degree of ξ is δ (ξ) = ∧ {d (u) | u∈V}. The maximum degree of ξ is Δ (ξ) = ∨ {d (u) | u∈V}. The total degree of a vertex u ∈ V is td (u) = d(u) + σ(u) .

Definition -5:

A fuzzy graph ξ = (V, σ, μ) is said to be regular if d(v) = k, a positive real number, for all v∈V . If each vertex of ξ has same total degree k, then ξ is said to be a totally regular fuzzy graph.

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Definition- 6:

A fuzzy graph is said to be irregular, if there is a vertex which is adjacent to vertices with distinct degrees. A fuzzy graph is said to be neighbourly irregular, if every two adjacent vertices of the graph have different degrees.

Bipolar fuzzy graphs

Definition 1.1

A fuzzy graph with V as the underlying set is a pair G = (σ, μ), where σ : V → [0, 1] is a fuzzy subset and μ : V × V → [0, 1] is a fuzzy relation on σ : μ(x, y) ≤ σ(x)∧ σ(y) for all x, y ∈ V where ∧ stands for minimum. The underlying crisp graph of G is denoted by G* = (σ *, μ*), where σ * = {x ∈ V : σ (x) > 0} and μ∗ = {(x, y) ∈ V × V : μ(x, y) > 0}. H = (σ ′, μ′) is a fuzzy sub graph of G if there exists X ⊆ V such that, σ′: X → [0, 1] is a fuzzy subset and μ′: X × X → [0, 1] is a fuzzy relation on σ′**:** μ′ (x, y) ≤ σ′(x) ∧ σ′(y) and σ′(x) ≤ σ (x), μ′(x, y) ≤ μ(x, y) for all x,y ∈ X.

Definition 1.2

Two fuzzy graphs $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ are isomorphic if there exists a bijection h : $V_1 \to V_2$ such that $\sigma_1(x) = \sigma_2(h(x))$ and $\mu_1(x, y) = \mu_2(h(x), h(y))$ for all x,y ∈ V_1, where V_1 and V_2 are the set of vertices of the underlying graphs of G_1 and G_2 respectively. The main objective of this paper is to study of bipolar fuzzy graph and this graph is based on the bipolar fuzzy set studied below.

Definition 1.3

Let X be a non-empty set. A bipolar fuzzy set B in X is an object having the form B = {(x, $\mu^P(x)$, $\mu^N(x)$) | x ∈ X}, where $\mu^P : X \to [0, 1]$ and $\mu^N : X \to [-1, 0]$ are mappings.

Definition 1.4

Let X be a non-empty set. Then we call a mapping A = (μ_A^P, μ_A^N) : X × X → [−1, 1] × [−1, 1] a bipolar fuzzy relation on X such that μ_A^P (x, y) ∈ [0, 1] and μ_A^N (x, y) ∈ [−1, 0].

Definition 1.5

Let A = (μ_A^P, μ_A^N) and B = (μ_B^P, μ_B^N) be bipolar fuzzy sets on a set X. If A = (μ_A^P, μ_A^N) is a bipolar fuzzy relation on a set X, then A = (μ_A^P, μ_A^N) is called a bipolar fuzzy relation on B = (μ_B^P, μ_B^N) if μ_A^P (x, y) ≤ min (μ_B^P(x), μ_B^P(y)) and μ_A^N(x, y) ≥ max (μ_B^N(x), μ_B^N(y)) for all x, y ∈ X. A bipolar fuzzy relation A on X is called symmetric if μ_A^P(x, y) = μ_A^P(y, x) and
μ_A^N(x, y) = μ_A^N(y, x) for all x, y ∈ X.

We denote G^* as a crisp graph, and G as a bipolar fuzzy graph.

Definition 1.6

A bipolar fuzzy graph of a graph G^*= (V,E) is a pair G = (A,B), where A = $[\mu_A^P, \mu_A^N]$ is a bipolar fuzzy set in V and B = $[\mu_B^P, \mu_B^N]$ is a bipolar fuzzy set in $\tilde{V}^2$ such that μ_B^P(xy) ≤ min {μ_A^P(x), μ_A^P(y)} for all x y ∈ $\tilde{V}^2$, μ_B^N(xy) ≥ min{μ_A^N (x), μ_A^N (y)} for all x y ∈ $\tilde{V}^2$ and μ_B^P (xy) = μ_B^N (xy) = 0 for all x y ∈∈ $\tilde{V}^2$ − E.

Definition 1.7

The complement of a strong bipolar fuzzy graph G = (A,B) of a graph G^*= (V,E) is a bipolar fuzzy graph $\overline{G}$ = ($\overline{A}$, $\overline{B}$) of $\overline{G*}$ = (V, V × V), where $\overline{A}$ = A =$[\mu_A^P, \mu_A^N]$ and $\overline{B}$ = $[\overline{\mu_B^P}, \overline{\mu_B^N}]$ is studied by $\overline{\mu_B^P}$ (xy) = min (μ_A^P(x), μ_A^P (y)) − μ_B^P (xy) for all x, y ∈ V, x y ∈ $\tilde{V}^2$, $\overline{\mu_B^N}$ (xy) = max (μ_A^N(x), μ_A^N(y)) − μ_B^N(xy) for all x, y ∈ V, x y ∈ $\tilde{V}^2$.

Definition 1.8

Let $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be two bipolar fuzzy graphs. A homomorphism f from G_1 to G_2 is a mapping f : $V_1 \to V_2$ which satisfies the following conditions:

(a) $\mu_{A1}^P(x_1) \le \mu_{A1}^P(f(x_1))$, $\mu_{A2}^N(x_1) \ge \mu_{A2}^N(f(x_1))$,
(b) $\mu_{B1}^P(x_1, y_1) \le \mu_{B2}^P(f(x_1)f(y_1))$, $\mu_{B1}^N(x_1 y_1) \ge \mu_{B2}^N(f(x_1)f(y_1))$, for all $x_1 \in V_1$, $x_1 y_1 \in \tilde{V}_1^2$.

Definition 1.9

Let G_1 and G_2 be the bipolar fuzzy graphs. An isomorphism f from $G_1$to G_2 is a bijective mapping f : $V_1 \to V_2$ which satisfies the following conditions:

(c) $\mu_{A1}^P(x_1) = \mu_{A2}^P f((x_1))$, $\mu_{A1}^N(x_1) = \mu_{A2}^N(f(x_1))$,
(d) $\mu_{B1}^P(x_1, y_1) = \mu_{B2}^P(f(x_1)f(y_1))$, $\mu_{B1}^N(x_1 y_1) = \mu_{B2}^N(f(x_1)f(y_1))$, for all $x_1 \in V_1$, $x_1 y_1 \in \tilde{V}_1^2$.

Definition 1.10

The semi-strong product of two fuzzy graphs G_1=(σ_1,μ_1) with crisp graphs G_1^* = (V_1,E_1) and G_2 = (σ_2,μ_2) with crisp graph G_2^* = (V_2,E_2) where we assume that $V_1 \cap V_2 = \emptyset$, is studied to be the fuzzy graph $G_1 \bullet G_2$ = ($\sigma_1 \bullet \sigma_2$, $\mu_1 \bullet \mu_2$) with crisp graph G^* = ($V_1 \times V_2$, E) such that
E = {(u ,v_1)(u ,v_2) | u ∈ V_1, (v_1, v_2) ∈ E_2}∪{(u_1, v_1)(u_2, v_2) | (u_1, u_2) ∈ E_1, (v_1, v_2) ∈ E_2},
($\sigma_1 \bullet \sigma_2$)(u, v) = σ_1(u) ∧ σ_2(v), for all (u, v) ∈ $V_1 \times V_2$,
($\mu_1 \bullet \mu_2$)((u ,v_1)(u ,v_2)) = σ_1 (u) ∧ μ_2 (v_1, v_2) and
($\mu_1 \bullet \mu_2$)((u_1, v_1)(u_2, v_2)) = μ_1 (u_1,u_2) ∧ μ_2 (v_1, v_2).

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Products on Bipolar Fuzzy Graphs

Definition 2.1

The direct product of two bipolar fuzzy graphs $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ with crisp graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively where we assume that $V_1 \cap V_2 = \emptyset$, is studied to be the bipolar fuzzy graph $G_1 \sqcap G_2 = (A_1 \sqcap A_2, B_1 \sqcap B_2)$ with crisp graph $G^* = (V_1 \times V_2, E)$ where, $E = \{(u_1, v_1)(u_2, v_2) \mid (u_1, u_2) \in E_1, (v_1, v_2) \in E_2\}$, $\subset \widetilde{V_1 \times V_2^2}$,

(i) $(\mu_{A1}^P \sqcap \mu_{A2}^P)(u, v) = \mu_{A1}^P(u) \wedge \mu_{A2}^P(v)$, for all $(u, v) \in V_1 \times V_2$

$(\mu_{A1}^N \sqcap \mu_{A2}^N)(u, v) = \mu_{A1}^N(u) \vee \mu_{A2}^N(v)$

(ii) $(\mu_{B1}^P \sqcap \mu_{B2}^P)((u_1, v_1)(u_2, v_2)) = \mu_{B1}^P(u_1, u_2) \wedge \mu_{B2}^P(v_1, v_2)$,

$(\mu_{B1}^N \sqcap \mu_{B2}^N)((u_1, v_1)(u_2, v_2)) = \mu_{B1}^N(u_1, u_2) \vee \mu_{B2}^N(v_1, v_2)$,

(iii) $(\mu_{B1}^P \sqcap \mu_{B2}^P)(w, x)(y, z) = 0 = (\mu_{B1}^N \sqcap \mu_{B2}^N)(w, x)(y, z)$

for all $(w, x)(y, z) \in (\widetilde{V_1 \times V_2^2} - E)$.

Theorem 1.1

If $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ are strong bipolar fuzzy graphs, then $G_1 \sqcap G_2$ is strong.

Proof. If $(u_1, v_1)(u_2, v_2) \in E$, then since G_1 and G_2 are strong we have

$(\mu_{B1}^P \sqcap \mu_{B2}^P)((u_1, v_1)(u_2, v_2)) = \mu_{B1}^P(u_1\ u_2) \wedge \mu_{B2}^P(v_1\ v_2)$

$= \mu_{A1}^P(u_1) \wedge \mu_{A1}^P(u_2) \wedge \mu_{A2}^P(v_1) \wedge \mu_{A2}^P(v_2)$

$= (\mu_{A1}^P \sqcap \mu_{A2}^P)(u_1, v_1) \wedge (\mu_{A1}^P \sqcap \mu_{A2}^P)(u_2, v_2)$.

$(\mu_{B1}^N \sqcap \mu_{B2}^N)((u_1, v_1)(u_2, v_2)) = \mu_{B1}^N(u_1\ u_2) \vee \mu_{B2}^N(v_1\ v_2)$

$= \mu_{A1}^N(u_1) \vee \mu_{A1}^N(u_2) \vee \mu_{A2}^N(v_1) \wedge \mu_{A2}^N(v_2)$

$= (\mu_{A1}^N \sqcap \mu_{A2}^N)(u_1, v_1) \vee (\mu_{A1}^N \sqcap \mu_{A2}^N)(u_2, v_2)$.

Now, semi-strong product between two bipolar fuzzy graphs is studied as follows. This product is helpful to construct certain bipolar fuzzy graphs.

Definition 2.2

The semi-strong product of two bipolar fuzzy graphs $G_1 = (A_1, B_1)$ of $G_1^* = (V_1, E_1)$ and $G_2 = (A_2, B_2)$ of $G_2^* = (V_2, E_2)$, where we assume that $V_1 \cap V_2 = \emptyset$, is studied to be the bipolar fuzzy graph $G_1 \bullet G_2 = (A_1 \bullet A_2, B_1 \bullet B_2)$ of $G^* = (V_1 \times V_2, E)$, where

$E = \{(u, v_1)(u, v_2) \mid u \in V_1, (v_1, v_2) \in E_2\} \cup \{(u_1, v_1)(u_2, v_2) \mid (u_1, v_1) \in E_1, (u_2, v_2) \in E_2\} \subset \widetilde{V_1 \times V_2^2}$,

(i) $(\mu_{A1}^P \bullet \mu_{A2}^P)(u, v) = \mu_{A1}^P(u) \wedge \mu_{A2}^P(v)$, for all $(u, v) \in V_1 \times V_2$

$(\mu_{A1}^N \bullet \mu_{A2}^N)(u, v) = \mu_{A1}^N(u) \vee \mu_{A2}^N(v)$

(ii) $(\mu_{B1}^P \bullet \mu_{B2}^P)((u, v_1)(u, v_2)) = \mu_{A1}^P(u) \wedge \mu_{B2}^P(v_1, v_2)$,

$(\mu_{B1}^N \bullet \mu_{B2}^N)((u, v_1)(u, v_2)) = \mu_{A1}^N(u) \vee \mu_{B2}^N(v_1, v_2)$,

(iii) $(\mu_{B1}^P \bullet \mu_{B2}^P)((u_1, v_1)(u_2, v_2)) = \mu_{B1}^P(u_1\ u_2) \wedge \mu_{B2}^P(v_1\ v_2)$,

$(\mu_{B1}^N \bullet \mu_{B1}^N)((u_1, v_1)(u_2, v_2)) = \mu_{B1}^N(u_1\ u_2) \vee \mu_{B2}^N(v_1\ v_2)$.

(iv) $(\mu_{B1}^P \bullet \mu_{B1}^P)(w, x)(y, z) = 0 = (\mu_{B1}^N \bullet \mu_{B2}^N)(w, x)(y, z)$

for all $(w, x)(y, z) \in (\widetilde{V_1 \times V_2^2} - E)$.

Theorem 1.2

If $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ are strong bipolar fuzzy graphs, then $G_1 \bullet G_2$ is strong.

Proof: If $(u, v_1)(u, v_2) \in E$, then

$(\mu_{B1}^P \bullet \mu_{B2}^P)((u, v_1)(u, v_2)) = \mu_{A1}^P(u) \wedge \mu_{B2}^P(v_1\ v_2)$

$= \mu_{A1}^P(u) \wedge \mu_{A2}^P(v_1) \wedge \mu_{A2}^P(v_2)$

$= (\mu_{A1}^P \bullet \mu_{A2}^P)(u, v_1) \wedge (\mu_{A1}^P \bullet \mu_{A2}^P)(u, v_2)$.

If $((u_1, v_1)(u_2, v_2)) \in E$, then since G_1 and G_2 are strong.

$(\mu_{B1}^P \bullet \mu_{B2}^P)((u_1, v_1)(u_2, v_2)) = \mu_{B1}^P(u_1\ u_2) \wedge \mu_{B2}^P(v_1\ v_2)$

$= \mu_{A1}^P(u_1) \wedge \mu_{A1}^P(u_2) \wedge \mu_{A2}^P(v_1) \wedge \mu_{A2}^P(v_2)$

$= (\mu_{A1}^P \bullet \mu_{A2}^P)(u_1, v_1) \wedge (\mu_{A1}^P \bullet \mu_{A2}^P)(u_2, v_2)$.

Similarly, we can show that

$(\mu_{B1}^N \bullet \mu_{B2}^N)((u, v_1)(u, v_2)) = (\mu_{A1}^N \bullet \mu_{A2}^N)(u, v_1) \vee (\mu_{A1}^N \bullet \mu_{A1}^N)(u, v_2)$

if $(u, v_1)(u, v_2) \in E$ and

$(\mu_{B1}^N \bullet \mu_{B2}^N)((u_1, v_1)(u_2, v_2)) = (\mu_{A1}^N \bullet \mu_{A2}^N)(u_1, v_1) \vee (\mu_{A1}^N \bullet \mu_{A2}^N)(u_2, v_2)$

if $(u_1, v_1)(u_2, v_2) \in E$.

The strong product between bipolar fuzzy graph is an important construction of bipolar fuzzy graph as it contains an edge between every pair of vertices.

Definition 2.3

The strong product of two bipolar fuzzy graphs $G_1 = (A_1, B_1)$ of $G_1^* = (V_1, E_1)$ and $G_2 = (A_2, B_2)$ of $G_2^* = (V_2, E_2)$, where we assume that $V_1 \cap V_2 = \emptyset$, is studied to be the bipolar fuzzy graph $G_1 \otimes G_2 = (A_1 \otimes A_2, B_1 \otimes B_2)$ of $G^* = (V_1 \times V_2, E)$, where

$E = \{(u, v_1)(u, v_2) \mid u \in V_1, (v_1, v_2) \in E_2\} \cup \{(u_1, w)(u_2, w) \mid \mid w \in V_2, (u_1, u_2) \in E_1\} \cup \{(u_1, v_1)(u_2, v_2) \mid (u_1, u_2) \in E_1, (v_1, v_2) \in E_2\} \subset \widetilde{V_1 \times V_2^2}$,

(i) $(\mu_{A1}^P \otimes \mu_{A2}^P)(u, v) = \mu_{A1}^P(u) \wedge \mu_{A2}^P(v)$, for all $(u, v) \in V_1 \times V_2$

$(\mu_{A1}^N \otimes \mu_{A2}^N)(u, v) = \mu_{A1}^N(u) \vee \mu_{A2}^N(v)$

(ii) $(\mu_{B1}^P \otimes \mu_{B2}^P)((u, v_1)(u, v_2)) = \mu_{A1}^P(u) \wedge \mu_{B2}^P(v_1, v_2)$

$(\mu_{B1}^N \otimes \mu_{B2}^N)((u, v_1)(u, v_2)) = \mu_{A1}^N(u) \wedge \mu_{B2}^N(v_1, v_2)$

(iii) $(\mu^P_{B1} \otimes \mu^P_{B2})$ $((u_1,w)(u_2,w)) = \mu^P_{A2}(w) \wedge \mu^P_{B1}(u_1, u_2)$,

$(\mu^N_{B1} \otimes \mu^N_{B2})$ $((u_1,w)\ (u_2,w)) = \mu^N_{A2}(w) \wedge \mu^N_{B1}(u_1, u_2)$,

(iv) $(\mu^P_{B1} \otimes \mu^P_{B2})$ $((u_1, v_1)(u_2, v_2)) = \mu^P_{B1}(u_1, u_2) \wedge \mu^P_{B2}(v_1,v_2)$,

$(\mu^N_{B1} \otimes \mu^N_{B2})$ $((u_1, v_1)(u_2, v_2)) = \mu^N_{B1}(u_1, u_2) \vee \mu^N_{B2}(v_1,v_2)$

(v) $(\mu^P_{B1} \otimes \mu^P_{B2})$ (w, x) (y, z) = 0 = $(\mu^N_{B1} \otimes \mu^N_{B2})$ (w, x) (y, z)

for all (w, x) (y, z) ∈ $(\widetilde{V_1 \times V_2^2}, - E)$.

The following example illustrates that strong product of bipolar fuzzy graphs is another bipolar fuzzy graph where between every pair of vertices, there exist an edge.

Theorem 1.3

If $G_1 = (A_1,B_1)$ and $G_2 = (A_2,B_2)$ are complete bipolar fuzzy graphs, then $G_1 \otimes G_2$ is complete.

Proof. As strong product of bipolar fuzzy graphs is a bipolar fuzzy graph and every pair of vertices are adjacent, E = $\widetilde{V_1 \times V_2^2}$. Now, for all $(u,v_1)\ (u,v_2) \in$ E,

$(\mu^P_{B1} \otimes \mu^P_{B2})$ $((u,v_1)(u,v_2)) = \mu^P_{A1}(u) \wedge \mu^P_{B2}(v_1,v_2)$

$= \mu^P_{A1}(u) \wedge \mu^P_{A2}(v_1) \wedge \mu^P_{A2}(v_2)$ (Since G_2 is complete)

$= (\mu^P_{A1} \otimes \mu^P_{A2})\ (u,v_1) \wedge (\mu^P_{A1} \otimes \mu^P_{A2})\ (u,v_2)$.

$(\mu^N_{B1} \otimes \mu^N_{B2})$ $((u,v_1)\ (u,v_2)) = \mu^N_{A1}(u) \vee \mu^N_{B2}(v_1,v_2)$

$= \mu^N_{A1}(u) \vee \mu^N_{A2}(v_1) \vee \mu^N_{A2}(v_2)$

(Since G_2 is complete)

$= (\mu^N_{A1} \otimes \mu^N_{A2})\ (u,v_1) \vee (\mu^N_{A1} \otimes \mu^N_{A2})\ (u,v_2)$.

If $((u_1,w)\ (u_2,w)) \in$ E, then

$(\mu^P_{B1} \otimes \mu^P_{B2})$ $((u_1,w)\ (u_2,w)) = \mu^P_{A2}(w) \wedge \mu^P_{B1}(u_1, u_2)$

$= \mu^P_{A2}(w) \wedge \mu^P_{A1}(u_1) \wedge \mu^P_{A1}(u_2)$ (Since G_1 is complete)

$= (\mu^P_{A1} \otimes \mu^P_{A2})\ (u_1,w) \wedge (\mu^P_{A1} \otimes \mu^P_{A2})\ (u_2,w)$.

Similarly, we can show that

$(\mu^N_{B1} \otimes \mu^N_{B2})$ $((u_1,w)(u_2,w)) = (\mu^N_{A1} \otimes \mu^N_{A2})\ (u_1,w) \vee (\mu^N_{A1} \otimes \mu^N_{A2})(u_2,w)$

If $(u_1,v_1)\ (u_2,v_2) \in$ E, then since G_1 and G_2 are complete

$(\mu^P_{B1} \otimes \mu^P_{B2})$ $((u_1,v_1)(u_2,v_2)) = \mu^P_{B1}(u_1,u_2) \wedge \mu^P_{B2}(v_1,v_2)$

$= \mu^P_{A1}(u_1) \wedge \mu^P_{A1}(u_2) \wedge \mu^P_{A2}(v_1) \wedge \mu^P_{A2}(v_2)$

$= (\mu^P_{A1} \otimes \mu^P_{A2})(u_1,v_1) \wedge (\mu^P_{A1} \otimes \mu^P_{A2})(u_2,v_2)$.

$(\mu^N_{B1} \otimes \mu^N_{B2})$ $((u_1, v_1)(u_2, v_2)) = \mu^N_{B1}(u_1, u_2) \vee \mu^N_{B2}(v_1, v_2)$

$= \mu^N_{A1}(u_1) \vee \mu^N_{A2}(u_2) \vee \mu^N_{A2}(v_1) \vee \mu^N_{A2}(v_2)$

$= (\mu^N_{A1} \otimes \mu^N_{A2})(u_1, v_1) \vee (\mu^N_{A1} \otimes \mu^N_{A2})(u_2, v_2)$.

Hence, $G_2 \otimes G_2$ is complete.

Theorem 1.4

If $G_1 = (A_1,B_1)$ and $G_2 = (A_2,B_2)$ are bipolar fuzzy graphs such that $G_1 \sqcap G_2$ is strong, then at least G_1 or G_2 must be strong.

Proof: Suppose that G_1 and G_2 are not strong. Then there exists at least one $(u_1,v_1) \in E_1$ and $(u_2,v_2) \in E_2$ such that

$\mu^P_{B1}(u_1,v_1) < \mu^P_{A1}(u_1) \wedge \mu^P_{A1}(v_1)$, $\mu^N_{B1}(u_1,v_1) > \mu^N_{A1}(u_1) \vee \mu^N_{A1}(v_1)$

And $\mu^P_{B2}(u_2,v_2) < \mu^P_{A2}(u_2) \wedge \mu^P_{A2}(v_2)$, $\mu^N_{B2}(u_2,v_2) > \mu^N_{A2}(u_2) \vee \mu^N_{A2}(v_2)$.

$(\mu^P_{B1} \sqcap \mu^P_{B2})((u_1,v_1)(u_2,v_2)) = \mu^P_{B1}(u_1,u_2) \wedge \mu^P_{B2}(v_1,v_2)$

$< \mu^P_{A1}(u_1) \wedge \mu^P_{A1}(u_2) \wedge \mu^P_{A2}(v_1) \wedge \mu^P_{A2}(v_2)$

(Since G_1 and G_2 are not strong).

Similarly,

$(\mu^N_{B1} \sqcap \mu^N_{B2})((u_1, v_1)(u_2, v_2)) > \mu^N_{A1}(u_1) \vee \mu^N_{A1}(u_2) \vee \mu^N_{A2}(v_1) \vee \mu^N_{A2}(v_2)$

But

$(\mu^P_{A1} \sqcap \mu^P_{A2})((u_1,v_1) = \mu^P_{A1}(u_1) \wedge \mu^P_{A2}(v_1)$ and

$(\mu^P_{A1} \sqcap \mu^P_{A2})(u_2,v_2) = \mu^P_{A1}(u_2) \wedge \mu^P_{A2}(v_2)$.

Thus

$(\mu^P_{A1} \sqcap \mu^P_{A2})\ (u_1,v_1) \wedge (\mu^P_{A1} \sqcap \mu^P_{A2})\ (u_2,v_2) = \mu^P_{A1}(u_1) \wedge \mu^P_{A1}(u_2) \wedge \mu^P_{A2}(v_1) \wedge \mu^P_{A2}(v_2)$.

$> (\mu^P_{B1} \sqcap \mu^P_{B2})\ ((u_1,v_1)(u_2,v_2))$

Similarly, we can show that

$(\mu^N_{A1} \sqcap \mu^N_{A2})(u_1,v_1) \vee (\mu^N_{A1} \sqcap \mu^N_{A2})\ (u_2,v_2) < (\mu^N_{B1} \sqcap \mu^N_{B2})\ ((u_1,v_1)(u_2,v_2))$. Hence, $G_1 \sqcap G_2$ is not strong, a contradiction. The next result can be proved in a similar manner as in the preceding theorem.

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