

Dynamic Analysis of Induction Machine in Various Reference Frame under Balanced and Unbalanced Voltage Conditions

*Jonah Sokipriala Solomon^{#1} and Nwaorgu.G.O^{#2}

1. Electrical Electronic Engineering Department, Rivers State University, Port Harcourt, Rivers State

2. Marine Engineering Department, Nigeria Maritime University, Okerenkoko, Delta State.

Submitted: 15-07-2021

Revised: 29-07-2021

Accepted: 31-07-2021

ABSTRACT

This paper analyses the dynamic behaviour of a three (3) phase induction machine in the various reference frames. It also compares torque behaviour, speed characteristics under balanced and unbalanced load conditions to determine which reference frame should be used during simulation under various conditions of voltage variations. The dynamic simulation of the various conditions is carried out in Matlab Simulink, using parks dq-axis transformation to simplify the analysis. In the stationary, Rotor and synchronous reference frame, the change in flux linkage from the various voltage equations is used in simulation of the dynamic model of the machine using an embedded matlab equation. To study the behaviour to predict which frames is most suitable when there is variation of in the symmetrical nature of the induction motor as a result of unbalance in volage magnitude and amplitude.

Keywords: induction motor, reference frame, balance and unbalanced voltage conditions, simulation

I. INTRODUCTION

Over the years various analysis have been carried out to study the behaviour of induction machine under balanced voltage condition. However certain factors such as line faults from supply might lead to unbalance in the voltage condition. Also analysis of machine for simplicity is carried using dq transformation [1]. The dq transformation reduces the complexity in the analysis of the machine behaviour of the machine [2].

The dynamic equation can be carried out using the stationary rotor or synchronous reference frame depending on the conditions of the voltage and other parameters[3]. There are various software packages used for the dynamic simulation of induction machine [4]. The reliability of the

performance of an induction motor is of major importance to design engineers. Hence a detailed study of the dynamic behaviour is always carried out. Matlab Simulink shown in appendix A is used for this work as and it is suitable for analysis of electrical drives [5]. An embedded Matlab program is simulated to study the behaviour of the machine from the various mathematical models in the various reference frames.

II. MATHEMATICAL MODEL OF THE INDUCTION MOTOR

The induction machine is modelled for simplicity using Parks transformation transform the voltage from the abc reference into the d-q reference frame. It can easily be analysed using conventional Proportional integral Controllers (PID) [6]

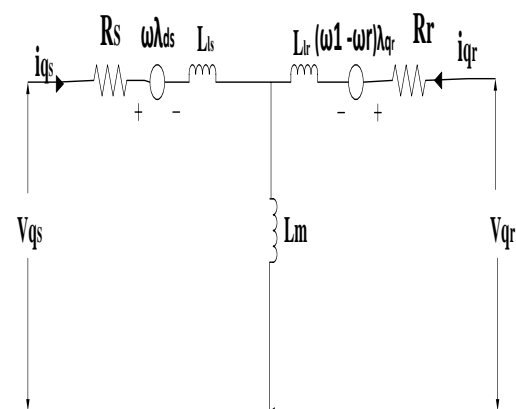


Figure (1) q-axis circuit representation

q-axis Voltage equation

$$V_{qs} = R_s i_{qs} + \omega \lambda_{ds} + P \lambda_{qs} \quad (1)$$

$$V_{qr} = R_r i_{qr} + (\omega - \omega_r) \lambda_{qs} + P \lambda_{qr} \quad (2)$$

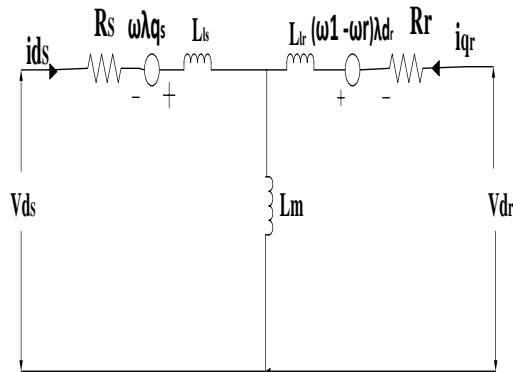


Figure (2) q-axis circuit representation

$$V_{ds} = R_s i_{ds} - \omega \lambda_{qs} + P \lambda_{ds} \quad (3)$$

$$V_{dr} = R_r i_{dr} + (\omega - \omega_r) \lambda_{qr} + P \lambda_{dr} \quad (4)$$

Flux linkage equation for the stator

$$\lambda_{qs} = i_{qs} L_{ts} + L_m (i_{qs} + i_{qr}) \quad (5)$$

$$\lambda_{ds} = i_{ds} L_{ts} + L_m (i_{ds} + i_{dr}) \quad (6)$$

Flux linkages for the Rotor

$$\lambda_{qr} = i_{qs} L_{tr} + L_m (i_{qr} + i_{qs}) \quad (7)$$

$$\lambda_{dr} = i_{dr} L_{tr} + L_m (i_{dr} + i_{qs}) \quad (8)$$

The flux linkage equation can be expressed in matrix form as

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{qr} \\ \lambda_{dr} \end{bmatrix} = \begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} \quad (9)$$

$$L_s = L_{ts} + L_m \quad (10)$$

$$L_r = L_{tr} + L_m \quad (11)$$

Torque equation

The torque equation is derived from the power input equation, summation of all terms neglecting terms independent of the speed and dividing by the rotor speed results in

$$T_e = \frac{3P}{4} (\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}) \quad (12)$$

Speed equation

$$\omega_r = \frac{P}{J} \int (T_e - T_L) dt \quad (13)$$

Balanced Voltage equation in the ABC reference frame

$$V_a = V_m \cos(\omega t) \quad (14)$$

$$V_b = V_m \cos(\omega t + 2\pi f) \quad (15)$$

$$V_c = V_m \cos(\omega t - 2\pi f) \quad (16)$$

Unbalanced Voltage condition

$$V_a = V_m \cos(\omega t) \quad (17)$$

$$V_b = 1.5 V_m \cos(\omega t + 2\pi f) \quad (18)$$

$$V_c = V_m \cos(\omega t - 2\pi f) \quad (19)$$

Parks transformation equation

$$k_{si} = \frac{2}{3} \begin{bmatrix} \cos \theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin \theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad (20)$$

III. ANALYSIS OF VOLTAGE EQUATIONS IN THE D IN THE VARIOUS REFERENCE FRAMES

The d-q axis rotation speed will now be substituted in the various reference frames. The angle of rotation substituted in parks transformation to give the equivalent dq-transformation in that reference frame.

Stationary Reference Frame

In the stationary reference frame, the d-q axis is static [7] as the speed is that of the stator hence $\omega = 0$, and $\theta = 0$ substituting $\omega = 0$ in eqn (1) to (4) we get eqn (21) to eqn(24) and θ is substituted in eqn (20)

q-axis Voltage equation

$$V_{qs} = R_s i_{qs} + P \lambda_{qs} \quad (21)$$

$$V_{qr} = R_r i_{qr} + \omega_r \lambda_{qs} + P \lambda_{qr} \quad (22)$$

d-axis Voltage equation

$$V_{ds} = R_s i_{ds} + P \lambda_{ds} \quad (23)$$

$$V_{dr} = R_r i_{dr} - \omega_r \lambda_{qr} + P \lambda_{dr} \quad (24)$$

The Rotor Reference Frame

In this reference frame the dq-axis is rotating with rotor speed $\omega = \omega_r$, substituting $\omega = \omega_r$ and $\theta = \theta_r$ in eqn (20) the transformation equation. Eqn(1) eqn (4) gives eqn (25) to eqn (28); $\theta_r = \int (\omega_r) dt$

$$\theta_r = \int (\omega_r) dt \quad (25)$$

q-axis Voltage equation

$$V_{qs} = R_s i_{qs} + \omega_r \lambda_{ds} + P \lambda_{qs} \quad (26)$$

$$V_{qr} = R_r i_{qr} + P \lambda_{qr} \quad (27)$$

d-axis Voltage equation

$$V_{ds} = R_s i_{ds} - \omega_r \lambda_{qs} + P \lambda_{ds} \quad (28)$$

$$V_{dr} = R_r i_{dr} + P \lambda_{dr} \quad (29)$$

Synchronous Reference Frame

The synchronous reference frame occurs when the d-q axis is rotating at synchronous speed and $\omega = \omega_s$ and $\theta = \theta_s$ substituting ω_s into eqn (1) to eqn(4) results in eqn(30) to eqn(33)

q-axis Voltage equation

$$V_{qs} = R_s i_{qs} + \omega_s \lambda_{ds} + P \lambda_{qs} \quad (30)$$

$$V_{qr} = R_r i_{qr} + (\omega_s - \omega_r) \lambda_{qs} + P \lambda_{qr} \quad (31)$$

d-axis Voltage equation

$$V_{ds} = R_s i_{ds} - \omega_s \lambda_{qs} + P \lambda_{ds} \quad (32)$$

$$V_{dr} = R_r i_{dr} + (\omega_s - \omega_r) \lambda_{qr} + P \lambda_{dr} \quad (33)$$

IV. SIMULATION RESULTS AND ANALYSIS

Table 1.1 Induction motor Parameters

Parameter	Value
Rr	26.37mΩ
Rs	14.14mΩ
Lm	6.94mH
Ls	36.76mH
Llr	1.74mH
Je	0.54kgm ²
V	155 Volts
P	2
TL	30Nm
F	50Hz

The simulated results include the main abc currents, and the d-q axis currents in the stator and rotor reference frame, the speed-torque characteristics under and during balance and unbalanced voltage conditions in the various reference frames as can be seen in the various diagrams

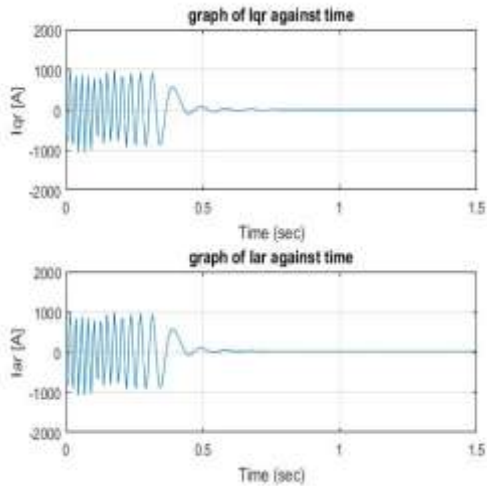


Figure 1.0 graph of iar and iqr in the rotor Reference frame

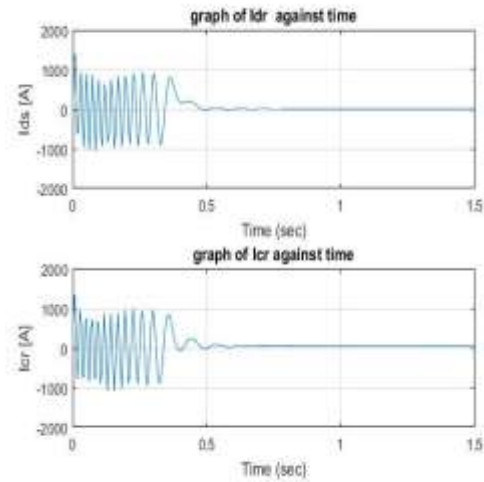


Figure 2 graph of icr and idr in the rotor Reference frame.

In the rotor reference frame the speed $\omega = \omega_r$, and the direct axis is moving in the same speed as the phase c current. The quadrature axis is moving in the same speed as that of the a-axis. Hence the direct axis current as shown in figure (2) coincides with the c axis current. The q-axis current coincides with that of the a axis as shown in figure (1)

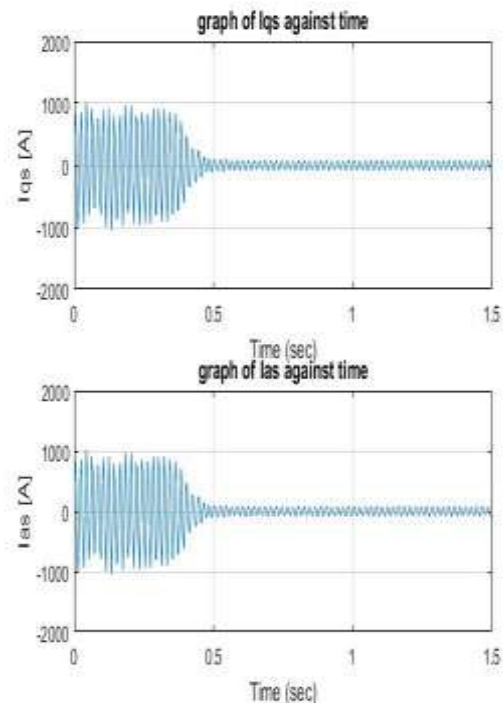


Figure 3 graph of iqs and ias in the rotor Reference frame.

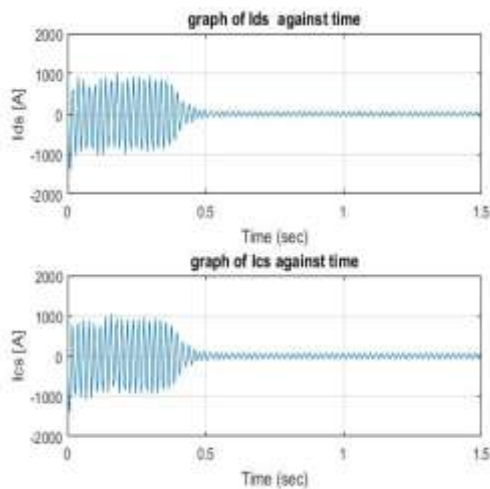


Figure 4 graph of ics and ids in the stator Reference frame.

In this reference frame the $\omega = 0$ hence the direct axis is fixed in the stator c axis. The quadrature axis is fixed at the stator a axis. It implies that the current in the stator a axis will have the same wave form as that in the q axis of the d-q transformation. Direct axis has the c axis in the abc reference frame of the stator as shown in figure (3) and (4), respectively.

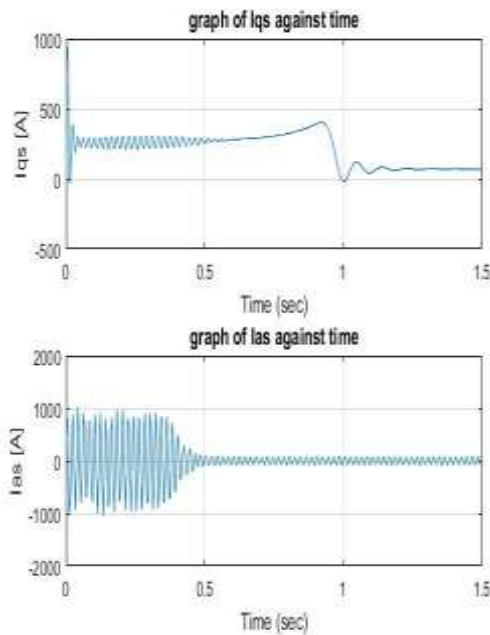


Figure 5 graph of iqs and ias in the Synchronous Reference frame.

At the synchronous reference frame, this frame is rotating at synchronous speed. The man

stator is rotating at the machine rated frequency, while the rotor is rotating at its equivalent slip frequency. The dq-axis current in this reference those not follow the same wave form in any of the abc axis as shown in figure (5)

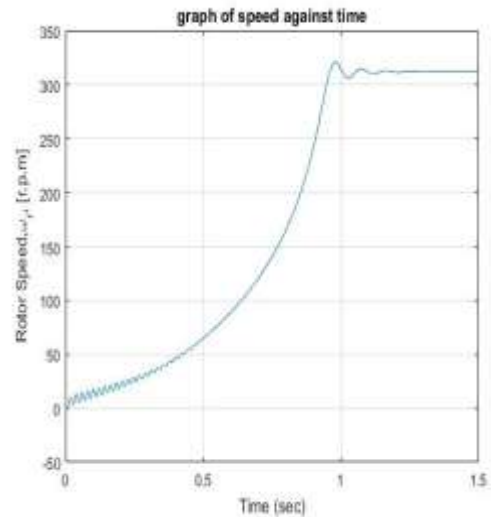


Figure 6 graph of speed against time.

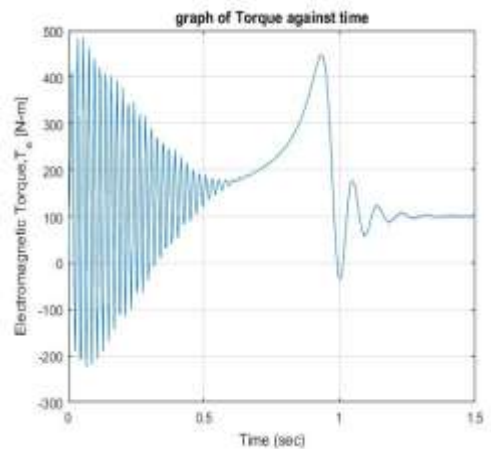


Figure 7 graph of Torque against time.

The torque and speed equation from eqn (12) and eqn(13) is independent of the speed. In the reference frames, hence the graph of speed against torque in the stationary, Rotor and Synchronous reference frame remains constant in all reference frames as can be seen in figure(6-8),

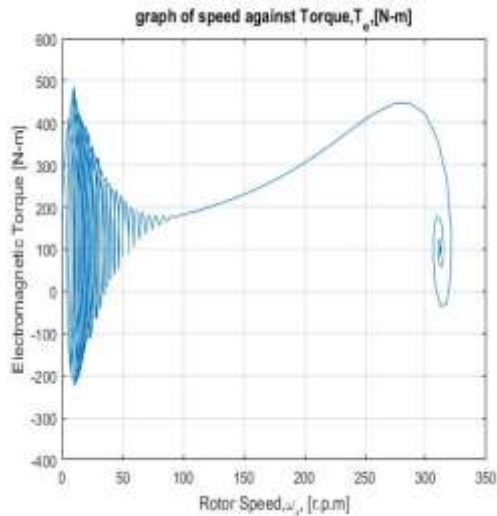


Figure (7) graph of Torque against time speed.

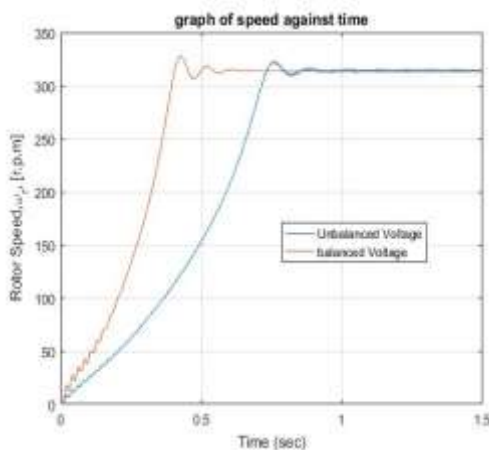


Figure (8) graph of speed against time speed.

Under balanced and unbalanced voltage

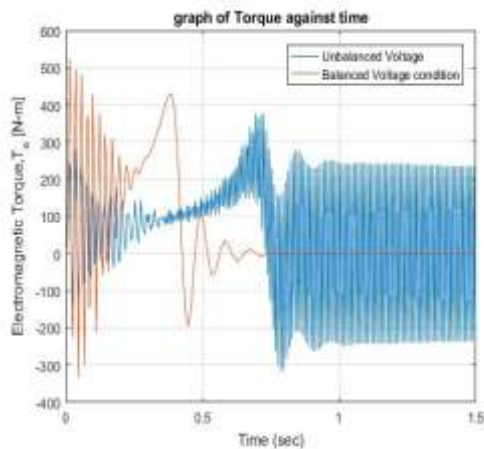


Figure (9) graph of torque against time speed.

Under balanced and unbalanced voltage

The speed torque equation in the various reference frames shows that under varying

magnitude and phase of the supply voltage. The Rotor reference frames maintained balanced conditions under varying rotor voltage. The stationary reference frame maintained a constant torque-speed characteristics and variation of voltage from any source affected the synchronous torque speed characteristics.

V. CONCLUSION

This work Analyses the dynamic performance of a three-phase induction machine. The stationary, rotor and synchronous reference frame using dq-transformation is also analysing the dynamic behaviour of the torque speed characteristics under balanced and unbalanced voltage conditions.

One of the contributions of this work is that it introduces a new pattern to the modelling of the induction motor using the flux linkage. The independent variable of voltage transformation equation will make the simulation of the motor behaviour easier in an embedded matlab simulation as shown in appendix (A).

It also helps in informing the suitability of reference frame to be used under unbalanced load conditions. This research shows that the synchronously rotating reference frame is most suitable to use under balanced voltage condition. The rotor reference frame is suitable under unbalanced stator voltage and the stationary reference frame under unbalanced rotor voltage. The dynamic simulation also shows that the torque speed characteristics remains constant in all reference frames under balanced voltage condition.

REFERENCE

- [1]. Paul C. Krause, O. Wasynczuk and S. D. Sudhoff. 2004. Analysis of Electric Machinery and Drive Systems. IEEE Press Series on power Engineering, John Wiley and Sons Inc. Publication.
- [2]. Okoro, O.I. 2004. "Generalised Program for the Dynamic Simulation of Symmetrical Induction Machine". Nigerian Journal of Tropical Engineering. 5(1 & 2):16 – 24.
- [3]. Faiz, J. 2004. "Influence of Unbalance Voltage on the Steady-State Performance of a Three-Phase Squirrel – Cage Induction Motor". IEEE Trans. Energy Conversion. 19(4):657-662.
- [4]. Dennis L. Feucht "Magnetic Reference-Frames" 2001
- [5]. T.K.A.Brekken, N. Mohan, "Control of a doubly fed Induction wind generator under unbalanced grid voltage conditions", IEEE

- trans. on Energy Conversion, vol.22, pp. 129 – 135, 2007
- [6]. Ching-Yui Lee: Effects of unbalanced voltage on the operation performance of a three-phase induction Motor, IEEE Trans on EC, Vol.14, No.2, pp.202-208
- [7]. C.M. Ong. 1998. Dynamic simulation of Electric Machinery. Prantice Hall PTR, Upper Saddle River, N. J. Publication.

APPENDIX A

```

SYNCHRONOUS function
[ppsiqs,ppsids,ias,ibs,ics,idr,iqr,ids,iqs,Te,pwr,ppsiqr,ppsidr,Me] =
fcn(psiqs,psids,wt,TL,wr,psiqr,psidr)
% This block is simulating an IM in the Synchronous ref frame.
%Parameters
P=2; f=50; Hp = 2250; vm= 155; rs=0.02637;
wb=2*pi*f;Lls=3.676e-4;Llr=1.174e-4;
xls=wb*Lls;xlr=wb*Llr;Lm=6.9397e-3;
xm=wb*Lm;rr=0.01414; Je=0.54; tt=wt;
vas=vm*cos(wt);
vbs=vm*cos(wt-2*pi/3);
vcs=vm*cos(wt+2*pi/3);
% abc to Q-D voltages
vqs = (2/3)*vas - (vcs+vbs)/3; vds=(vcs-
vbs)/sqrt(3);
vos = (vas+vbs+vcs)/3; vqr = 0; vdr = 0;
L1=Lls+Lm;
L2=Llr+Lm;
a1=(Lm*Lm)/(L1*L2);
a2=1-a1;
a3=(1-a2)/(a2*Lm);
b1=L2/Lm;
b2=L1/Lm;
K1=(1)/(L1*L2-Lm*Lm);
% Derivatives of the flux linkage per second are
obtained here
Vq=vm;
Vd=0;
ids=((psids*b1)-psidr)*a3;
iqs=((b1*psiqs)-psidr)*a3;
idr=(-psids+(b2*psidr))*a3;
iqr=(-psiqs+(b2*psidr))*a3;
Te=(1.5*P)*(psids*iqs-psiqs*ids);
ppsids=Vd-rs*ids+wb*psiqs;
ppsiqs=Vq-rs*iqs-wb*psids;
ppsidr=-rr*idr+(wb-wr)*psidr;
ppsiqr=-rr*iqr-(wb-wr)*psidr;
    
```

```

% Electromagnetic Torque and Rotor Speed
pwr = (P/(Je))*(Te-TL);
Me=(wr*60)/(P*2.*pi);
% ***Phase currents*****
ias=iqs;
ibs=-0.5*iqs-0.5*sqrt(3)*ids;
ics=-0.5*iqs+0.5*sqrt(3)*ids;
    
```

