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Fatigue Analysis of Subsea Wellhead Systems under Dynamic Loading Conditions

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ABSTRACT:This study investigates the fatigue behavior of subsea wellhead systems under dynamic loading by integrating analytical models with MATLAB-based simulations. The methodology combines a suite of governing equations—from the elastic stress–strain relationship and modified Remberg–Osgood formulations for cyclic loading, to Neuber's rule for stress concentration, fatigue life predictions using Basquin's and Manson–Coffin models, and crack propagation analysis via Paris' law. Simulation results reveal that under cyclic loading, the material exhibits a steeper strain response and earlier onset of plasticity compared to monotonic loading. The S–N curve shows that while stress amplitudes exceed 200 MPa at lower cycle counts, they reduce to below 50 MPa by 10^8 cycles, highlighting the trade-off between stress intensity and fatigue life. Combined stress analyses pinpoint critical stress concentrations near the inner radius of the wellhead, and damage accumulation based on the Palmgren–Miner rule indicates a nonlinear increase in damage with rising stress amplitudes. Moreover, rainflow cycle histograms reveal a predominance of low-amplitude cycles with sporadic high-stress events, and crack growth predictions illustrate an extension from 1.0 mm to 1.8 mm under sustained cyclic loads. These comprehensive findings provide essential design insights and proactive maintenance strategies for enhancing the reliability and longevity of offshore subsea wellhead systems.

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KEYWORDS: Subsea Wellhead, Dynamic loading, Fatigue analysis, Cyclic loading, crack propagation, damage accumulation, stress-strain relationship, MATLAB simulation

I. INTRODUCTION

Subsea wellhead systems are critical components in offshore oil and gas production, serving as vital junctions that maintain structural integrity under severe and fluctuating load conditions. These structures face high internal pressures, variable external pressures, cyclic dynamic loads, and wave-induced forces that can lead to fatigue-induced damage and, if unchecked, catastrophic failure. Ensuring their durability is essential for safe and efficient offshore operations.

The understanding of fatigue behavior in such systems has deep historical roots. Early foundational studies by Basquin introduced the S–N curve concept linking stress amplitude to the number of cycles to failure in high-cycle fatigue[1]. Coffin andManson developed strain–life relationships that remain critical for low-cycle fatigue analysis, while Stephens et al. provided a comprehensive framework on metal fatigue phenomena [2],[4],[5]. In offshore engineering, Hopper addressed specific challenges related to subsea wellheads, and industry standards such as those from DNVGL andNORSOK have underscored the necessity of integrated analytical approaches [6],[7],[8].

Recent literature has built upon these classical theories by incorporating modern simulation techniques and refined modeling strategies. For example, Chen et al. (2019) presented advanced numerical methods that merge traditional fatigue theories with contemporary computational tools for offshore applications. Wang et al. investigated dynamic loading effects on subsea structures, offering detailed insights into stress distributions and damage accumulation under realistic conditions[9]. More recently, Kim et al. [10] developed innovative fatigue modeling

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techniques that account for both elastic and plastic deformation behaviors, and Singh et al. [11] demonstrated the benefits of integrating machine learning with conventional fatigue models to predict crack propagation more accurately.

Building on both historical foundations and these recent advancements, our study employs a comprehensive analytical–numerical framework to assess the fatigue behavior of subsea wellhead systems under dynamic loading conditions. The methodology begins with the basic elastic stress–strain relationship and extends to encompass nonlinear and cyclic loading effects via modified Remberg–Osgood relations. Local stress concentrations are corrected using Neuber's rule, further refined by fatigue-specific stress concentration factors. Fatigue life is predicted by combining the Manson–Coffin strain–life equation, Basquin's S–N curve, and low-cycle fatigue models. Cumulative damage is quantified using the Palmgren–Miner rule, while the combined effects of internal and external pressures are modeled with both the thin-wall approximation and Lamé's equation for thick-walled cylinders. Finally, Paris' law is applied to forecast crack growth under cyclic loading.

This integrated approach not only reflects the evolution of fatigue modeling from classical theories to state-of-the-art computational methods but also provides a practical framework for designing and maintaining robust subsea wellhead systems. The insights obtained from our MATLAB simulations offer valuable design guidance and proactive maintenance strategies to ensure that these offshore structures can withstand the challenging dynamic loading conditions encountered in real-world operations.

II. METHODOLOGY

We build our fatigue analysis framework by starting with the basic elastic behavior and progressing through increasingly complex models that capture cyclic effects, local stress corrections, and crack propagation.

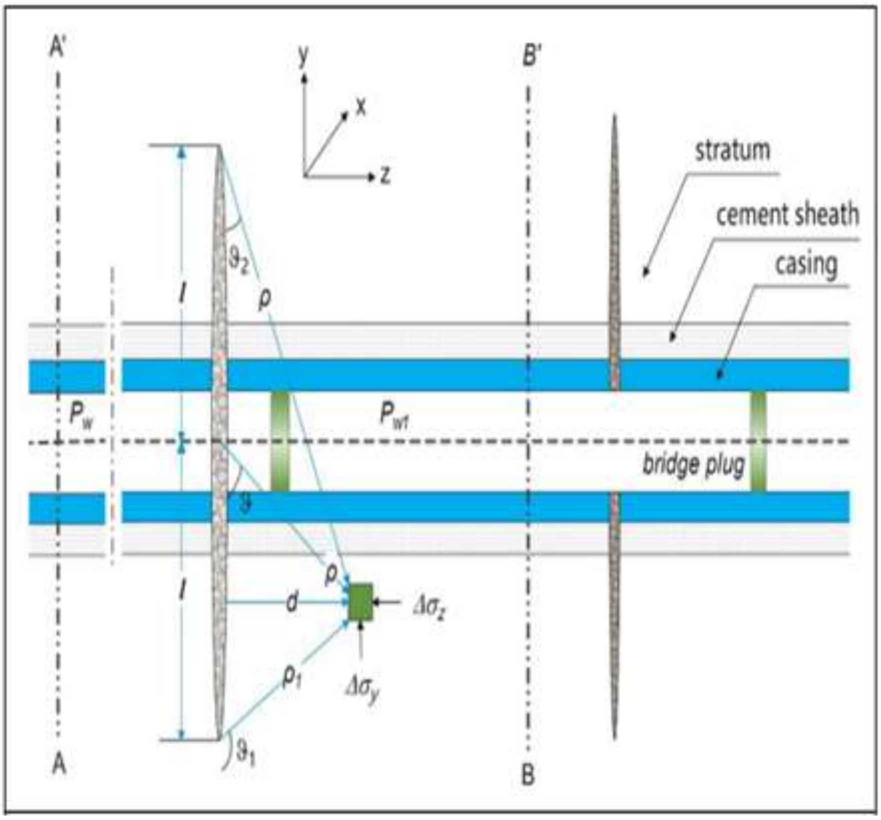

Figure 1: cross-sectional view of a wellbore [9]

Considering the wellbore representation of a cross- section of a wellhead as shown in figure 1, and initializing the methodologies, we consider the following formulations.

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The initial linear response of the material is described by the elastic stress–strain relationship in equation (1)

$$\sigma = E\,\epsilon \tag{1}$$

where σ is the stress, E is Young's modulus, and ϵ is the strain.

Beyond the elastic limit, the Remberg–Osgood relation captures the nonlinear behavior during monotonic loading, this is estimated using equation (2)

$$\epsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^n \tag{2}$$

Here, K is the strength coefficient and n is the strain hardening exponent. For cyclic loading conditions, the Remberg–Osgood equation is modified to include cyclic effects:

$$\epsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K_c}\right)^{n_c} \tag{3}$$

where K_c and n_c denote the cyclic strength coefficient and cyclic hardening exponent, respectively.

To account for stress concentrations at geometric discontinuities, Neuber's rule is applied:

$$\sigma_{\text{local}}\,\epsilon_{\text{local}} = K_t^2\,\sigma_{\text{nominal}}\,\epsilon_{\text{nominal}} \tag{4}$$

K_t is the stress concentration factor. The stress concentration factor is defined as in equation (5)

$$K_t = \frac{\sigma_{\max}}{\sigma_{\text{nominal}}} \tag{5}$$

which quantifies the peak stress relative to the nominal value.

To reflect the effect of repeated loading on stress amplifications, the fatigue stress concentration factor, K_f, is used. Although it is not expressed as a unique formula here, it is computed by adjusting K_t based on material fatigue properties.

$$K_f = \frac{\sigma_{\text{fatigue}}}{\sigma_{\text{nom}}} \tag{6}$$

where K_f adjusts K_t for material fatigue properties.

This equation relates the strain amplitude to the fatigue life, accounting for both elastic and plastic strain components:

$$\frac{\Delta\epsilon}{2} = \epsilon_f^{'}\left(2N_f\right)^c + \frac{\sigma_f^{'}}{E}\left(2N_f\right)^b \tag{7}$$

where $\Delta\epsilon/2$ is the strain amplitude, N_f is the number of cycles to failure, $\epsilon_f^{'}$ and $\sigma_f^{'}$ are the fatigue ductility and strength coefficients, and c and b are material exponents.

For high-cycle fatigue, Basquin's equation relates the stress amplitude to fatigue life; see equation (8).

$$\sigma_a = \sigma_f^{'}\left(2N_f\right)^b \tag{8}$$

In the low-cycle regime where plastic deformation dominates, the plastic strain component is given by:

$$\Delta\epsilon_p = \epsilon_f^{'}\left(2N_f\right)^c \tag{9}$$

which is effectively the second term of the Manson–Coffin equation.

The cumulative damage due to repeated load cycles is assessed using:

$$D = \sum \frac{n_i}{N_i} \tag{10}$$

where n_i is the number of cycles at a particular stress level and N_i is the number of cycles to failure at that level.

The internal pressure produces a hoop stress in the wellhead, estimated by:

$$\sigma_h = \frac{p\,r}{t} \tag{11}$$

where p is the internal pressure, r is the radius, and t is the wall thickness.

Similarly, external pressure induces stress on the wellhead, which can be expressed in equation(12)

$$\sigma_e = \frac{p_e\,r}{t} \tag{12}$$

where p_e is the external pressure.

The net stress experienced by the wellhead is obtained by combining the internal and external effects.

$$\sigma_{\text{net}} = \sigma_h - \sigma_e \tag{13}$$

For a more detailed stress distribution in thick-walled cylinders, Lamé's equation is used:

$$\sigma_\theta(r) = \frac{p_i\,\mathrm{r_i^2} - \mathrm{p_o\,r_o^2}}{\mathrm{r_o^2 - r_i^2}} + \frac{(\mathrm{p_i - p_o})\,\mathrm{r_i^2\,r_o^2}}{(\mathrm{r_o^2 - r_i^2})\,\mathrm{r}^2} \tag{14}$$

where $\mathrm{p_i}$ and $\mathrm{p_o}$ are the internal and external pressures, and $\mathrm{r_i}$ and $\mathrm{r_o}$ are the inner and outer radii.

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The rate of crack propagation under cyclic loading is predicted by Paris' law in equation (15)

$$\frac{da}{dN} = C\,(\Delta K)^m \quad (15)$$

where $\frac{da}{dN}$ is the crack growth rate per cycle, C and m are material constants, and ΔK is the stress intensity factor range.

III. RESULTS AND DISCUSSIONS

In this study, the fatigue behavior of a subsea wellhead system is analyzed using well-defined material, geometric, and loading parameters as adapted from OD Damilola[3]

Table 1: Wellhead Parameter [3]

Parameter	Value	Units
Young's Modulus	210×10^9	Pa
Strength Coefficient (Monotonic)	800×10^6	Pa
Strain-hardening exponent (Monotonic)	0.15	–
Cyclic Strength Coefficient	600×10^6	Pa
Cyclic Strain-hardening Exponent	0.12	–
Inner Radius	0.2	m
Outer Radius	0.3	m
Wall Thickness	0.01	m
Internal Pressure	10×10^6	Pa
External Pressure	5×10^6	Pa
Fatigue Strength Coefficient	600×10^6	Pa
Fatigue Strength Exponent	-0.12	–
Fatigue Ductility Coefficient	0.02	–
Fatigue Ductility Exponent	-0.6	–
Material Constant	1×10^{-12}	–
Paris Law Exponent	3	–
Seawater Density	1025	kg/m^3
Gravitational Acceleration	9.81	m/s^2
Wave Height	2	m
Drag Coefficient	0.5	–

Figure 2 shows the Stress–Strain relationship for Monotonic and cyclicloading.

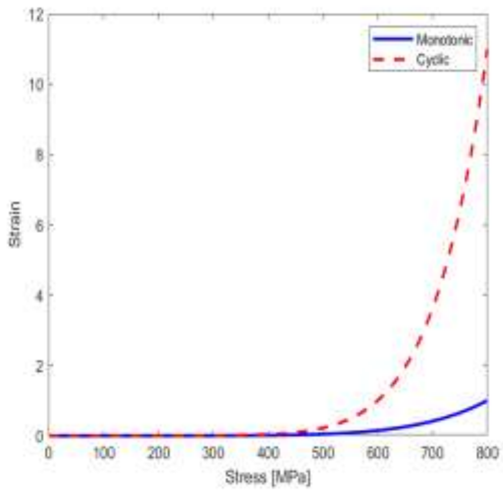

Figure 2: Stress–Strain Curves: Monotonic vs. Cyclic Loading

The Monotonic load initially follows a linear elastic path, then transitions to a mild nonlinear region at higher stress levels. The Cyclic exhibits a steeper strain response for the same stress level in the plastic region, indicating different hardening/softening behavior under

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repeated loading. Under monotonic loading, the material follows the typical elastic–plastic transition.

Under cyclic loading, the material can exhibit cyclic softening or hardening, depending on its microstructural response. Here, the curve suggests a faster rise in strain with increasing stress once plasticity sets in. For wellhead design, accounting for cyclic material properties is crucial, since real offshore operations often involve repeated loading rather than a single monotonic load.

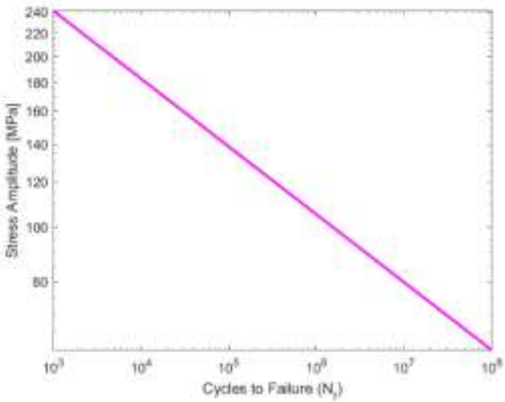

Figure 3: S-N curve based on the Basquin's law

Figure 3 depicts the S-N curve based on the Basquin's law. The S–N curve on a log–log scale shows stress amplitude decreasing with an increasing number of cycles to failure. At 10^3 cycles, the stress amplitude is relatively high (over 200 MPa), while by 10^8 cycles, the stress amplitude is below 50 MPa. The curve highlights the trade-off between stress amplitude and fatigue life. Higher stress amplitudes quickly reduce the number of cycles the material can sustain. For subsea wellheads, which may experience millions of cycles over their service life, designing for lower stress amplitudes is essential to extend fatigue life.

Figure 3 shows the combined stress distribution in the wellhead wall.

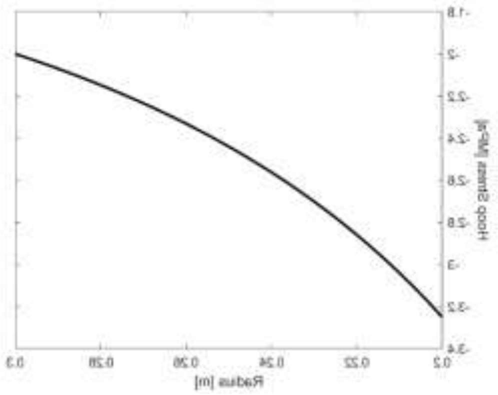

Figure 3 : Combined Stress

The hoop stress distribution (negative values here may indicate a net compressive stress or a reference shift) decreases in magnitude as radius increases from 0.2 m to 0.3 m. The highest magnitude of hoop stress is near the inner radius, aligning with typical thick-walled cylinder theory. Stress concentrations near the inner radius can lead to localized fatigue damage. Proper thickness selection and material grading are important to handle these stress gradients. Verifying that the peak hoop stress stays within safe limits under operational pressures is essential to prevent crack initiation from the inner wall.

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The damage accumulation can be seen in figure 4.

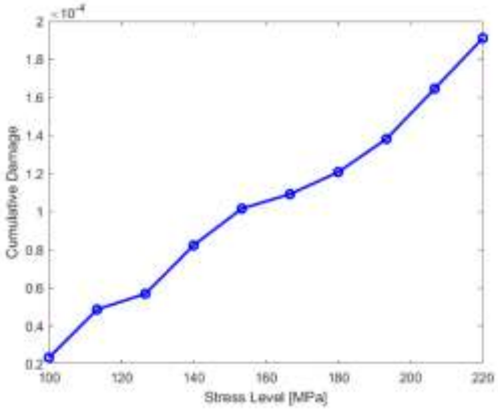

Figure 4:The Palmgren–Miner Damage Accumulation

The cumulative damage rises from around 2×10^{-5} at 100 MPa up to about 14×10^{-5} at 220 MPa. The damage increments become larger at higher stress levels, indicating a nonlinear relationship between stress amplitude and damage accumulation. Since total damage remains well below unity (D < 1), no immediate fatigue failure is predicted under the simulated conditions. The trend underscores that high stress amplitudes accelerate fatigue damage, even if they occur relatively infrequently. In real operations, if these high-amplitude events become more frequent (e.g., storms, sudden pressure surges), the cumulative damage could approach unity more quickly.

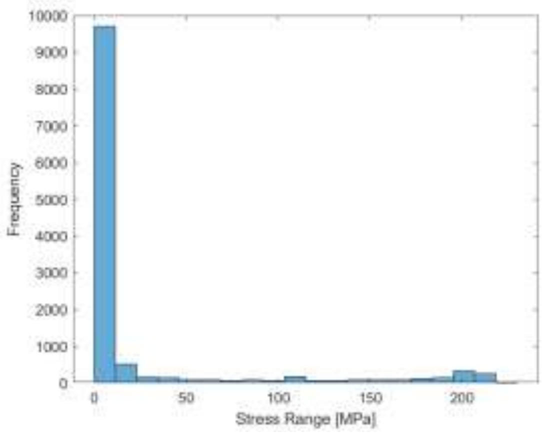

Figure 5: Rainflow Histogram of Synthetic Stress History

Figure 5 shows the stress history. The histogram is heavily skewed toward low stress ranges (0–10 MPa), with very few cycles exceeding 100 MPa. The largest bin, near 0 MPa, contains several thousand cycles, while only a small number of cycles exceed 200 MPa. The wellhead experiences predominantly low-amplitude cycles, which contributes to relatively slow fatigue damage accumulation. A small fraction of high-amplitude cycles can still drive crack initiation if stress concentrations exist.

Monitoring the frequency and magnitude of these large cycles is crucial; occasional spikes can have a disproportionate effect on fatigue life.

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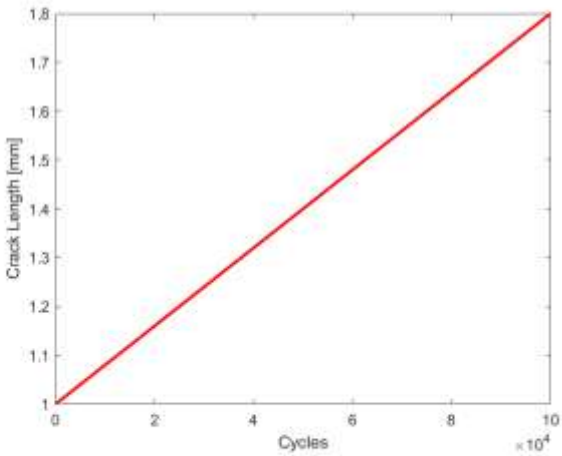

Figure 6 ; Crack propagation along the wellhead

Figure 6 depicts the crack Growth Prediction via Paris' Law.The crack length grows linearly from 1.0 mm to around 1.8 mm over 10^4 cycles at a constant stress intensity range (ΔK). The slope indicates a significant crack extension if these conditions persist. Even a moderate ΔK (here 20 MPa$\sqrt{m}$) can lead to a notable crack growth rate. Subsea wellheads with pre-existing flaws or initiated cracks could see rapid propagation once the crack size and stress intensity surpass critical thresholds. Regular in-service inspection (e.g., nondestructive testing) is recommended to detect crack growth early, especially after known high-stress events.

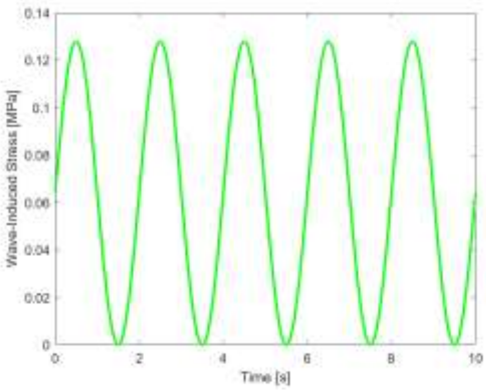

Figure 7 : Wave stress history

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Figure 7 shows Wave-Induced Load Effect on Wellhead Stress . The stress fluctuates sinusoidally between about 0.0 MPa and 0.14 MPa over a period of roughly 2 seconds (0.5 Hz). Although the amplitude is relatively small compared to internal pressure stresses, it is a repeated dynamic load. Repeated wave-induced stresses can accumulate fatigue damage, especially if the structure operates for extended durations in rough seas. Even small cyclic stresses can be critical over millions of cycles. Mitigation can involve optimizing riser design, wellhead support, or scheduling drilling/completion in calmer weather windows.

IV. CONCLUSION

The above results illustrate how a subsea wellhead system responds to various fatigue-related loads. The difference between monotonic and cyclic stress–strain curves underscores the importance of using cyclic material properties in design. The S–N curve confirms that higher stress amplitudes drastically reduce the number of cycles to failure. Hoop stress is greatest near the inner radius, a typical outcome in thick-walled cylinders, making it a critical zone for inspection. Although the simulated Palmgren–Miner damage values remain below unity, localized high-amplitude cycles could rapidly increase damage if they become more frequent. Under repeated loading, even moderate stress intensity factors can lead to significant crack extension over thousands of cycles. Although smaller than internal pressure loads, cyclic wave loads add another layer of fatigue stress that must be accounted for over time.

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