International Journal of Advances in Engineering and Management (IJAEM)
Volume 3, Issue 7 July 2021, pp: 1623-1629 www.ijaem.net ISSN: 2395-5252

Controller Design of a Stirred Mixing Tank Using the Model Reference Adaptive Control Scheme and Mit Rule

**Madu, C,Fadayini, O.M.,Folami, N.A. andItamomoh, A.
Department of Chemical Engineering, Lagos State Polytechnics, Ikorodu, Lagos, Nigeria.

Submitted: 01-07-2021 Revised: 10-07-2021 Accepted: 13-07-2021

ABSTRACT

Due to variations in process dynamics as a result of nonlinear actuators, changes in environmental conditions and variations in the character of disturbances, feedback controllers may not perform optimally when used online. In this paper, the design of a controller using a second order system with Model Reference Adaptive Control (MRAC) scheme and MIT Rule for adaptive mechanism was carried out to overcome the challenges. The aim was to ensure that plant process output data tracked the reference model. The performance of the adaptive controller scheme was evaluated through simulationusing MATLAB 18.0 SIMULINK.The simulation was carried using MIT rule for different values of adaptation gain. The result was satisfactory and showed very sensitive to changes in adaptation gain parameter. It was observed that the response of the system improvedwith increment in adaptation gain, but beyond a certain limit (0.5> γ >2.0), the performance of the system became poor.The optimum value of the adaptation gain was γ =1.0, with a peak time of 2.65 seconds and zero overshoot.

Key Word: Controller Design, Adaptative Control, MIT Rule, Mixing Tank, Peak Time

I. INTRODUCTION

Every process has operating conditions which must be maintained during working time in order to achieve the goal of the process (Gupta and Nigam, 2020). The goal of such systems is to obtain quality product at optimum operating conditions.The violation of operating conditions may be hazardous and even may cause human death (Arshad et al., 2013).Chemical process industry is very vast and complex with several operating systems. It is difficult to handle such complex processes without controllers. The design of a mixing tank deals with multiple aspects of chemical engineering (Ahmed et al., 2016) and it belongs to a class of nonlinear systems. Their models are derived and described inthe works of Schmidt (2005),Corriou (2004) and Ogunnaike and Ray (1994). The process nonlinearities may cause difficulties when controlling using conventional controllers with fixed parameters. One possible method to cope with this problem is using adaptive strategies basedon appropriate choice of an external linear model (ELM) with recursively estimated parameters. These parameters are consequently used for parallel updating of controller's parameters(Dostail, et al., 2004).The control itself can be either continuous-time or discrete.

Mixing tank is widely used in many process industries in handling liquid- liquid mixture, slurries etc. The P1 and PID controllers are widely used in many industrial control systems because of its simple structure and robustness. Tuning of the PID is commonly done using the classical controller tuning methods of Ziegler-Nichols (Z-N) and Cohen-Coon methods, since they are easier than other methods. Internal model control (IMC) tuning offers an alternative tuning to increase the controller's overall performance. A tuning system of an adaptive control senses parametervariations and tune the controller parameters in order to compensate for it(Boiling et al., 2007). The parametric variation may be due to the inherent non-linearity of the system. In MRAC a reference model is used to describe the system's performance. The adaptive controller is then designed to force the system (or plant) to behave like the reference model. Model output is compared to the actual output and the difference is used to adjust feedback controller parameters (Chatterjee et al., 2017).

The major disadvantage of non-adaptive control systems is that these control systems cannot cope with fluctuations in the parameters of the process. The solution to this problem is to develop a control system that adapts to changes in the process(Chatterjee et al., 2011). The controller parameters are adjusted to give a desired closed-

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loop performance. Adaptive controller changes the control algorithm coefficients in real time to compensate for variations in the environment or in the system itself and hence is suitable for online control of processes. Gain scheduling is one form of adaptive control but it requires knowledge about the process to be effective.

Our aim in this work is to design adaptive control system which has the ability to adjust itself to handle unknownmodel uncertainties. This is a technique that provides a systematic approach for automatic adjustment of the controller parameters in order to achieve orto maintain a desired level of control system performance when the parameters of the system dynamic model are unknown and/or change in time. The various types of adaptive control systems differ only in the way the parameters of the controller are adjusted (Swathi et al., 2017). The adaptive controller needs an objective(cost) function that will guide the adaptation mechanism to the "best" adjustment of the controller parameters. Any of the performance criteria could be used such as one - quarter decay ratio, integral of square error or gain or phase margin(Chatterjee et al., 2011).

II. THEORY

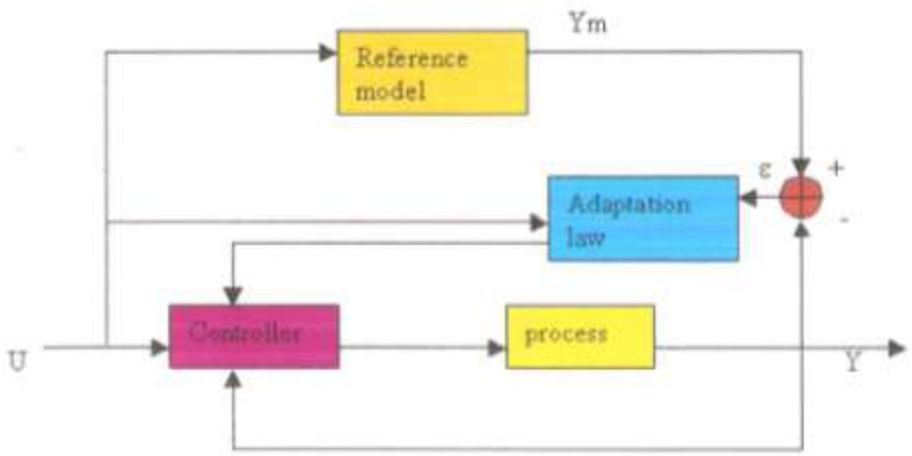

Figure 1: Basic Model Reference Adaptive Control (MRAC)Structure

The standard MRAC based systems contain four key blocks as shown in Figure 1.The reference model defines the desired performance characteristics of the process being controlled. The adaptation law uses the error between the process and the model output, to vary the parameters of the control system(Petre, 2010). These parameters are varied so as to minimize the error between the process and the reference model. The control system can be anything from a simple gain-based controller to a more complicated parameter-based transfer function or plant matrix. Whatever type of control system is used the parameters of the controller will be varied by the adaptation law. The final element of the MRAC system is the process that is being controlled.

The typical controller structure used for adaptive control-based solutions is shown in Figure 2. This controller combines both feed forward and feedback control elements. These are typically just gain values; however, they can be complete transfer functions. They have parameters that can be modified by the adaptation law.

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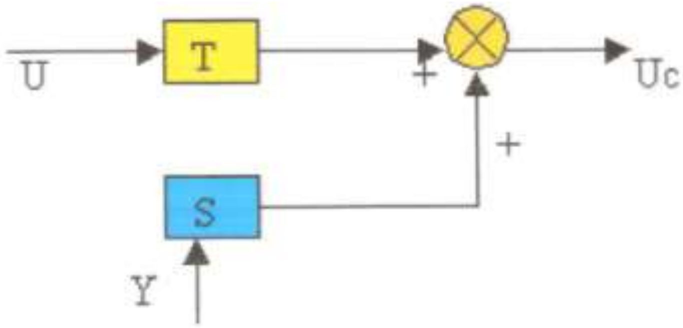

Figure 2: Standard Controller's Structure

2.1 Adaptation Law: The adaptation law attempts to find a set of parameters that minimizes the error between the plant and the model outputs. To do this, the parameters of the controller are incrementally adjusted until the error has reduced to zero. The gradient method, also known as the MIT rule*, changes the parameters based upon the gradient of the error, with respect to that parameter. The parameters are changed in the direction of the negative gradient of the error. This means that if the error, with respect to a specified parameter, is increasing then by the MIT rule the value of that parameter will be decreased(Chatterjee et al., 2017) as shown by Equation 2. below.

Let $y_m(t)$ be the output of the reference model and y(t) the output of the actual plant. The difference is the tracking error e(t).

$$e(t) = y(t)-y_m(t) \quad 1$$

We now apply the MIT rule. In this rule, a function is defined as

$$J(\theta) = \frac{e^2}{2} \quad 2$$

e is the error between the outputs of plant and the model, and θ is the adjustable parameter. θ is adjusted in such a way that the cost function is minimized to zero. Hence the change in the parameter θ is kept in the direction of negative gradient of J.

$$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} \quad 3$$

From Equ.2 $$\frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \gamma} = -\gamma e \frac{\partial e}{\partial \theta} \quad 4$$

$\frac{\partial e}{\partial \theta}$ is the sensitivity derivative of the system. It indicates how the error is changing with respect to the parameter. Equation 3 describes the change in parameter wrt time so that the cost function J(θ) can be reduced to zero. γ is a positive quantity which is the adaption gain of the controller.

This approach has one major disadvantage.It does not guarantee stability. The adaptation gain must be made small and the initial values of parameters must be stable for the adaptation law to operate correctly. A rule developed by researchers at Massachusetts Institute of Technology (MIT).TheModel Reference Adaptive Controller is shown in Figure 3

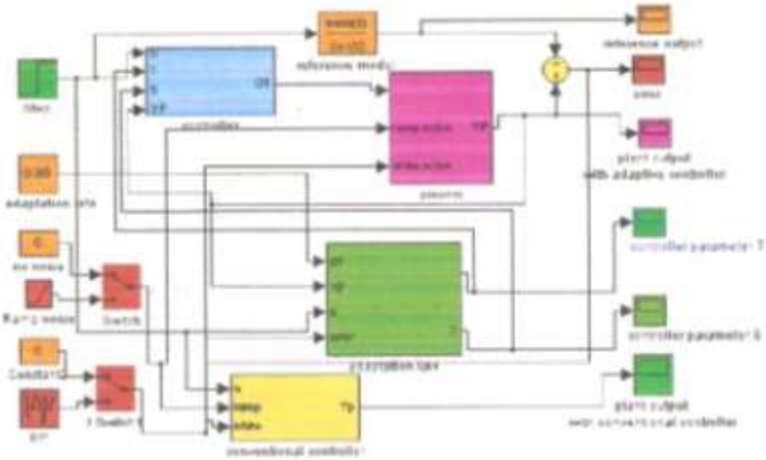

Figure 3: Simulink Implementation of Model Reference Adaptive Control

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The diagram shows the controller and adaptation law systems given previously(Abubeker, 2004). The plant and the model are also shown. Two blocks (A_p Noise and B_p Noise) are used to vary the parameters of the control system, forcing the adaptation law to continually adapt to the changes in the process parameters. A number of scopes are provided to monitor the output of the model, the output of the plant and the error between the two(Dostal et al., 2007). The final element of the system is the adaptation rate (gamma) which can be adjusted manually. For instance, to obtain rapid adaptation to changes in the system we have to set higher adaptation rate, however that may also cause instability(Swathi, 2017).

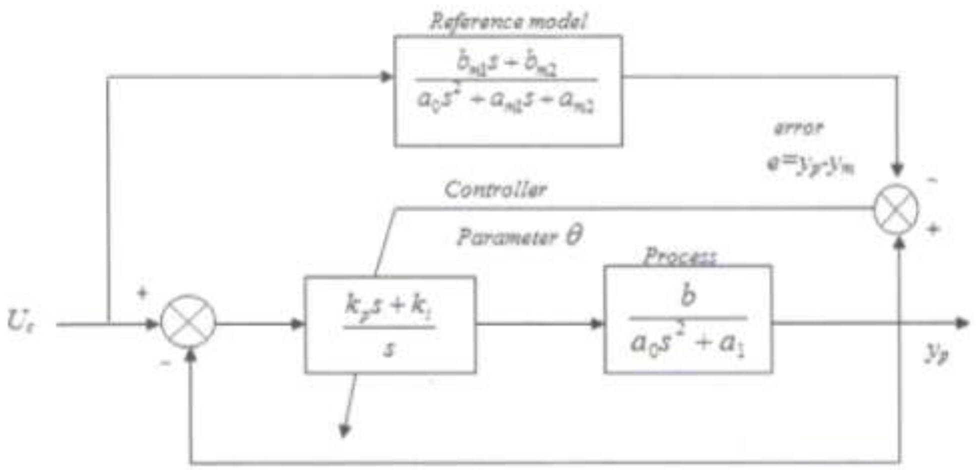

Figure 4: Block diagram of MRAC Strategy

III. CONSTRUCTING THE MODEL REFERENCE ADAPTIVE CONTROLLER

MRAC has two loops: an inner loop (or regulator loop) that is an ordinary control loop consisting of the plant and the regulator, and an outer (or adaptation) loop that adjusts the parameters of the regulator in such a way as to drive the error between the model output and plant output to zero. (Chatterjee et al., 2017). Block diagram of MRAC is shown in Figure 4.

Comparing θ with the P1 controller parameters we have

$$k_p = \frac{1}{s}\left[-\gamma_\rho^e\left[\frac{bs}{a_o s^2+[as+bkp]s+bki}\right]u_c-\gamma_\rho\right] \quad 5$$

$$k_i = \frac{1}{s}\left[-\gamma_\rho^e\left[\frac{b}{a_o s^2+[as+bkp]s+bki}\right]u_c-\gamma_\rho\right] \quad 6$$

IV. METHODOLOGY

Transfer function for a mixing tank

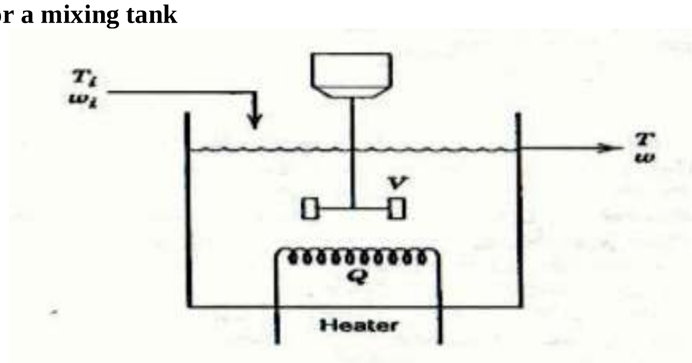

Figure 5: Stirred Tank Heating Process with Constant Holdup, V

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Let:
V = volume of the tank, 5 m^3
w = mass flow rate of the liquid, 500 kg/hr
ρ = density of the liquid = 1,000 kg/m^3
T_i = Initial temperature of the liquid, 20 °C
T = Final temperature of the liquid, 90 °C
Q = quantity of heat flow, 2 x 10^5 J

Assuming constant liquid holdup and flow rates, and the process is initially at steady state. The energy balance of the process gives:

$$V\rho C\frac{dT}{dt} = wC(T_i - T) + Q \quad 7$$

For steady state

$$T(0) = T_s\,,\ T_i(0) = T_{is},\ Q(0) = Q_s \quad 8$$

Subscript s indicates steady state.
At steady state Equation 7 becomes

$$0 = wC(T_i - T) \quad 9$$

Subtracting Equation 9 from Equation 7, we have

$$V\rho C\frac{dT - T_s}{dt} = wC(T_i - T_{is} - (T - T_{i)} + (Q - Q_s) \quad 10$$

Simplifying, and putting Equ.10 in deviation variable we get

$$V\rho C\frac{dT'}{dt} = wC(T_i' - T' + Q' \quad 11$$

Where $T' = T - T_s$ etc. 12

The Laplace Transform of Equation 12 is

$$V\rho C(sT'(s) - T'(t = 0)) = wC(T_i'(s) - T'(s)) - Q'(s) \quad 13$$

Thus, at t = 0, $T'(0)$ = T(0) - T_s 14

Re-arranging Equation 13 and dropping the prime, we obtain

$$\mathrm{T(s)} = \frac{\mathrm{K}}{\tau s+1}\mathrm{Q(s)} + \frac{1}{\tau s+1}\mathrm{T_i(s)} \quad 15$$

Where $K = \frac{1}{wC}$ and $\tau = \frac{V\rho}{w}$

Assume Q is constant, at its steady state value, then

$$\mathrm{Q(t)} = \mathrm{Q_s} \quad \therefore \quad \mathrm{Q'} = 0 \text{ and } \mathrm{Q'(s)} = 0 \quad 16$$

We substitute this condition in Equation 15 to get

$$\frac{\mathrm{T(s)}}{\mathrm{T_i(s)}} = \frac{1}{\tau s+1}$$

This implies that

$$\mathrm{y_1(s)} = \frac{\mathrm{T(s)}}{\mathrm{T_i(s)}} = \frac{1}{10s+1} \quad 17$$

Similarly, assume $\mathrm{T_i}$ is constant at steady state, then $\mathrm{T_i(t)} = \mathrm{T_{is}}$ 18

$$\mathrm{T_i} = \mathrm{T} = \mathrm{T'(t)} = 0 = \mathrm{T'(s)} \quad 19$$

From Equation 19, we get

$$\frac{\mathrm{T(s)}}{\mathrm{Q(s)}} = \frac{\mathrm{K}}{\tau s+1} \quad 20$$

Hence $$\mathrm{y\ (s)} = \frac{\mathrm{T\ (s)}}{\mathrm{Q(s)}} = \frac{\mathrm{K}}{10s+1} \quad 21$$

Or

$$\mathrm{y_2(s)} = \frac{4.7846 \times 10^{-4}}{10s+1} \quad 22$$

Combining the two transfer functions, $\mathrm{y_1(s)}$ and $\mathrm{y_2(s)}$ we have for the plant transfer function

$$\mathrm{y\ (s)} = \frac{1.00047846}{100s^2+20\,s+1} \cong \frac{1}{100s^2+20\,s+1} \quad 23$$

Which is a second order system.

Choice of a reference function was made by considering the fact that the plant model is a second order function, Equ.23.Also underdamped system with ζ< 1 gives a very good processcharacteristic (Ogunnaike et al, 2010). Our aim is to design a controller so that plant process tracks the reference model, $\mathrm{G_m(s)}$, which is a second order model given by

$$\mathrm{G_m(s)} = \frac{10}{5s^2+5s+10} = \frac{\mathrm{K}}{\tau^2 s^2+2\tau\zeta s+1} \quad 24$$

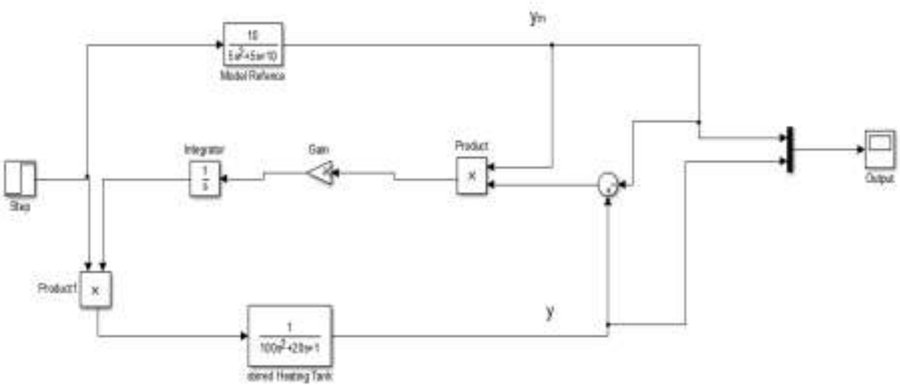

Figure 6: Block Diagram for Adaptive Control of A Stirred Mixing Tank using MIT Rule

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Equating the denominators and solving we get, $\tau = 2.24$ and $\zeta = 0.5$. The reference model is therefore suitable as a reference model. The block diagram is shown in Figure 6.Now let us assume that the process is linear with transfer functionKG(s), where K is anunknown parameter and G(s) is a second order known transfer function. The reference model transfer function is

$G_m(s) = K_0 G(s)$ where $K_{0,}$ is a known parameter. From Equation 1,

$$e(s) = KG(s)U(s) - K_0 G(s) \quad 25$$

Defining a control law as

$$U(t) = \theta^* U_c \quad 26$$

From Equation 25 -26, take a partial derivative to get

$$\frac{\partial e)s)}{\partial \theta} = KG(s)U_c(s) = \frac{K}{K_0}\gamma_m \quad 27$$

From Equation. 4 andEquation 27

$$\frac{d\theta}{dt} = -\gamma e \frac{K}{K_0}\gamma_m = -\gamma' e\gamma_m 28$$

Equation 28 gives the law for adjusting the parameter θ. Larger values of γ can cause instability so the selection of it is very critical.

V. RESULTS AND DISCUSSION

Table 1:Simulation Results –Comparison between the Responses of the System for Various Values of Adaptation Gain

Adaptation gain (γ)	Peak time (sec)	Overshoot	Tracking error
0.5	2.70	0.00	0.00
1.0	2.65	0.00	0.00
2.0	3.4	0.00	0.00
3.0	3.5	0.2	20
4.0	3.6	0.2	20
5.0	4.8	0.2	20

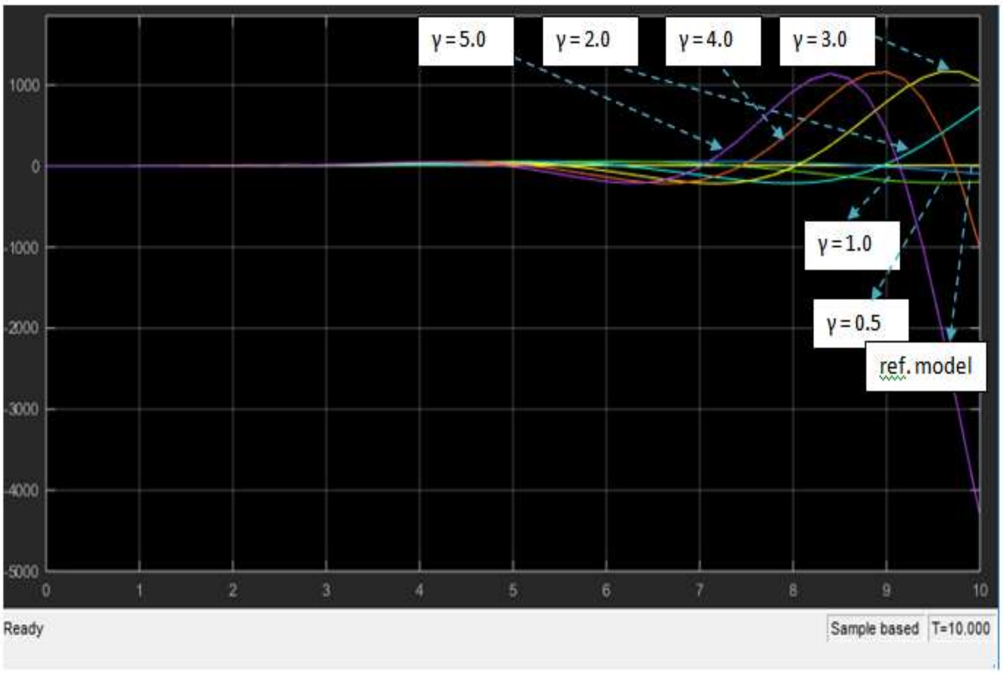

Figure 7: Combined Simulation Result of Marc with MIT Rule for the various Adaptation Gains

Table1 summarizes the dynamic behaviour of the system in terms of time domain parameters for various values of γ.Figure7 shows the response of actual plant and reference model for different values of adaptation gain γ. It is clear from Figure 7 that, for values of γ> 3.0, the system respondedslowly, but with larger errors (deviation of the model at different γ values) and high values of settling time, and for small values of γ (0.5, 1.00 and 2.00 system responded faster with small overshoot. In this project, the span of gain γ is chosen was from 0.5 to 5 for the given system. Beyond this span the system performance was not satisfactory, the system exhibited instability, the system continued without an actual peak value. Therefore, it can be seen in this work that for suitable values of adaptation gain, the MIT rule was able to make the plant follow the model as accurately as possible.Changing the adaptive gain produced different characteristics of the plant in times of peak time, overshoot and settling time. From the simulation results presented in Table1, and graph of Figure7, the optimum parameters of the system were to be: adaptive gain (γ) 1.0, peak time 2.65 sec, with zero overshoot.In this work, the MRAC approach was applied to a second order system with MIT rule, and thesimulation results are shown above. The MIT rule helped to design the adaptive controller, so that the plant process derived in terms of transfer function would track the reference model.

VI. CONCLUSION

Process dynamics are usually characterized with nonlinear actuators, variation in the conditions of the prevailing environment and other characteristic disturbances. As such, feedback controllers may not perform optimally when used online. In view of these challenges, a simulated work on MRAC scheme using MIT rule was carried out in this work and the performance evaluation was done using simulations on MATLAB 18.0 SIMULINK. The results of MIT scheme for different values of adaptation gain were compared. It was observed that the response of the system improves with the increment in adaptation gain but beyond a certain limit (0.5> γ >2.0), the performance of the system becomes very poor.Therefore, for suitable values of adaptation gain, the MIT rule with normalization can make the plant to follow the model as accurately as possible, with an optimum adaptive gain (γ) of 1.0 of peak time of 2.65 sec and zero overshoot.

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