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Continuous Monotonic Cube Decomposition of Coconut Tree Graph CT_m

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ABSTRACT: Let G be a finite, connected, undirected simple graph with p vertices and q edges. If $G_1, G_2, \ldots, G_n$ are connected edge – disjoint subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)$, then $\{G_1, G_2, \ldots, G_n\}$ is said to be a decomposition of G. In this paper, we introduce a new concept called Continuous Monotonic Cube Decomposition. A graph G is said to have a Continuous Monotonic Cube Decomposition if G can be decomposed into subgraphs $\{G_1, G_2, \ldots, G_n\}$ such that each G_i is connected and $|E(G_i)| = i^3$, for $1 \le i \le n$. Clearly, $q = \left[\frac{n(n+1)}{2}\right]^2$. Also, we obtained the characterization for a coconut tree graph CT_m to admit Continuous Monotonic Cube Decomposition.

Keywords : Decomposition of graph, Continuous Monotonic Decomposition, Continuous Monotonic Cube Decomposition.

I. INTRODUCTION

Let $G = (V, E)$ be a simple, connected graph with p vertices and q edges. If $G_1, G_2, \ldots, G_n$ are connected edge – disjoint subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)$, then $\{G_1, G_2, \ldots, G_n\}$ is said to be a decomposition of G. Different types of decomposition of G have been studied in the literature by imposing suitable conditions on the subgraphs G_i. In this paper, we introduce a new concept called Continuous Monotonic Cube Decomposition. Terms not defined here are used in the sense of Harary [2].

Definition 1.1. Let $G = (V, E)$ be a simple graph of order p and size q. If $G_1, G_2, \ldots, G_n$ are edge - disjoint subgraphs of G such that $E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)$, then $\{G_1, G_2, \ldots, G_n\}$ is said to be a Decomposition of G.

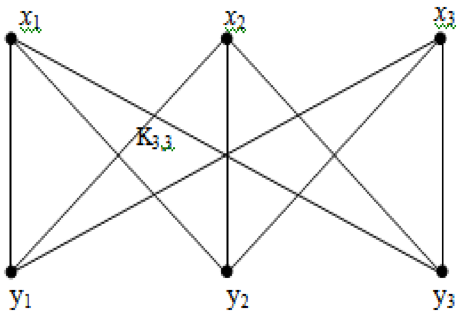
Definition [3] 1.2. A decomposition $\{G_1, G_2, \ldots, G_n\}$ of a connected graph G is said to have Continuous Monotonic Decomposition if each G_i is connected and $|E(G_i)| = i$, for $1 \le i \le n$.

Definition [5] 1.3. A Coconut Tree graph CT(m,n) is the graph obtained from the path P_n by appending m new pendant edges at an end vertex of P_n. If m = n, then a coconut tree graph CT(m,n) is denoted as CT_m.

II. CONTINUOUS MONOTONIC CUBE DECOMPOSITION OF GRAPHS

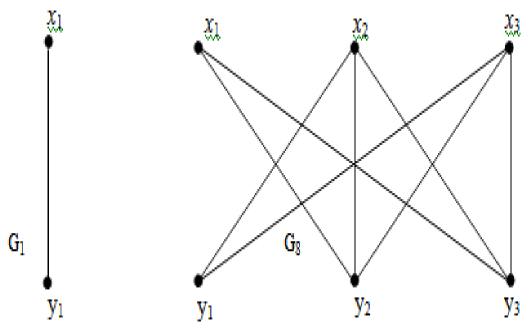
Definition 2.1. A connected graph G admit Continuous Monotonic Cube Decomposition $\{G_1, G_2, \ldots, G_n\}$ if each G_i is connected and $|E(G_i)| = i^3$, $\forall$ i = 1, 2, . . . , n. Clearly $q = \left[\frac{n(n+1)}{2}\right]^2$ is the sum of the cubes of first n natural numbers. We denote the Continuous Monotonic Cube Decomposition as $\{G_1, G_8, G_{27}, \ldots, G_{n^3}\}$.

Example 2.2. Let us consider the complete bipartite graph $K_{3,3}$. The graph $K_{3,3}$ is given in the following figure.



e

The complete bipartite graph $K_{3,3}$ admit Continuous Monotonic Cube Decomposition $\{G_1, G_8\}$. The Continuous Monotonic Cube Decomposition of $K_{3,3}$ is given in the following figure.



Theorem 2.3. A connected graph G admit Continuous Monotonic Cube Decomposition $\{G_1, G_8, G_{27}, ..., G_{n^3}\}$ if and only if q = $\left[\frac{n(n+1)}{2}\right]^2$, ∀ n ∈ N.

III. CONTINUOUS MONOTONIC CUBE DECOMPOSITION OF CT_M

Lemma 3.1. Let k + 2 ≡ 0(mod 4). Then G can be decomposed into $\{G_{1^3}, G_{2^3}, ..., G_{k^3}\}$. Here $\left[\frac{k(k+1)}{2}\right]^2 = 2m - 1.$

Proof. We have k + 2 ≡ 0(mod 4). Then k = 4r - 2, r ≥ 1 and r ∈ ℤ. Proof is by induction on r. When r = 1, k = 2. Then 2m - 1 = 9 can be decomposed into $\{G_{1^3}, G_{2^3}\}$. Hence the result is true for r = 1.

Assume the result is true for r-1. Then k = 4r - 6. Then $q' = 2m - 1 = \left[\frac{(4r-6)(4r-5)}{2}\right]^2$ can be decomposed into $\{G_{1^3}, G_{2^3}, \ldots, G_{(4r-6)^3}\}$.

Now, to prove the result is true for r. We have to prove that q = 2m - 1 = $\left[\frac{(4r-2)(4r-1)}{2}\right]^2$ can be decomposed into $\{G_{1^3}, G_{2^3}, \ldots, G_{(4r-2)^3}\}$.

Define q = q' ∪ (4r-5) ∪ (4r-4) ∪ (4r-3) ∪ (4r-2). Then q = q' + $(4r - 5)^3 + (4r - 4)^3 + (4r - 3)^3 + (4r - 2)^3$ = $\left[\frac{(4r-2)(4r-1)}{2}\right]^2$ can be decomposed into $\{G_{1^3}, G_{2^3}, \ldots, G_{(4r-2)^3}\}$. Hence by induction hypothesis, the result is true for all r.

Lemma 3.2. Let k + 3 ≡ 0(mod 4). Then G can be decomposed into $\{G_{1^3}, G_{2^3}, ..., G_{k^3}\}$. Here $\left[\frac{k(k+1)}{2}\right]^2 = 2m - 1.$

Proof. We have k + 3 ≡ 0(mod 4). Then k = 4r - 3, r ≥ 1 and r ∈ ℤ. Proof is by induction on r. When r = 1, k = 1. Then 2m - 1 = 1 can be decomposed into $\{G_{1^3}\}$. Hence the result is true for r = 1.

Assume that the result is true for r - 1. Then k = 4r - 7. Then $q' = 2m - 1 = \left[\frac{(4r-7)(4r-6)}{2}\right]^2$ can be decomposed into $\{G_{1^3}, G_{2^3}, \ldots, G_{(4r-7)^3}\}$.

Now, to prove the result is true for r. We have to prove that q = 2m - 1 = $\left[\frac{(4r-3)(4r-2)}{2}\right]^2$ can be decomposed into $\{G_{1^3}, G_{2^3}, \ldots, G_{(4r-3)^3}\}$. Define q = q' ∪ (4r - 6) ∪ (4r - 5) ∪ (4r - 4) ∪ (4r - 3). Then q = q' + $(4r - 6)^3 + (4r - 5)^3 + (4r - 4)^3 + (4r - 3)^3$ = $\left[\frac{(4r-3)(4r-2)}{2}\right]^2$ can be decomposed into $\{G_{1^3}, G_{2^3}, \ldots, G_{(4r-3)^3}\}$. Hence by induction hypothesis , the result is true for all r.

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Theorem 3.3. For any odd integer m, CT_m has Continuous Monotonic Cube decomposition [k-decompositions] denoted by $\{G_{1^3}, G_{2^3}, ..., G_{k^3}\}$ if and only if there exists an integer k satisfying the following properties :

1. k = 4r - 2 or 4r - 3, r ≥ 1 and r ∈ ℤ, where k denotes the total number of decompositions.
2. $\left[\frac{k(k+1)}{2}\right]^2 = 2m-1.$

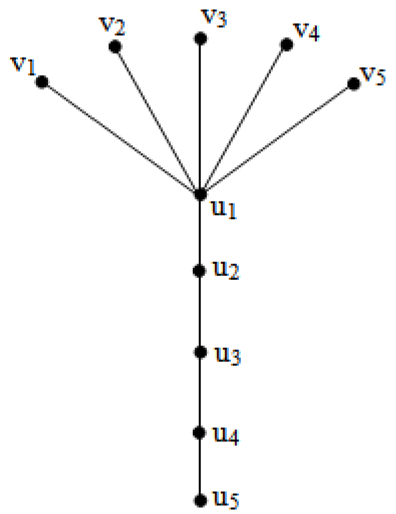
Proof. Let G = CT_m. By the definition of G, q = 2m - 1. Assume that G accept Continuous Monotonic Cube Decomposition. By the definition, q = $\left[\frac{k(k+1)}{2}\right]^2$ where k denotes the total number of decompositions. Hence $\left[\frac{k(k+1)}{2}\right]^2 = 2m-1$.

Then m = $\frac{[k(k+1)]^2+4}{8}$. Clearly m is an odd integer. Since m is an odd integer, k = 4r - 2 or 4r - 3, r ≥ 1 and r ∈ ℤ.

Conversely, assume that k = 4r - 2 or 4r - 3, r ≥ 1 and r ∈ ℤ. Also $\left[\frac{k(k+1)}{2}\right]^2 = 2m-1$.

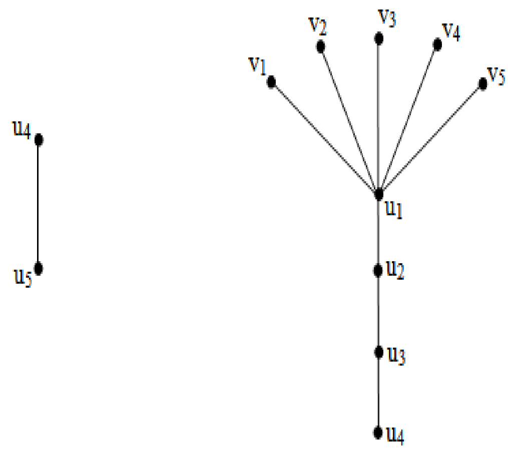
By Lemma 3.1 and 3.2, 2m - 1 = $\left[\frac{k(k+1)}{2}\right]^2$ can be decomposed into $\{G_{1^3}, G_{2^3}, ..., G_{k^3}\}$. Hence G admit Continuous Monotonic Cube Decomposition.

Illustration 3.4. As an illustration, let us decompose CT_5. The graph CT_5 is given in the following figure.



CT_5

Let G = CT_5. Here m = 5. Then k = 2. Thus k + 2 ≡ 0(mod 4). Hence G admit Continuous Monotonic Cube Decomposition. The decomposition $\{G_{1^3}, G_{2^3}\}$ of G is given in the following figure.



G_{1^3} G_{2^3}

Table 3.5. List of first 10 CT_m's that accept Continuous Monotonic Cube decomposition and their decompositions are given in the following table.

m	Continuous Monotonic Cube Decompositions
1	G_{1^3}
5	G_{1^3}, G_{2^3}

113	$G_{1^3}, G_{2^3}, ..., G_{5^3}$
221	$G_{1^3}, G_{2^3}, ..., G_{6^3}$
1013	$G_{1^3}, G_{2^3}, ..., G_{9^3}$
1513	$G_{1^3}, G_{2^3}, ..., G_{10^3}$
4141	$G_{1^3}, G_{2^3}, ..., G_{13^3}$
5513	$G_{1^3}, G_{2^3}, ..., G_{14^3}$
11705	$G_{1^3}, G_{2^3}, ..., G_{17^3}$
14621	$G_{1^3}, G_{2^3}, ..., G_{18^3}$

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