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Comparison of the Pseudo-Rigid Body (PRB) Method with Euler–Bernoulli, BCM, And Abaqus Models in Cantilever Beam Deflection Analysis

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Abstract
This paper presents a comprehensive comparative study of the Pseudo-Rigid Body (PRB) method with the Euler–Bernoulli beam theory, the Beam Constraint Model (BCM), and finite element simulations conducted in ABAQUS for the analysis of deflection in a cantilever beam subjected to a concentrated load at the free end. The PRB method models an elastic beam as a chain of rigid links connected by torsional springs, allowing geometric nonlinearity and large rotations to be captured with low computational cost. In particular, this study implements PRB in the form of PRB–FSM (Finite Segment Method) combined with midpoint curvature integration, which improves algorithmic transparency and reproducibility from a mechanics perspective. The results show that the deviation between PRB and ABAQUS remains below 0.3% along the entire beam length while accurately preserving the deformation shape and elastic energy. These findings confirm that the PRB method is an efficient and reliable approach for the analysis and design of compliant structures operating in the large-deformation regime.

Keywords: Pseudo-Rigid Body, PRB–FSM, cantilever beam, large deformation, compliant mechanism, geometric nonlinearity.

I. Introduction

The deformation analysis of elastic beams is a fundamental problem in structural mechanics and mechanical design. The Euler–Bernoulli beam theory is widely used due to its simplicity and the availability of analytical solutions; however, this model relies on the assumptions of small displacement and small rotation, and therefore remains accurate only within the small-deflection regime [1,2].

In modern applications such as compliant mechanisms, microelectro mechanical systems (MEMS), soft robotics, and flexible structures, beams often operate in the large-deformation regime where geometric nonlinearities dominate the structural response [3]. In such cases, the Euler–Bernoulli model tends to underestimate structural stiffness and beam deflection, leading to noticeable discrepancies compared with experimental results and finite element simulations [4].

To address this limitation, several nonlinear modeling approaches have been developed. Among them, the Beam Constraint Model (BCM) provides an important framework for improving the prediction of large-deflection behavior in compliant mechanisms [5]. However, BCM typically relies on empirical correction factors derived from experiments or FEM simulations, which may reduce the general applicability of the model.

The Pseudo-Rigid Body (PRB) method, originally proposed by Howell and co-workers, approaches the problem from a mechanism design perspective: the flexible beam is replaced by a series of rigid links connected by equivalent torsional springs [3,6]. This representation allows geometric nonlinearities to be captured effectively while maintaining a simple mechanical structure. The present paper focuses on a detailed investigation of the PRB approach, particularly in the form of PRB–FSM, and evaluates its accuracy through comparison with Euler–Bernoulli theory, BCM, and ABAQUS finite element simulations.

II. Modeling and Computational Methods

2.1 Cantilever Beam Model

A cantilever beam of length L=0.1 m is considered, where one end is fixed and a concentrated load F is applied at the free end. The deflection is evaluated at equally spaced positions along the beam length (from 0% to 100% of the span). This configuration is a standard benchmark problem commonly used to validate beam-bending models [2,4].

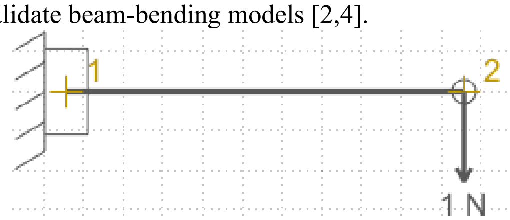

Figure 1: cantilever beam

2.2 Euler–Bernoulli Beam Model

According to Euler–Bernoulli beam theory, the curvature of the beam is given by [1]:

$$\kappa(x) = \frac{M(x)}{EI}$$

where M(x) denotes the bending moment, E is the Young's modulus, and I is the second moment of area of the beam cross-section.

This formulation neglects shear deformation and geometric nonlinear effects.

2.3 Beam Constraint Model (BCM)

The Beam Constraint Model (BCM) can be considered an extension of Euler–Bernoulli theory designed to describe large-deflection behavior in compliant mechanisms [5]. The model introduces nonlinear correction functions for rotational and translational distributions along the beam, improving accuracy relative to linear beam theory. Nevertheless, BCM relies on empirical coefficients derived from experimental measurements or FEM simulations.

2.4 Finite Element Simulation Using ABAQUS

Finite element simulations were conducted using ABAQUS with the B31 beam element while fully accounting for geometric nonlinearity. The ABAQUS results are treated as reference solutions for evaluating the accuracy of the analytical models [8].

2.5 PRB–FSM Algorithm Based on Midpoint Curvature Integration

To improve the clarity and reproducibility of the Pseudo-Rigid Body method, this study adopts a specific implementation known as the PRB–FSM (Finite Segment Method). In this approach, the beam is divided into multiple finite segments and curvature integration is performed using the midpoint rule.

This approach combines the discrete nonlinear modeling capability of PRB with the second-order numerical accuracy of midpoint integration, which is commonly used in large-deflection beam analysis [6,9].

2.5.1 Basic Concept of PRB–FSM

The cantilever beam of length LLL is divided into nnn equal segments, each having length

$$\Delta x = L/n$$

Instead of assuming continuous curvature distribution, the curvature within each segment is approximated by its value at the midpoint. The nodal rotation and coordinates are then updated according to nonlinear geometric relationships.

This formulation enables:

- consideration of large rotations and large displacements,
- elimination of the small-angle assumption of Euler–Bernoulli theory,
- a simple and computationally efficient algorithm.

2.5.2 Update Equations for Each Segment

For segment j (j=0,1,…,n−1), the quantities are defined as:

$$x_{mid} = (j + 1/2)\Delta x$$

$$M_{mid} = F(L - x_{mid})$$

$$\kappa_{mid} = \frac{M_{mid}}{EI}$$

$$\Delta\theta = \kappa_{mid}\Delta x$$

$$\theta_{j+1} = \theta_j + \Delta\theta$$

$$\theta_{avg} = \theta_j + \frac{1}{2}\Delta\theta$$

$$x_{j+1} = x_j + \Delta x \cos(\theta_{avg})$$

$$y_{j+1} = y_j + \Delta x \sin(\theta_{avg})$$

Initial conditions at the fixed support are:

$$x_0 = 0, \quad y_0 = 0, \quad \theta_0 = 0$$

$$\Delta x = 10\,mm, \quad EI = 87\,500\,N \cdot mm^2$$

Physically, each segment Δx behaves as a rigid link of the PRB model, while Δθ represents the rotation of the equivalent torsional spring.

2.5.3 Numerical Illustration for Each Segment

In this study, the beam is divided into n=10 segments with l=100mm, b=5mm, h=1mm,

Segment 0 (j=0)

- x_mid = 5 mm
- M_mid = 1 × (100 − 5) = 95 N·mm
- κ = 95 / 87500 = 0.001085714286 mm^{-1}
- Δθ = 0.001085714286 × 10 = 0.010857142857 rad
- θ_1 = 0.010857142857 rad
- θ_avg = 0.005428571429 rad
- x_1 = 0 + 10 cos(0.00542857) ≈ 9.99985 mm
- y_1 = 0 + 10 sin(0.00542857) ≈ 0.054285 mm

Segment 1 (j=1)

- x_mid = 15 mm, M_mid = 85 N·mm
- κ = 85 / 87500 = 0.0009714285714
- Δθ = 0.009714285714 → θ_2 ≈ 0.02057142857 rad

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• θ_avg ≈ 0.01571428571
• x_2 ≈ 19.99862 mm
• y_2 ≈ 0.211420 mm
Segment 2 (j=2)
• x_mid = 25 mm, M_mid = 75 N·mm
• κ = 75 / 87500 = 0.0008571428571
• Δθ = 0.008571428571 → θ_3 ≈ 0.02914285714 rad
• x_3 ≈ 29.99554 mm
• y_3 ≈ 0.460959 mm
Segment 3 (j=3)
• x_mid = 35, M_mid = 65
• κ = 0.0007428571429, Δθ = 0.007428571429 → θ_4 ≈ 0.03657142857 rad
• x_4 ≈ 39.99013 mm
• y_4 ≈ 0.789469 mm
Segment 4 (j=4)
• x_mid = 45, M_mid = 55
• κ = 0.0006285714286, Δθ = 0.006285714286 → θ_5 ≈ 0.04285714286 rad
• x_5 ≈ 49.98224 mm
• y_5 ≈ 1.186501 mm
Segment 5 (j=5)
• x_mid = 55, M_mid = 45
• κ = 0.0005142857143, Δθ = 0.005142857143 → θ_6 ≈ 0.04799999999 rad
• x_6 ≈ 59.97190 mm
• y_6 ≈ 1.640634 mm
Segment 6 (j=6)
• x_mid = 65, M_mid = 35
• κ = 0.0004, Δθ = 0.004 → θ_7 ≈ 0.052 rad
• x_7 ≈ 69.95940 mm
• y_7 ≈ 2.140424 mm
Segment 7 (j=7)
• x_mid = 75, M_mid = 25
• κ ≈ 0.0002857142857, Δθ ≈ 0.002857142857 → θ_8 ≈ 0.05485714286 rad
• x_8 ≈ 79.94511 mm
• y_8 ≈ 2.674488 mm
Segment 8 (j=8)
• x_mid = 85, M_mid = 15
• κ ≈ 0.0001714285714, Δθ ≈ 0.001714285714 → θ_9 ≈ 0.05657142857 rad
• x_9 ≈ 89.92959 mm
• y_9 ≈ 3.231399 mm
Segment 9 (j=9)
• x_mid = 95, M_mid = 5
• κ ≈ 5.714285714 × 10^{-5}, Δθ ≈ 0.0005714285714 → θ_{10} ≈ 0.05714285714 rad
• x_{10} ≈ 99.91342 mm
• y_{10} ≈ 3.799758 mm

Table 1: List of all solutions obtained using the PRB method.

node	x (mm)	y (mm)	θ (deg, ≈)
0	0.000	0.0000	0.0000
1	9.99985	0.05429	0.6226
2	19.9986	0.21142	1.1782
3	29.9955	0.46096	1.6692
4	39.9901	0.78947	2.0968
5	49.9822	1.18650	2.4562
6	59.9719	1.64063	2.7496
7	69.9594	2.14042	2.9806
8	79.9451	2.67449	3.1468
9	89.9296	3.23140	3.2549
10	99.9134	3.79976	3.2730

• Final PRB–FSM solution: δ_FSM ≈ 3.79976 mm and θ_FSM ≈ 0.05714285714 rad (3.273°).

2.6 Finite Element Simulation and Result Visualization in ABAQUS

To validate the accuracy of the PRB–FSM method, finite element simulations were performed in ABAQUS using B31 beam elements with full geometric nonlinearity enabled (NLGEOM = ON). Figure 2 illustrates the deformation shape of the cantilever beam and the nodal deflections obtained from ABAQUS. Deflection values were extracted at evenly spaced locations along the beam, corresponding exactly to the nodes used in the PRB–FSM model (10 segments, 11 nodes), ensuring a consistent comparison.

Simulation conditions in ABAQUS:

- **Element type:** Beam B31
- **Boundary condition:** Fully fixed at the left end (Encastre); concentrated transverse load at the free end
- **Analysis type:** Static, General with geometric nonlinearity
- **Coordinate system:** identical to the PRB–FSM model

The nodal deflection values indicate that deflection increases nonlinearly from the fixed support toward the free end. The maximum deflection at the free end reaches **3.80757 mm**, which is used as the reference value for evaluating the other models.

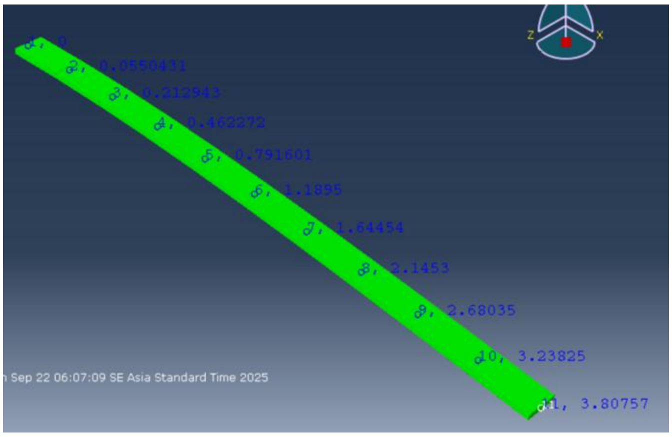

Figure 2. *Deformed shape and nodal deflection of the cantilever beam obtained from ABAQUS simulation.*

Figure 2 illustrates the magnified deformed shape of the cantilever beam subjected to a concentrated load at the free end. The blue labels indicate the deflection in the loading direction at the nodes along the beam length. The ABAQUS results are used as the reference solution for comparison with the PRB–FSM, Euler–Bernoulli, and BCM methods.

III. Results and Discussion

3.1 Deflection Results and Quantitative Comparison

The deflection values along the beam obtained from four methods (PRB, Euler–Bernoulli, BCM, and ABAQUS) are summarized in Table 1 and illustrated in Figure 2. The PRB deflection curve almost perfectly overlaps with the ABAQUS results along the entire beam length.

Table 2: *Deflection data along the beam length (mm)*

X (mm)	PRB	Euler–Bernoulli	BCM	ABAQUS
0	0.00000	0.00000	0.00000	0.00000
10	0.05429	0.05523	0.05523	0.05504
20	0.21142	0.21333	0.21333	0.21294
30	0.46096	0.46286	0.46285	0.46227
40	0.78947	0.79238	0.79238	0.79160
50	1.18650	1.19048	1.19048	1.18950
60	1.64063	1.64571	1.64571	1.64454
70	2.14042	2.14666	2.14666	2.14530
80	2.67449	2.68190	2.68190	2.68035
90	3.23140	3.24000	3.23999	3.23825
100	3.79976	3.80952	3.80520	3.80757

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3.2 Discussion on Curvature, Stiffness, and Energy

The Euler–Bernoulli theory assumes curvature proportional to bending moment and constant bending stiffness EIEIEI, which tends to underestimate geometric stiffness under large deformation [1,2].

In contrast, PRB–FSM updates beam geometry using the average rotation within each segment, enabling the influence of large rotations on deformation shape to be captured.

From an energy perspective, under monotonic static loading, the strain energy may be approximated by the external work:

$$U \approx \frac{1}{2} F\,\delta(L)$$

The strong agreement between PRB and ABAQUS in predicting the tip deflection implies a consistent estimation of global elastic energy, further confirming the mechanical validity of the PRB method in geometrically nonlinear regimes [6,7].

3.3 Direct Node-to-Node Comparison Between PRB–FSM and ABAQUS

The discretization of the beam into ten segments in the PRB–FSM model corresponds directly to eleven nodes in the ABAQUS model, enabling node-to-node comparison without interpolation.

This direct comparison demonstrates that:

a. the deformation shape predicted by PRB–FSM closely matches the ABAQUS results,

b. nodal deflection differences remain below 0.3%,

c. the smallest discrepancies occur near the free end where bending moment decreases.

These observations confirm that midpoint curvature integration in PRB–FSM not only reproduces displacement magnitudes accurately but also preserves the global curvature distribution of the beam.

IV. Conclusions

This study demonstrates that the Pseudo-Rigid Body method, particularly in the PRB–FSM formulation with midpoint curvature integration, is an effective and reliable tool for analyzing cantilever beam deflection in the large-deformation regime.

With an error below **0.3%** relative to ABAQUS finite element simulations and significantly lower computational cost, the PRB method is highly suitable for rapid analysis and design of compliant mechanisms, soft robots, and flexible structures.

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