International Journal of Advances in Engineering and Management (IJAEM)
Volume 3, Issue 11 Nov 2021, pp: 336-346 www.ijaem.net ISSN: 2395-5252

Building the Optimal Structure Controlling the SoC Balance of Lithium-Ion Battery Cells

Le Thi Quynh Trang
Lecturers, Thai Nguyen University Of Technology, Vietnam

Submitted: 05-11-2021 Revised: 12-11-2021 Accepted: 15-11-2021

ABSTRACT: In the article [1], we built anenhanced self corect circuit (ESC) model of a lithium-ion battery to describe the dynamic characteristics of the charge/discharge process of a lithium-ion battery(LiB), determine the parameters of the ESC model, and use Use a kalman filter to observe the state of change (SoC) of a LiB. In this article, we will build the optimal structure controlling the SoC balance for LiB cells.

Keywords: Control, Control structure, Optimal control, SoC balance.

I. INTRODUCE

The cell balancing problem is voltage balancing and SoC balancing between LiB cells when they are fully charged. Figure 1 depicts LiB cells in series, with the average SoC value being equal, but the SoC of each cell may not be equal when the SoC between different cells leads to an imbalance between the cells, which requires must perform cell balancing to bring the cells to the same SoC value when the LiBP is active (a set of LiBs consisting of many cells joined together is called a LiBP).

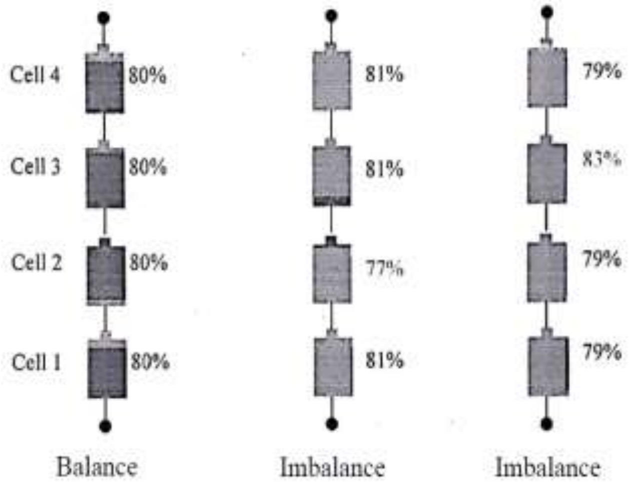

Figure 1. Description of SoC changes on serial cells

[2] Research has shown that, currently, there have been many studies offering different methods of performing cell balancing. Each method has its own advantages and disadvantages, depending on the application that the appropriate cell balancing technique can be selected. The performance criteria of the cell balancing system are expressed in aspects such as the influence on SoH, equalization time, efficiency, control complexity, etc.

[3, 4] The article has presented the current cell balancing techniques that can be divided into two main methods: passive balancing method and active balancing method (active method). With current cell balancing techniques, there are certain advantages, but the methods of performing cell balancing problems using voltage-based algorithms should have limitations such as equalization time, efficiency as well as efficiency. as the level of control complexity.

The content of this paper will present an active cell balancing method for serial cells based on SoC cells with the aim of optimizing the balancer operation considering the SoC constraints, voltage, charge/discharge current, balance current, temperature and capacity of the LiBP.

International Journal of Advances in Engineering and Management (IJAEM)
Volume 3, Issue 11 Nov 2021, pp: 336-346 www.ijaem.net ISSN: 2395-5252

II. BUILD A BALANCE MODEL SOC SERIES CELLS

Balanced model of two cells in series

The equalization technique using a bidirectional Cuk circuit (a circuit using a bidirectional DC-DC converter) has the advantage of being able to transfer energy between two DC sources without causing or losing energy, because So in this paper, the author applies a bidirectional Cuk circuit to perform the SoC balancing function for two adjacent cells. Suppose there are two battery cells i and i + 1 in series and adjacent as shown in Figure 2.

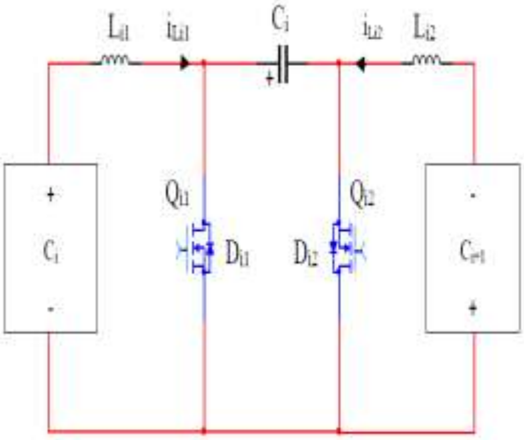

Figure 2. SoC balance diagram for two LiB cells using Cuk converter

Where: $L_{i1}$and L_{i2} are the two inductances of the balanced circuit, C_i is the capacitor, Q_{i1}, $Q_{i2}$are two switching MOSFETs, D_{i1}, $D_{i2}$are two MOSFET switching diodes.

Two MOSFETs are controlled opening and closing by Pulse Width Modulation (PWM) width modulation, with a large frequency of 10Khz or more. This Cuk converter is designed to operate in discontinuous inductor conduction mode to reduce switching losses in MOSFET[5]. This Cuk circuit is also responsible for transferring energy between two battery cells i and i + 1 through the opening and closing action of the MOSFET to control the discharge and charge process of the capacitor C_i and the two inductors L_{i1}, L_{i2}. The switching frequency of the MOSFET is very high, so the Cuk circuit always works in transient mode.

The operating principle of this cell balancing circuit is as follows [6].

Under normal conditions, the circuit has not performed the balancing operation (both MOSFETs are locked), the voltage across the capacitor is:

$$V_{C_i} = V_{B_i} + V_{B_{i+1}} \qquad (1)$$

First of all, we consider the case that the Cuk circuit works in the power mode that needs to be transferred from cell i to cell i + 1 ($V_{B_i} > V_{B_{i+1}}$, SoC_i> SoC_{i+1}). Call the PWM period given to MOSFET Q_{i1} as T_p, the pulse width (or Duty) as D_{i1}. Calling the beginning of the PWM at the t_0, the working process of the Cuk circuit takes place in two cases as follows:

**** Case 1:*** In case of pulse ($t_0 \le t \le D_{i1}T_p$), Q_{i1} is open, capacitor C_i transmits cell energy i + 1, Li is stored energy (as magnetic field) during this period. The currents are denoted in figure 3, the kinematic equation of the equivalent circuit is:

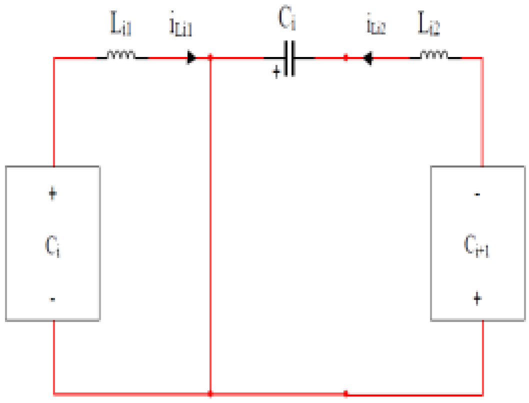

Figure 3. The equivalent circuit in the case 1

$$V_{B_i} = L_{i1}\frac{di_{L_{i1}}}{dt}, \quad i_{L_{i1}}(t_0) = I_0 \qquad (2)$$

$$V_{B_{i+1}} = -L_{i2}\frac{di_{L_{i2}}}{dt} + \frac{1}{C_i}\int_{t_0}^{D_{i1}T_p} i_{L_{i2}}dt, \quad i_{L_{i2}}(t_0) = I_0 \qquad (3)$$

**** Case 2:*** In case there is no pulse ($D_{i1}T_p< t \le T_p$), $Q_{i1}$is locked, D_{i2} is through, the capacitor $C_i$is charged with energy from cell i, the energy stored on L_{i+1}continues to be charged to cell i + 1. The currents are denoted as shown in Figure 4:

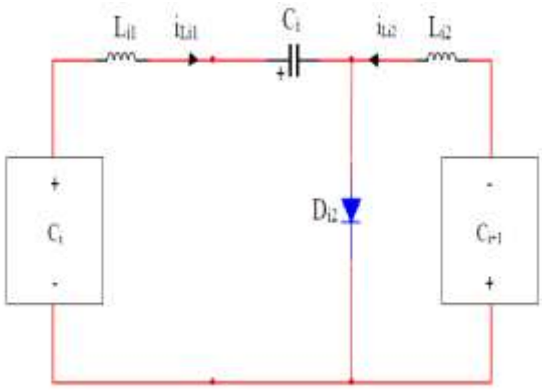

Figure 4. Equivalent circuit in the case 2

The kinematic equation of the equivalent circuit in this case is:

$$V_{B_{i+1}} = -L_{i2}\frac{di_{L_{i2}}}{dt}, \quad i_{L_{i2}}(t_0) = I_p \quad (4)$$

$$V_{B_i} = L_{i1}\frac{di_{L_{i1}}}{dt} + \frac{1}{C_i}\int_{D_{i1}T_p}^{T_p} i_{L_{i2}}dt, \quad i_{L_{i2}}(D_{i1}T_p) = I_p$$

$$V_{C_i}(D_{i1}T_p) = V_{B_i} + V_{B_{i+1}} \quad (5)$$

According to the voltage balance principle and the charge balance principle [7], during a PWM cycle T_p the average currents $I_{L_{i1}}, I_{L_{i2}}$ through L_{i1}, L_{i2} and the average voltage across the capacitor C_i by constant. The average current through L_{i1}, L_{i2} during a PWM pulse period fed into Q_{i1} is determined as follows [8]:

$$I_{L1} = \varphi_{i1}D_{i1} = \frac{1}{2}\frac{V_{B_i}T_pD_{i1}^2}{L_{i1}}$$

$$I_{L2} = \varphi_{i2}D_{i1} = \frac{1}{2}\frac{T_pV_{C_i} - V_{B_{i+1}}D_{i1}^2}{L_{i2}} \quad (6)$$

We define the ratio $\beta_i > 0$ between two currents $I_{L_{i1}}, I_{L_{i2}}$ as follows:

$$\beta_i = \begin{cases} \frac{\varphi_{i2}(D_{i1})}{\varphi_{i1}(D_{i1})}, & \varphi_{i1}(D_{i1}) \neq 0 \\ 0 & \varphi_{i1}(D_{i1}) = 0 \end{cases} \quad (7)$$

Similarly, for the case where energy needs to be transferred from cell i+1 to cell i. Call the PWM period given to the Q_{i2} MOSFET as T_p, the pulse width as D_{i2}. The average current $I_{L_{i1}}, I_{L_{i2}}$ through L_{i1}, L_{i2} is written as:

$$I_{L_{i1}} = \varphi'_{i1}D_{i2} = \frac{1}{2}\frac{T_pV_{C_i} - V_{B_i}D_{i2}^2}{L_{i2}}$$

$$I_{L_{i2}} = \varphi'_{i2}D_{i2} = \frac{1}{2}\frac{T_pV_{B_{i+1}}D_{i2}^2}{L_{i1}} \quad (8)$$

Similarly the current ratio $I_{L_{i1}}, I_{L_{i2}}$ in this case is as follows:

$$\beta_i = \begin{cases} \frac{\varphi'_{i1}(D_{i2})}{\varphi'_{i2}(D_{i2})}, & \varphi'_{i2}(D_{i2}) \neq 0 \\ 0 & \varphi'_{i2}(D_{i2}) = 0 \end{cases} \quad (9)$$

So in general for both cases where energy is transferred from cell i to cell i + 1 and vice versa where energy is transferred from cell i + 1 to cell i, the average current through L_{i1}, L_{i2} is written as:

$$I_{L_{i1}} = \begin{cases} \varphi_{i1}(D_{i2}) & \text{cell i} \rightarrow \text{cell i} + 1 \\ \varphi'_{i1}(D_{i2}) & \text{cell i} + 1 \rightarrow \text{cell i} \end{cases}$$

$$I_{L_{i2}} = \begin{cases} \varphi_{i2}(D_{i2}) & \text{cell i} \rightarrow \text{cell i} + 1 \\ \varphi'_{i2}(D_{i2}) & \text{cell i} + 1 \rightarrow \text{cell i} \end{cases} \quad (10)$$

For the Cuk circuit at a time, only one of the two MOSFETs is active, so we have the condition between the two Duty of the two MOSFETs is:

$$D_{t1}(k).D_{t2}(k) = 0 \quad (11)$$

Where: k is the sampling period of the control circuit and calculation.

SoC balance model of multiple serial cells

Assuming there are n cells connected in series, we have n - 1 cell equalizer for two adjacent cells designed as shown in Figure 5.

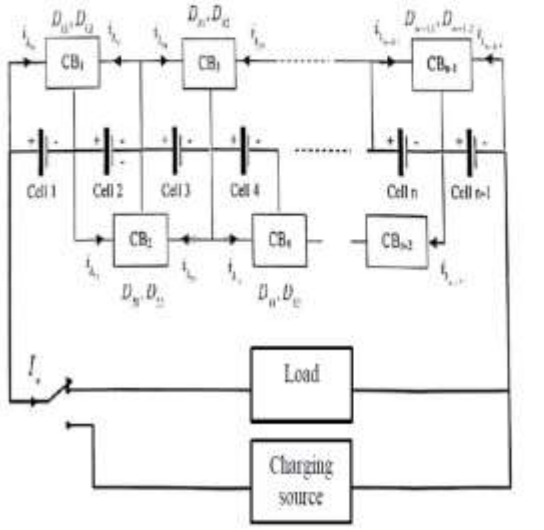

Figure 5. Balanced circuit structure for n series LiB cells

Let the equilibrium current of cell i, 1 < i < n is:

$$I_{eq_i} = I_{L_{i1}} - I_{L_{i2}} \quad (12)$$

Because a cell participates in two equalization circuits except the first and last cell. The balance current of cell 1 and cell n is:

$$I_{eq_1} = I_{L_{i1}}$$

$$I_{eq_n} = I_{L_{n-1,2}} \quad (13)$$

Call the current through the i[th] cell I_{B_i}, the charge (discharge) current of all n cells connected in series is I_s, we have:

$$I_{B_i} = I_s + I_{eq_i} \quad (14)$$

Assume that at sampling time k, or time kT, k = 0, 1, 2 → ∞ (for sampling period T, this sampling period must be larger and integer times T_p of PWM) of the cell balancing controller, the SoC values of all cells have been determined through the SoC estimation stage. Thus, the preliminary SoC value of the cells during the next sampling time k + 1 can be determined through the integral in one sampling period of the cell current ratio and the cell capacity. Thus, the SoC value of cell i, 1 ≤ i ≤ n can be updated by the following formula:

$$SoC_i(k+1) = SoC_i(k) - \Delta SoC_i(k) - \Delta SoC_s(k) \quad (15)$$

Where ΔSoCi(k) is the amount of SoC variation of cell i caused by receiving or transmitting energy to its two adjacent cells (cell i - 1, i + 1), defined as:

$$\Delta SoC_i(k) = -\Delta SoC_{it}(k) - \Delta SoC_{ir}(k) \quad (16)$$

International Journal of Advances in Engineering and Management (IJAEM)
Volume 3, Issue 11 Nov 2021, pp: 336-346 www.ijaem.net ISSN: 2395-5252

With $\Delta SoC_{it}(k)$, $\Delta SoC_{ir}(k)$ is the amount of SoC given and SoC received, respectively. The amount of SoC given and received by cell i, $1 \le i \le n$ is calculated as follows:

$$\Delta SoC_{it}(k) = \frac{I_{L_{(i-1),2}}(k)T}{Q}$$

$$\Delta SoC_{ir}(k) = \frac{I_{L_{i1}}(k)T}{Q} \qquad (17)$$

For the first cell $\Delta SoC_{1t}(k) = 0$, for the last cell $\Delta SoC_{nr}(k) = 0$. $\Delta SoC_s(k)$ is the amount of variation the SoC causes to the load (or charge) current through all n cell, since the I_s line is the same for all cells, $\Delta SoC_s(k)$ is the same for all cells.

$$\Delta SoC_s(k) = \frac{I_s(k)T}{Q} \qquad (18)$$

For n cells, the SoC update formula is generally defined as:

Cell 1: $SoC_1(k+1) = SoC_1(k) - \frac{\varphi_{11}(D_{11})T}{Q} - \frac{\varphi'_{11}(D_{12})T}{Q} - \frac{I_s(kT)T}{Q}$ (19)

Cell j: 1 < j < n

$SoC_j(k+1) = SoC_j(k) + \frac{\varphi_{i2}(D_{i1})T}{Q} - \frac{\varphi_{i1}(D_{i1})T}{Q} + \frac{\varphi'_{i2}(D_{i2})T}{Q} - \frac{\varphi'_{i1}(D_{i2})T}{Q} - \frac{I_s(kT)T}{Q}$ (20)

Cell n: $SoC_n(k+1) = SoC_n(k) + \frac{\varphi_{(n-1),1}(D_{(n-1),1})T}{Q} - \frac{\varphi'_{(n-1),1}(D_{(n-1),2})T}{Q} - \frac{I_s(kT)T}{Q}$ (21)

$SoC(k) \in R^* = [SoC_1(k) \quad SoC_2(k) \cdots \quad SoC_n(k)]^T$

$u_1(k) \in R^{n-1} = [D_{11} \quad D_{21} \cdots \quad D_{n-1,1}]^T$ (22)

$u_2(k) \in R^{n-1} = [D_{12} \quad D_{22} \cdots \quad D_{n-1,2}]^T$

Let the system matrices be:

$$B_1(k) \in R^{nx(n-1)} \begin{bmatrix} -1 & 0 & 0 & \cdots & 0 \\ \beta_1(k) & -1 & 0 & \cdots & 0 \\ 0 & \beta_2(k) & -1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & 0 & \cdots & \beta_{n-1}(k) \end{bmatrix} \qquad (23)$$

$$B_2(k) \in R^{nx(n-1)} = \begin{bmatrix} \beta'_1(k) & 0 & 0 & \cdots & 0 \\ 1 & \beta'_2(k) & 0 & \cdots & 0 \\ 0 & 1 & \beta'_3(k) & \cdots & 0 \\ \cdots & \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \qquad (24)$$

Equations (19) to (21) are models representing the SoC variation of n series cells, the SoCs of the cells depend on the Duty of the cell balancing circuits (n - 1 equalizer) and load current through the cell. To represent the relationship model between the SoC of cells and the Duty of the cell balancing circuits, we set the SoC variable vector and the input vectors as follows:

$f_1 u_1(k) \in R^{n-1} = [f_{11}D_{11} \quad f_{21}D_{21} \cdots \quad f_{n-1,1}D_{n-1,1}]^T$

$f_2 u_2(k) \in R^{n-1} = [f'_{12}D_{12} \quad f'_{22}D_{22} \cdots \quad f'_{n-1,2}D_{n-1,2}]^T$

$I_s(k) \in R^{n-1} = [I_s(k)T \quad I_s(k)T \cdots \quad I_s(k)T]^T$ (25)

The model of a cell-balanced system is generally written as follows:

$SoC(k+1) = SoC(k) + Q^{-1}B_1(kT)\, f_1\, u_1(k) + Q^{-1}B_2(kT)\, f_2\, u_2(k) + Q^{-1}I_s(k)$ (26)

Looking at the model, we see that the SoC, the parts $Q^{-1}B_1(k)f_1u_1(k)$, $Q^{-1}B_2(k)f_2u_2(k)$ are the amount of SoC transmitted to neighboring cells and the amount of SoC received from neighboring cells, respectively. Neighboring cell SoC of a cell at the next time will be equal to the current SoC plus the amount of SoC transferred from neighboring cells minus the amount of SoC transmitted to neighboring cells and the amount of SoC received/reduced when charge/discharge.

Thus, from this model, we can adjust the terminals $u_1(k)$, $u_2(k)$ to adjust the process of transmitting and receiving energy of each cell in order to receive the cell's SoC at the next time.

It should be noted that the current through the cells is limited by the operating characteristics of the cell, the current through the cell is equal to the sum of the balance current and the charge/discharge current, so the designs of the cell balancing circuit should be limited such that In the case of the maximum charge/discharge current, the current through the cell does not violate this condition. Therefore, the limit of the balanced currents needs to be set and the limits of $u_1(k)$, $u_2(k)$ also need to be considered in the control problem.

This is a model describing the SoC relationship between time k and time k + 1, SoC at time k is taken from SoC estimation algorithm, SoC value at time k + 1 is used for calculation purposes only the control signals $u_1(k)$, $u_2(k)$ at time k, and the SoC at time k through this model will not be used for time k + 1, but will be taken from the SoC estimator.

III. OPTIMAL CONTROL OF CELL BALANCE FOR LITHIUM-ION BATTERIES

Setting up the balanced optimal control problem SoC

The optimal control problem of cell balance is performed to ensure that n cells connected in series have relatively equal SoCs and allow a difference of an acceptable limit. At the same time, it is necessary to ensure that the cell current conditions are within the allowable limits in the normal operating conditions of the cells, and to ensure the duty limit of the PWMs sent to the MOSFETs of the cell balancing circuits. The constraints set in terms of limits in the optimal control problem are defined as follows:

The firstly, the SoC of all cells must be within operating limits, represented by the set Ω and defined:

$\Omega = [SoC_i \in R | SoC_{min} \le SoC_i \le SoC_{max},\ i = 1, 2,..., n$ (27)

In which: SoC_{min}, SoC_{max} are the SoCs at the lowest level and the highest level, respectively.

Secondly, since the Cuk converter circuit is designed to operate in DICM mode, the balance current should not be greater than the maximum allowable balance current, the duty of the PWM control signal sent to the MOSFETs must satisfy the following requirements. binding as follows:

$D_{i1}, D_{i2} \in \emptyset,\ i = 1, 2, \ldots, n$ (28)

In which:

$$\emptyset = \begin{cases} D_{i1} \in R, & 0 \le D_{i1} \le D_{max} \\ D_{i2} \in R, & 0 \le D_{i2} \le D_{max} \end{cases} \quad (29)$$

With D_{max} being the maximum allowed duty of the PWM pulse.

Thirdly, the current through the cells must be within the appropriate operating limits, which means:

$I_{B_{cmax}} \le I_{B_i} \le I_{B_{dmax}},\ i = 1, 2, \ldots, n$ (30)

In which: $I_{B_{cmax}}, I_{B_{dmax}}$ is the value of maximum charge current and maximum allowable discharge current of LiB (negative charge current and positive discharge current), respectively.

The goal of the optimal control problem is to control the SoC balance for the cells so that the energy loss is minimal. To achieve this goal, it is necessary to control the SoC balance of all series cells so that the squared deviation of the SoC of the cells from the average SoC value of the cells in the LiBP is minimized. This goal is expressed by the following formula:

$min \sum_{i=1}^{n} SoC_i - \overline{SoC}^2$ (31)

Where $\overline{SoC}$ is the average SoC of serial cells, defined as:

$\overline{SoC} = \frac{1}{n} \sum_{j=1}^{n} SoC_i$ (32)

In order to ensure normal working conditions, the currents through the inductances in the balanced circuits are not too large, causing harm to the cells, so in the following target cell balance control problem related to n - 1 cell equalizer needs to be implemented:

$min \sum_{i=1}^{n-1} [I_{L_{i1}} - I_{L_{i2}}]^2 = min \sum_{i=1}^{n-1} [\varphi_{i1}(D_{i1}) - -\varphi i2'(Di2)2$ (33)

Thus, the cell balancing problem for n cells needs to ensure the calculation of the Duty of the cell balancers so that the minimization conditions (31) and (33) are satisfied and the bound constraints are met. of working conditions (27), (28) (30)

To implement this optimal control problem, we use the following objective function:

J SoC(k), u_1(k), u_2(k) = $\sum_{i=1}^{n} p_i [SoC_i(k+1) - SoC(k+1)]^2 + \sum_{j=1}^{n-1} q_j [\varphi_{i1}(u_1(k)) - \varphi_{i2}'(u_2(k))]^2$ (34)

Where: $p_i > 0$, $q_j > 0$, i = 1, 2,…, n, j = 1, 2,…, n - 1 are the positive weights in the objective function, respectively. The objective function (34) can be written in quadratic form as follows:

J SoC(k), u_1(k), u_2(k) = SoC(k+1) – $I_{nx1}\overline{SoC}$(k+1) P SoC(k+1) – $I_{nx1}\overline{SoC}(k+1)^T$+ f_1 (u_1(k)) – f_2 u_2(k)Qf_1(u_1(k)) – f_2 (u_2(k))^T (35)

Where: I_{nx1} is a 1- column vector, n - 1 row and has elements equal to 1, the weight matrices P, Q have the form:

$$P \in R^{nxn} = \begin{bmatrix} p_1 & 0 & 0 & 0 \\ 0 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & p_n \end{bmatrix},$$

$$Q \in R^{(nx1)x(n-1)} = \begin{bmatrix} p_1 & 0 & 0 & 0 \\ 0 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & p_n \end{bmatrix} \quad (36)$$

The solution of the optimization problem is to find the control signal u_1(k), u_2(k) and Duty of the PWM pulse leading to n - 1 cell equalizer, by solving the following optimization problem:

J SoC(k), u_1(k), u_2(k) ⟶ min (37)

With the SoC kinematics of the cells being:

SoC(k +1) = SoC(k) + $Q^{-1}B_1$(kT) f_1 u_1(k) + $Q^{-1}B_2$(kT) f_2 u_2(k) + $Q^{-1}I_s$(k) (38)

Satisfy the following conditions:

$I_{min} \le -B_1(kT)\, f_1\, u_1(k) - B_2(kT)\, f_2\, u_2(k) + I_s(k) \le I_{max}$ (39)

SoC (k+1) ∈ Ω (40)

$u_1(k)\ x\ u_2(k) = 0$ (41)

$u_1(k), u_2(k) \in \emptyset$ (42)

With:

$I_{min} = [I_{B_{cmax}} \cdots I_{B_{cmax}}]^T \in R^n$

$I_{max} = [I_{B_{dmax}} \cdots I_{B_{dmax}}]^T \in R^n$ (43)

And SoC(k) is the SoC estimate of the cells at time k.

Algorithm to solve the cell-balanced optimal control problem

See the cell-balanced optimal control problem to determine the duties that lead to the control of MOSFETs in n - 1 cell-balanced circuits with the objective function (37) satisfying the unbalanced nonlinear constraints (39) and (40), balanced nonlinearity (41) and satisfying the limit (42) of the Duty. In general, this is an optimal control problem with balanced and unbalanced

International Journal of Advances in Engineering and Management (IJAEM)
Volume 3, Issue 11 Nov 2021, pp: 336-346 www.ijaem.net ISSN: 2395-5252

nonlinear constraints. Considered in one sampling cycle of the cell-balanced controller, this optimization problem is considered as a static nonlinear optimal control problem, because during this computation period the SoCs of the cells are considered to be zero. change, because the kinetics of the battery is slow. The solution of this optimization problem depends on many factors such as: frequency of control pulses fed into cell-balanced controllers $f_p = 1/T_p$period T, limits Φ, Ω, currents charge/discharge currents allowed through cells $I_{B_{cmax}}$,$I_{B_{dmax}}$,charge/discharge current I_s and number of cells in series n. On the other hand, this is a nonlinear optimization problem, so the convergence problem of the algorithm also depends a lot on the starting point of finding the solution.

There are many methods that can be applied to solve the cell balancing optimal control problem [9] – [18]: Interior-Point method, Trust-Region-Reflectivemethod, Active-Set method, Sequential Quadratic Programming (SQP) method, SQP-Lagary method.

The author applies the SQP method to the cell balance optimal control problem. Algorithm to solve the nonlinear optimization problem SQP can be found in the documents [6], [9]. Express the balanced optimization problem for n cells connected in series in standard form as follows:

(u) → min (44)

Satisfy the following conditions:

$$c(u) \le 0$$
$$c_{eq}(u) = 0$$
$$A\,u \le b$$
$$A_{eq}\,u = B_{eq}$$
$$u_{min} \le u \le u_{max} \quad (45)$$

With: u is the solution vector of the optimization problem; b and B_{eq} are vectors, A and Aeq are the matrices of the two linearly balanced and unbalanced constraints; image c(u) and $c_{eq}(u)$ are functions, the output of the function is vectors; J(u) is a function whose head is a scalar, J(u), c(u), $c_{eq}(u)$ are nonlinear functions; u_{min}, $u_{max}$are the lower and upper limit vectors of the optimal solution.

Specific representations of the quantities in (44) and (45) are as follows:

$$u = \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix}_{2(n-1)x1}$$

$$u_1(k) = \begin{bmatrix} u_{11}(k) \\ u_{21}(k) \\ \vdots \\ u_{n-1,1}(k) \end{bmatrix} = \begin{bmatrix} D_{11}(k) \\ D_{21}(k) \\ \vdots \\ D_{n-1,1}(k) \end{bmatrix},$$

$$u_2(k) = \begin{bmatrix} u_{21}(k) \\ u_{22}(k) \\ \vdots \\ u_{n-1,2}(k) \end{bmatrix} = \begin{bmatrix} D_{21}(k) \\ D_{22}(k) \\ \vdots \\ D_{n-1,2}(k) \end{bmatrix} \quad (46)$$

$$J(u) = SoC(k+1) - I_{nx1}\overline{SoC}(k+1)\,P\,SoC(k+1) - I_{nx1}\overline{SoC}(k+1)^T + f_1(u_1(k)) - f_2\,u_2(k)Qf_1(u_1(k)) - f_2\,u_2(k))^T \quad (47)$$

With SoC(k + 1) calculated according to (38).

The matrices and vectors A, b, A_{eq}, $B_{eq}$are zero because there are no balanced and imbalanced linear constraints.

Unbalanced nonlinear constraints:

$$c(u) = \begin{bmatrix} -B_1(kT)f_1u_1(k) - B_2(kT)f_2u_2(k) + I_s(k) - I_{max} \\ I_{min} + B_1(kT)f_1u_1(k) + B_2(kT)f_2u_2(k) + I_s(k) \\ SoC(k+1) - SoC_{max}\,I_{nx1} \\ SoC_{min}\,I_{nx1} - SoC(k+1) \end{bmatrix} \le 0_{4nx1} \quad (48)$$

Balanced nonlinear constraints:

$$c_{eq}(u) = \begin{bmatrix} u_{11}(k)u_{n-1,2}(k) \\ u_{21}(k)u_{n,2}(k) \\ \cdots \\ u_{n-1,1}(k)u_{2n-2,2}(k) \end{bmatrix} = 0_{(n-1)x1} \quad (49)$$

Limits:

$$u_{min} = 0_{n-1,1} \quad u_{max} = D_{max}\begin{bmatrix} D_{max} \\ D_{max} \\ \vdots \\ D_{max} \end{bmatrix}_{n-1,1} \quad (50)$$

The principle of the SQP algorithm to solve the cell balance control optimization problem (44) with constraints and limits (45) is described as follows [6]:

First, we approximate the optimization problem (44) with constraints and limits (45) based on the approximation transform of the quadratic Lagrange function:

$$L\,u,\lambda = J(u) + \sum_{a=1}^{n}\lambda_a g_a(u) \quad (51)$$

into a quadratic programming subproblem:

$$\min_{d\in R^n} \frac{1}{2}d^T H_i d + \nabla J(u(i)^T)d$$
$$\nabla g_j(u(i)^T)d + g_j(u(i)) = 0,\quad j = 1,2,\dots,n \quad \nabla g_j(u(i)^T)d + g_j(u(i)) \le 0,\quad j = 1,2,\dots,n \quad (52)$$

Where: i is the sequence number of steps in the solution, $\lambda_a, a = 1, 2, \dots, 2n-2$is the Lagrange multiplication operator, H_i is the approximate positive definite matrix of the Hessian matrix of the

International Journal of Advances in Engineering and Management (IJAEM)
Volume 3, Issue 11 Nov 2021, pp: 336-346 www.ijaem.net ISSN: 2395-5252

Lagrange function at the ith iteration, g(u(i)) is the function of the solution at the i iteration defined in the Lagrange function (25), d is the search direction vector.

The quadratic programming problem (26) can be solved using any quadratic programming algorithm. The solution of the problem at the (i + 1)th iteration is usually of the form:

$u(i+1) = u(i) + \alpha_i d_i$ (53)

Where α_i is the step length parameter determined by the search procedure so that the value of the objective function decreases accordingly.

To solve the quadratic programming problem (52) using the SQP technique, the main steps are as follows:

1) Update the Hessian matrix

2) Solving the problem of dimensional planning

3) Search and determine value

***Step 1:** Update matrix H_i

At each iteration, the matrix H_i is updated using the BFGS method:

$$H_{i+1} = H_i + \frac{q_i q_i^T}{q_i^T s_i} - \frac{H_i s_i s_i^T H_i^T}{s_i^T H_i s_i} \quad (54)$$

With:

$s_i = u(i+1) - u(i)$

$$q_1 = \left[\nabla J\, u(i+1) + \sum_{a=1}^{n} \lambda_a g_a\big(u(i+1)\big)\right] - \left[\nabla J\, u(i) + \sum_{a=1}^{n} \lambda_a g_a\big(u(i)\big)\right] \quad (55)$$

During the calculation, H_i must be positive. To ensure that the matrix H_i is positive definite, $q_i^T s_i$ must be positive definite and the matrix H must be initialized with a positive definite matrix. When $q_i^T s_i$ is positively undefined then q_i must be changed element-by-element such that $q_i^T s_i > 0$. If after this process $q_i^T s_i$ is still positively undefined then change q_i by adding into a vector *v* multiplied by a scalar ω as follows:

$$q_i = q_i + \omega v \quad (56)$$

In which:

$$v_a = \begin{cases} = \nabla g_a\big(u(i+1)\big) g_a\big(u(i+1)\big) - \nabla g_a\big(u(i)\big) g_a\big(u(i)\big) \\ khi\ (q_i)_n.w < 0 \text{ và } (q_i)_n.(s_i)_n < 0, a = 1, 2, \ldots, 2n-2 \\ = 0 \quad Other\ case \end{cases}$$

(57)

And increase ω slowly until $q_i^T s_i > 0$.

***Step 2:** Solve the square planning problem

At each iteration, solve the following quadratic programming problem:

$$\min_{d \in R^n} q(d) = \frac{1}{2} d^T H_i d + c^T d$$

$A_j d = b_j$, j = 1, 2,…, n_e (58)

$A_j d \le b_j$, j = n_e,…, n

The process of solving the square plan is divided into two cases. Case 1 is to compute a possible point, case 2 is to generate a sequence of iterations of possible points that converge on the solution of the problem. Let $\overline{A}_i$ be the "active set", which is an estimate of the positive constraints at the solution of the problem. $\overline{A}_i$ is updated at each iteration i, $\overline{A}_i$ is used to determine the possible search direction d_i, denote the possible search direction di to distinguish it from the search direction d_i in the main iteration. The search direction di is calculated and minimizes the objective function while ensuring positive constraints. The possible space for the divisible search direction is determined from the matrix Z_i whose columns are orthogonal to the positive set $\overline{A}_i$ (that is, $\overline{A}_i Z_i = 0$) the matrix Z_i is formed from m - 1 column of the QR analysis of the $\overline{A}_i^T$ matrix, where l is the number of positive constraints and *l* < *m* as follows:

$$Z_k = Q[:, l+1 : m] \quad (59)$$

In which:

$$Q^T \overline{A}_i^T = \begin{bmatrix} R \\ 0 \end{bmatrix} \quad (60)$$

The search direction is defined as:

$d_i = Z_i p$ (61)

The quadratic function q is a function of the vector p of the form:

$$q(p) = \frac{1}{2} p^T H_i Z_i^T p + c^T Z_k p \quad (62)$$

The partial derivative with respect to p we have:

$$\nabla q(p) = Z_i^T H_i Z_i p + Z_i^T c \quad (63)$$

Where: ∇q(p) is considered as the Gradient of the quadratic function by the gradient projected onto the subspace defined by Z_i. The component $Z_i^T H_i Z_i$ is called the projection Hessian matrix. Assuming that the matrix Z_i is positively defined, then the minimum of the function q(p) in the subspace Z_i is determined when ∇q(p)=0, which is the solution of the following linear equation:

$$Z_i^T H_i Z_i p = -Z_i^T c \quad (64)$$

So the corresponding optimal solution is:

$u_{i+1}(k) = u_i(k) + \alpha\, \hat{d}_i$, $\hat{d}_i = Z_i p$ (65)

***Step 3:** Search and determine the value

The solution of the quadratic programming problem generates the vector di, which is used to establish a new calculation step:

$u_{i+1}(k) = u_i(k) + \alpha\, \hat{d}_i$, $\hat{d}_i = Z_i p$ (66)

Where: The step length parameter αi is determined to effectively reduce the value function. The value function is defined as follows:

$$\psi(u) = J(u) + \sum_{j=1}^{m_e} r_j g_j(u) + + \sum_{j=m_e+1}^{n} r_j . max[0, g_i(u)]$$

(67)

$$r_j = r_{i+1} = \max\left\{\lambda_j, \frac{(r_i)_j + \lambda_j}{2}\right\}, \text{ j = 1, 2,…, n}$$

(68)

International Journal of Advances in Engineering and Management (IJAEM)
Volume 3, Issue 11 Nov 2021, pp: 336-346 www.ijaem.net ISSN: 2395-5252

Select the starting value of the optimal problem solving process

The nonlinear programming optimization problem to determine different Duty leads to n - 1 cell equalizer at each control cycle. The initial starting point of the optimization problem plays an important role in the process of finding the optimal Duty solution and completing it in the shortest possible time in a control cycle. We know, based on the SoC of each cell, we can preliminarily determine the starting Duty set in the direction of satisfying the constraints and limitations. To adjust the adaptive starting value, the author uses the following algorithm:

for j = 1 : n-1
  if $|SoC_0(j) - SoC_0(j+1)| = 0$
    $u_0(j) = 0$; $u_0(j + (n-1)) = 0$
  else
    if $SoC_0(j) > SoC_0(j+1)$
      $u_0(j) = \mu . D_{max}$; $u_0(j + (n-1)) = 0$
    else
    end
  end
end

Where: μ is the fitness coefficient determined as follows:

$$\mu = \frac{J_k u(k)}{J_1(u(1)-J_f} \quad (69)$$

With: J_1, J_f, $J_k$are the objective function value at the time of the first calculation, desired objective function value when cell balancing is stopped, and objective function value at the current computation time k.

Simulation results for LiB cell of Samsung battery type ICR18650-22P

Cell parameters:

Q = 2200 Ma; U_{min} = 2,6 V; U_{max} = 4,2 V

The parameters of the equalizer circuit:

L = 0,1 mH; C = 470 μF; T = 1s

f_{req} = 10kHz; $T_p = 1/ f_{req}$

Binding parameters:

SoC_{low} = 5; SoC_{high} = 95

$I_{c_{max}}$ = - 0,5A; $I_{d_{max}}$ = 1,5 A

The relationship between SoC and OCV cell terminal voltage is assumed to be linear with the equation:

$V_B = [(4,2 - 2,6)*SoC_0/ 100 + 2,6$

Simulation of multiple cells in series of a Samsung battery type ICR18650-22P

In this case perform balanced optimal control simulation for 11 cells in series, current I_s = 0 A. In the general case, equalization can be performed for n cells. The initial SoC of the cells is random with specific values as shown in Table 1.

***Table 1.** Original SoC of 11 serial cells*

	Cell 1	Cell 2	Cell 3	Cell 4	Cell 5	Cell 6	Cell 7	Cell 8	Cell 9	Cell 10	Cell 11
SoC_0 (%)	79,29	18,91	48,97	44,78	64,62	70,93	75,02	27,82	67,75	65,49	16,48

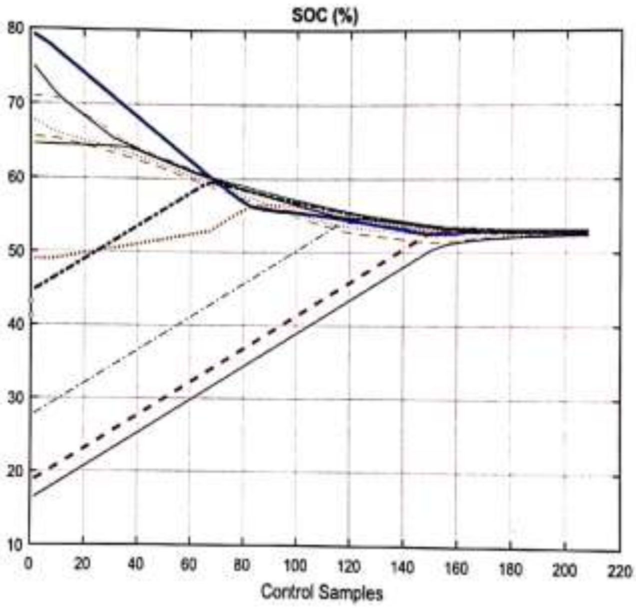

Figure 6. 11-cell SoC balancing at optimal balance control with random SoC (I_s = 0A)

International Journal of Advances in Engineering and Management (IJAEM)
Volume 3, Issue 11 Nov 2021, pp: 336-346 www.ijaem.net ISSN: 2395-5252

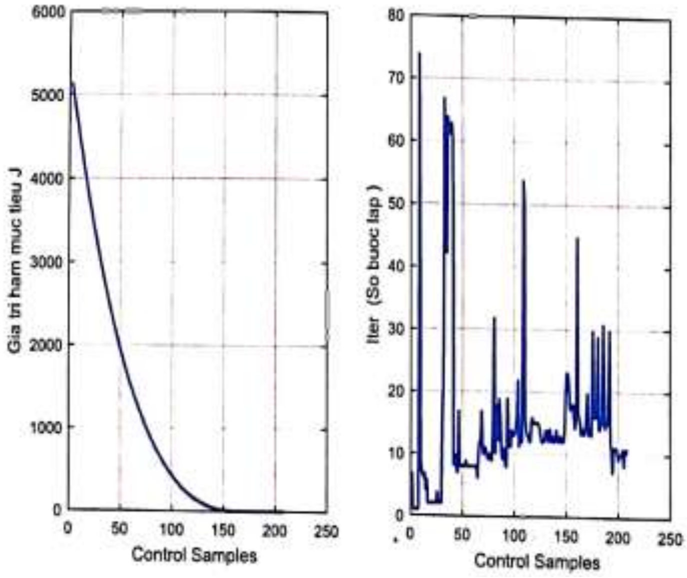

Figure 7. Objective function value J and number of iterations with random SoC (1 cell, I_s = 0.1 A)

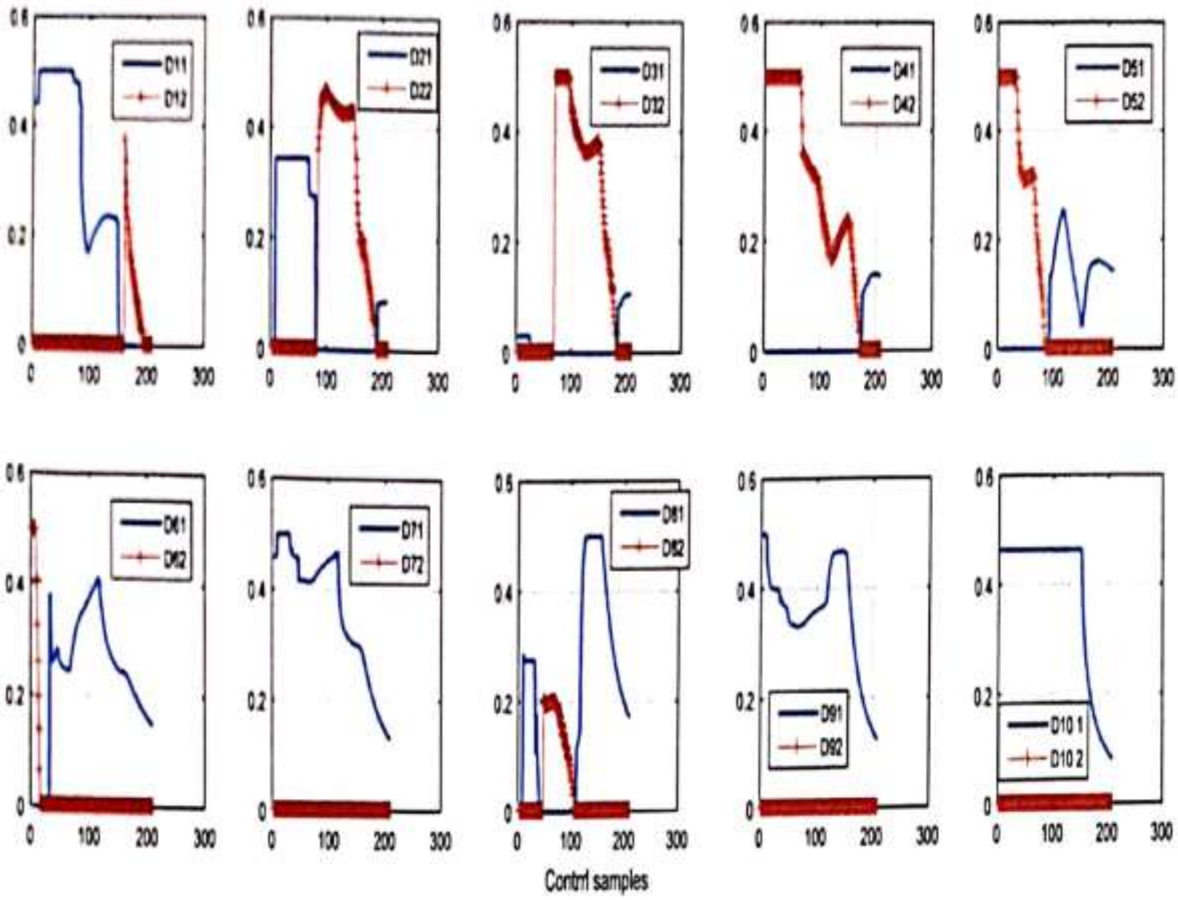

Figure 8. Duty of PWM signal balanced circuit with random SoC(11 cells, I_s = 0 A)

Comment: The optimal control results of SoC balance for 11 Cells in series are shown in Figure 6 to 8. In the above cases, the optimal control of SoC balance is performed between two adjacent cells according to the rule that the cell with the higher SoC will transfer energy to the cell with the lower until the SoC of all cells is balanced. equal. The power transfer between two adjacent cells is accomplished through the optimal duty control of the two PWM control signals for the two MOSFETs in each balanced circuit.

IV. CONCLUDE

The content of the article implements the problem of optimal control of SoC balance for serial LiB cells based on active cell balancing technique. The article has shown the efficiency of cell balancing methods, thereby selecting the appropriate cell balancing method. Build SoC balance model for serial cells using bidirectional Cuk transform based on active balancing technique. Set up the cell-balanced optimal control problem considering the balanced and unbalanced nonlinear constraints, using the SQP method to solve the nonlinear optimization problem.

The simulation results for Samsung LiB cells of type ICR18650-22P with equalization circuits using bidirectional Cuk transform show that the time to perform cell balancing in the cases is quite short, the convergence speed of the objective function fast with small number of control iterations.

VII. ACKNOWLEDGEMENTS

This research was supported by the Thai Nguyen University of Technology, Vietnam (http://www.tnut.edu.vn/).

REFERENCES

[1]. Le Thi Quynh Trang, Phan Vinh Thuy, "Use Kalman Filter to Estimate the State Of Change of Lithium-Ion Battery", International Journal of Advances in Engineering and Management (IJAEM), Volume 3, Issue 9 Sep 2021, pp: 1817-1828 www.ijaem.net, ISSN: 2395-5252

[2]. C. Zhang, L. Y. Wang, X. Li, W. Chen, G. G. Yin, and J. Jiang, "Robust and adaptive estimation of state of charge for lithium-ion batteries" IEEE Transactions on Industrial Electronics, vol. 62, pp. 4948-4957, 2015.

[3]. G. L. Plett, "Recursive approximate weighted totalleasts quares estimation of battery cell total capacity", Journal of Power Sources, vol. 196, pp. 2319-2331, 2011

[4]. [4]. R. Arablouei, S. Werner, and K. Doğançay, "Analysis of the gradient-descent total least-squares adaptive filtering algorithm," IEEE Transactions on Signal Processing, vol. 62, pp. 1256-1264, 2014.

[5]. Y. Luo, Y. Kan, Yanli Yin, Li Liu, H. Cui, and Fei Wu, "Study on a High-Accuracy Real-Time Algorithm to Estimate SoC of Multiple Battery Cells Simultaneously",Journal of Control Science and Engineering, 2017

[6]. Rui Xiong, H. He, F. Sun, and Kai Zhao, " Evaluation on State of Charge Estimation of Batteries With Adaptive Extended Kalman Filter by Experiment Approach",IEEE Transactions on Vehicular Technology, Vol. 62, pp.108-117, 2013

[7]. Hicham Chaoui, A. Elmejdoubi, and H. Gualous, "Online Parameter Identification of Lithium-Ion Batteries with Surface Temperature Variations",IEEE Transactions on Vehicular Technology, 2016

[8]. A.Ito, A. Kawashima, T. Suzuki S. Inagaki, and Z. Zhou, "Model Predictive Charging Control of In-Vehicle Batteries for Home Energy Management Based on Vehicle State Prediction",IEEE Transactions on control systems technology, 2017

[9]. Xiaosong Hu, S. Eben Li, Y. Yang "Advanced Machine Learning Approach for Lithium-Ion Battery State Estimation in Electric Vehicles", IEEE Transactions on Transportation Electrification, 2015

[10]. Il-Song Kim, "A Technique for Estimating the State of Health of Lithium Batteries Through a Dual-Sliding-Mode Observer", IEEE Transactions on Power Electronics, Vol. 25, pp.1013-1022, 2010

[11]. Rui Xiong, X. Gong, C. Chris Mi, and F. Sun "A robust state-of-charge estimator for multiple types of lithium-ion batteries using adaptive extended Kalman filter", Journal of Power Sources, vol 243, pp. 805-816, 2013

[12]. Satadru Dey, Sara Mohon, P. Pisu, and B. Ayalew, "Sensor Fault Detection, Isolation, and Estimation in Lithium-Ion Batteries",IEEE Transactions on control systems technology, 2016

[13]. S. Hong, J. Jiang, Xi Li, "A battery management system with two-stage equalization",IEEE, 29th Chinese Control And Decision Conference, pp.4109-4113

[14]. Ji Liu, Guang Li, and H. K. Fathy, "An Extended Differential Flatness Approach for the Health-Conscious Nonlinear Model Predictive Control of Lithium-Ion

Batteries",IEEE Transactions on control systems technology, 2016
[15]. Krishnan S. Hariharan, Sanoop Ramachandran, Piyush Tagade, "Mathematical Modeling of Lithium Batteries", Springer International Publishing, 2018
[16]. Jiuchun Jiang,Caiping Zhang, "Fundamentals and Applications of Lithium-Ion Batteries in Electric Drive Vehicles", John Wiley & Sons publishers, 2015
[17]. Whittingham MichaelStanley,"Electrical Energy Storage and Intercalation Chemistry", Science,vol 192, pp. 1126–1127, 1976.
[18]. Doublet Marie-Liesse, ‘'Batteries Li-ion:Conception théorique", Techniques de l'ingénieur, vol. AFP4, 2009