

# Flow Analysis and Its Prediction Methods for Compound Channels

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**ABSTRACT:** Compound channels are a common configuration of rivers. Dependable estimation of discharge capacity in a compound channel helps specialists to obtain reliable information regarding flood mitigation, construction of hydraulic structures, prediction of sediment loads and capacity of reservoir basin etc. which in turn assess the flow variation during catastrophic situations so as to minimize the loss. Many researchers have studied the traditional methods for estimating the discharge capacity of a compound channel based on the standard uniform flow formulas such as Chezy's, Manning's, and Darcy-Weisbach's equations, by either treating the cross-section as a whole or by dividing it vertically, horizontally or diagonally into non-interacting subareas. Discharge predictions can be done by estimating the flow resistance of a compound channel. Various methods to evaluate the roughness coefficient for a compound section have been suggested by Lotter (1930), Krishnamurthy and Christensen (1970), Cox (1970) etc. which in-turn helpful to estimate the discharge capacity of the channel. Discharge calculations can also be done by the Coherence Method (COHM) suggested by Ackers (1992) for a compound open channel. It is a function of geometry where the geometrical wetted parameters of the channel is used to find discharge deficit factor (DISDEF). However, this method gives satisfactory results as compared to other methods but involves difficult formulae constituting a complex procedure. So, this discharge prediction method is not always suited in all sort of data sets. Conventional discharge prediction models such as Divided Channel Methods, Lotter, Cox, and COHM are used to analyzed the experimental data sets reported by other researcher, such as the large scale channel data of FCF (Flood Channel Facility) (1991), Straight channel data of Knight and Demetriou (1983), Myers (1984), Knight (1989), and the data observed at SIRDA Institute of Technology, Sundernagar by Khatua (2008) and

Mohanty (2013). The paper provides a distinct comparison among the different discharge predictions methods and their suitability among the different ranges of data sets.

**Keywords:** Discharge Prediction, Straight Compound Channel, Coherence Method,, Error Analysis

## I. INTRODUCTION:

A major area of uncertainty in river channel analysis is that of accurately predicting the capability of river channels with floodplains, which are termed compound channels. Cross-sections of these compound channels are generally characterized by a deep main channel, bounded on one or both sides by a relatively shallow floodplain, which is rougher, often vegetated and has slower velocities than the main channel. When the water inundates the floodplains, there are a bank of vertical vortices along the vertical interface between the main channel and its floodplains, which are found in many experiments. At low depths, when the flow is only in the main channel, conventional methods are used to assess discharge capacity. However, when overbank flow occurs, for instance for a river in flood, the classical formulae for discharge capacity estimation do not yield reliable solutions and may lead either to overestimation of discharge capacity, which is

dangerous, or to underestimation of capacity, which may cause waste of resources. This problem has led to a thorough investigation of the flow mechanism in compound channels, leading to studies involving either improvement of the classical discharge estimation methods, or development of new computational methods for an accurate prediction of discharge capacity.

Many practical problems in river engineering require accurate prediction of discharge capacity in compound channels as it is extremely essential to imply in flood mitigation schemes. Sellin (1964) showed that Large-scale

turbulence associated with significant momentum transfer leads to the decrease in total conveyance of the section. Several attempts have been made at quantifying the interaction between the main channel and floodplain. Yen and Overton, 1973; Myers, 1978; Knight and Demetriou, 1983; Ackers, 1991; Zhonghua Yang Wei Gao et al., (2011) based on the energy concept developed a model to estimate discharge in which energy loss and the transition mechanism were analyzed.

A number of experimental results have shown that the Manning equation and the Darcy-Weisbach equation are not suitable for compound channels. If a compound channel is treated as a single channel, the computed discharge will be less than the actual one; if it is divided into many subsections and the standard equations are adopted in every subsection, then the sum of subsections' discharges will be larger than the actual one. Thus, it is necessary to clarify the flow resistance features in compound channels. Research concerning resistance to flow in compound open channel has been studied by many scholars, such as Lotter (1933), Krishnamurthy and Christensen (1972), Myers and Elsayy (1975) developed models for composite friction factor. Wormleaton (1982) have experimentations and observed that the Manning's equation and the Darcy-Weisbach equation are not suitable for compound channels. Knight and Hamed (1984) extended the work of Knight and Demetriou (1983) to rough floodplains. Pang (1998) conducted experiments on compound channel in straight reaches under isolated and interacting conditions. It was found that the distribution of discharge between the main channel and floodplain was in accordance with the flow energy loss, which can be expressed in the form of flow resistance coefficient. Yang et al. (2005) presented the study of Mannings' and Darcy-Weisbach equation and through vast number of collected experimental data indicated that Darcy-Weisbach resistance factor is a function of Reynolds number but the functional relationship is different from single channel

As the channel sections are having varying laterally along the wetted perimeter and most importantly covered with different vegetation including grasses, brush, trees and grass materials on the bottom and sides. Sometimes, one wants to know the discharge that passes through a cross-section with a certain slope. Hence, it is necessary to develop a method to predict the composite roughness. In 1930s, Lotter proposed a concept that total channel discharge equals the sum of subareas discharge by considering different roughness into account. Since then, Krishnamurthy and

Christensen (1970), Cox (1970), Yen (1990), have proposed different formulae separately. These methods are valid for a single channel. When these methods are used to determine the composite roughness of compound channels. it is suggested to divide the cross section into subsections in a vertical, horizontal, diagonal or bisectonal manner. Finally, the discharge can be obtained from Manning's equation taking into consideration of the above composite roughness.

In addition to this, there are methods such as Single Channel Method(SCM), Double Channel Method(DCM) used to calculate the flow in the channel cross section based on channel prospect how one can bisect the channel into subsections effectively without much error. However, a different method has been laid to predict the discharge in the channel which is improved the result given by DCM, called Coherence Method (COHM). In this paper, the main objective is to find the composite roughness by using different methods and therefore calculating the required discharge. There should be a comparison among all methodologies to propose a detailed report on Error analysis.

## II. METHODOLOGIES FOR ESTIMATING DISCHARGE CAPACITY OF AN OPEN CHANNEL

### Channel Division Methods:

The compound channels can be divided into sub-areas homogenously by horizontal or vertical lines. There are mainly 2 types of division methods

- 1) Single Channel Method
- 2) Divided Channel Method
  - a) Horizontal Division Method (I, II)
  - b) Vertical Division Method (I, II)

### Single Channel Method (SCM)

The simplest model of computing uniform flow in a compound channel is the single channel method (SCM), in which the channel is treated as a single unit with some appropriate averaging for the friction coefficient. In this SCM, the composite character of the channel is discarded and the velocity is assumed to be uniform in the whole cross-section. It has been shown by Myers and Brennen (1990) that with the application of this model the discharge capacity is significantly underestimated at low overbank flow depths due to the uniform velocity assumption.

$$Q = \frac{1}{n} AR^{2/3} S_0^{1/2}$$

(1)

$$Q = AC\sqrt{RS_0} = \sqrt{\frac{8g}{f}} A\sqrt{RS_0}$$

(2)

here Q is the overall discharge of the compound channel, A is the cross-sectional area of the compound channel, R is the hydraulic radius for the compound channel, S<sub>0</sub> is the slope of the channel, n is the composite Manning's coefficient for the compound channel while C and f are the Chezy's constant and Darcy-Weisbach's friction factor for the compound channel respectively.

Divided Channel Method (DCM)

The most commonly used method for calculating discharge in compound channels is the DCM, in which the compound cross-section is divided into hydraulically homogeneous sub-areas, in such a way that the velocity in each subsection can be assumed to be uniform. The division lines between the sub-sections can either be vertical, horizontal or diagonal, with the most common and practical choice being the vertical ones (Bousmar and Zech, 1999). In VDM, the division of main channel and flood plain are carried out by putting the vertical division lines on the interface of both but the difference of VDM-I from VDM-II is consideration of vertical line while calculating the wetted perimeter. Similarly the HDM-I and HDM-II are unlikely differs same as others. These imaginary division interfaces were originally assumed to be shear-free and therefore were not included in the wetted perimeters of the adjacent subdivisions when discharge was computed. Using boundary shear stress measurements, Myers (1978) showed that these division planes were not shear free, due to a turbulent interaction between the main channel and floodplain, and an apparent shear force must be present to produce a balance between the gravitational and boundary resistance forces.

Divided channel methods solve the issue of composite roughness as distinct roughness values of the main channel and floodplains can be used in their individual calculations. With the help of these division interfaces, the compound channel section is divided into subsections whose individual discharges are calculated by using Manning's or Chezy's equations and summed up to give the total discharge carried by the compound channel section. Generally, Manning's formula is used in the calculations and is given by the equation

$$Q_{total} = \sum_{i=1}^{i=N} Q_i = \frac{1}{n_i} A_i R_i^{2/3} S_0^{1/2}$$

(3)

where i refers to the subsections of the compound channel and the other variables have their usual denotation and N is the number of subdivisions.

## 2.2. Representative Methods for Predicting the Composite Roughness:

In the natural compound channels, the channel section roughness varies laterally along the wetted perimeter. The composite roughness is needed to determine the conveyance capacity in such a compound channel. Since the Manning, Chezy, Darcy-Weisbach coefficients and the relative roughness can be correlated, the equations are summarized and discussed for predicting the composite Manning coefficient only. Though the equations have a variety of structures and forms, they can be classified into the following groups according to different criterions.

All the methods are briefly described below with mathematical expression to calculate the composite roughness in turn to obtain the discharge. In the following equations, P is the wetted perimeter; P<sub>i</sub>, n<sub>i</sub>, R<sub>i</sub>, d<sub>i</sub>, are the wetted perimeter, the Manning coefficient, the hydraulic radius and the mean flow depth for subsection, respectively; N is the number of the subsections

Lotter Method (1930)

It was developed in 1930 by G. K. Lotter who assumes that the friction slope in all subsection is same. The total discharge equals the sum of the constituent discharges. This method holds good for irregularly shaped open channels such as natural floodplain.

$$n_c = PR^{5/3} \left( \sum_{i=1}^{i=N} \frac{P_i R_i^{5/3}}{n_i} \right)^{-1}$$

(4)

Cox Method (1970)

This method, also known as the U.S army Corps of Engineers Los Angeles District Method, was developed in the 1970s by R. G. Cox for the U.S army Corps of Engineers. He assumed: (1) the total shear force equals the sum of the constituent subsection shear force, (2) the friction slope is the same for all subsections and (3) the subsection velocities vary in proportion to the depth to a seven

sixth power law. By these assumptions, he derived the following equation for the composite roughness.

$$n_c = \frac{1}{A} \sum_{i=1}^{i=N} A_i n_i \quad (5)$$

Krishnamurthy and Christensen Method (KCM) (1970)

This method is also proposed in early 1970s jointly by Muthusamy Krishnamurthy, a graduate research assistant, and Professor Bent Christensen at the University of Florida. They assume (1) the friction slope is the same for all subsections, (2) the vertical velocity distribution for all subsections follows the logarithmic law and the mean flow velocity in the subsection occurs at a vertical location of 0.368di from the bed, (3) there is no lateral momentum exchange between the adjoining subsections, (4) the hydraulic radius for every subsection may be represented by the corresponding mean depth di. Thus, the composite roughness was expressed as

$$n_c = \exp \left[ \left( \sum_{i=1}^{i=N} P_i d_i^{3/2} \ln n_i \right) \left( \sum_{i=1}^{i=N} P_i d_i^{3/2} \right)^{-1} \right]$$

Yen Method (1990)

Ben Chie Yen, a professor at the University of Illinois at Urbana Campaign, proposed a number of different methods in the early 1990s. He assumed that the total shear velocity should be equal to a weighted sum of subarea shear velocities. This results in a relationship between the velocities and hydraulic radii of the subdivided areas. The expression shows

$$n_c = \frac{\sum_{i=1}^{i=N} \left( \frac{P_i n_i}{R_i^{1/6}} \right)}{\frac{P}{R^{1/6}}}$$

After calculating the composite Manning roughness, the required Discharge for the compound channel section can be obtained easily by taking Manning's Equation.

$$Q_{DCM} = \frac{1}{n_c} AR^{2/3} S_0^{1/2}$$

Here QDCM is the total discharge of the channel and nc is the composite roughness calculated from all the methods.

### 2.3. Coherence Method (COHM) (1991-1992):

Ackers (1991, 1992) proposed an approach using the traditional DCM with a vertical division plane and a large amount of previously published experimental data, with which he developed empirical correction coefficients he termed "discharge adjustment factors". He applied these coefficients to discharge given by the DCM to correct for the momentum interaction effects.

The coherence (COH) is the relationship between the discharge obtained by the equation but assuming only one section (SCM, average roughness coefficient and velocity for the whole cross section) and the DCM.

$$COH = \frac{Q_{SCM}}{Q_{DCM}}$$

The value closest to 1 is this coefficient, the most appropriate is to treat the channel as a single one. When this coefficient is significantly less than 1 it is necessary to apply a different coefficient, called DISADF in order to correct the discharge in each subsection. In extreme cases, COH may be as low as 0.5. When coherence is much less than unity, discharge deficit factors DISDEF are required to correct the individual discharges in each sub-area and calculations are similar to divided channel method. (Fig. 1 presents an example of one flow condition with the division of the flow into 4 regions according to its relative depth, I. e, floodplain/main channel water depth ratio).



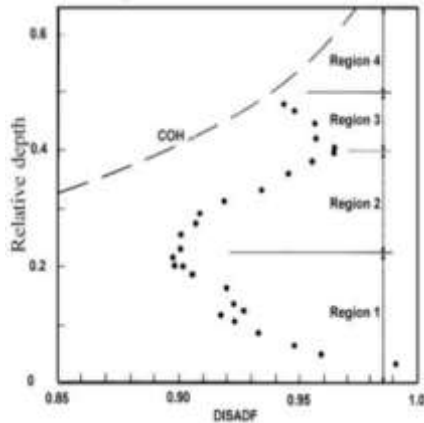


Figure 1: DISADF Coefficient

The coherence method was developed by Ackers by basically taking into consideration the experimental investigations of the flood channel facility (FCF) phase A, He has presented the formulas for DISADF in each flow region. The discharge is then obtained by the following equations.

Region 1:

The depth of flow is quite low in this region; hence, the velocities in floodplains and the main channel are very dissimilar. This region is characterized by relative depth,  $H_r < 0.2$ .

$$H_r = \frac{H - h}{H} \quad (10)$$

where  $H$  is the water level above channel bottom and  $h$  is the bank-full level of the main channel

$$Q_1 = Q_{DCM} - DISDEF \quad (11)$$

Region 2:

This zone is of a higher depth where interaction effect is still not dominant and the flow computation depends on discharge adjustment factor, DISADF in the channel under consideration.

$$Q_2 = Q_{DCM} * DISADF_2 \quad (12)$$

where DISADF2 is the discharge adjustment factor for region 2. Region 3:

This zone appears when the relative depth is around 0.5 and the interference effect affects the discharge capacity and hence the discharge adjustment factor is different in this region and is termed as DISADF3.

$$Q_3 = Q_{DCM} * DISADF_3 \quad (13)$$

Region 4:

This zone has a greater value of relative depth i.e. above 0.6 and behaves as a single unit due to the coherence character that obeys both the main channel and the floodplains. The discharge adjustment factor for this region DISADF4 is dependent on the aforesaid coherence character of flow.

$$Q_4 = Q_{DCM} * DISADF_4 \quad (14)$$

Here QDCM is the total discharge calculated using zones separated by vertical divisions. The discharge deficit factor for zone 1 and the discharge adjustment factors for the others zones are

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calculated by individual methodologies and then the predicted discharge is estimated from the choice of region.

### III. DATA SETS FOR ANALYSIS

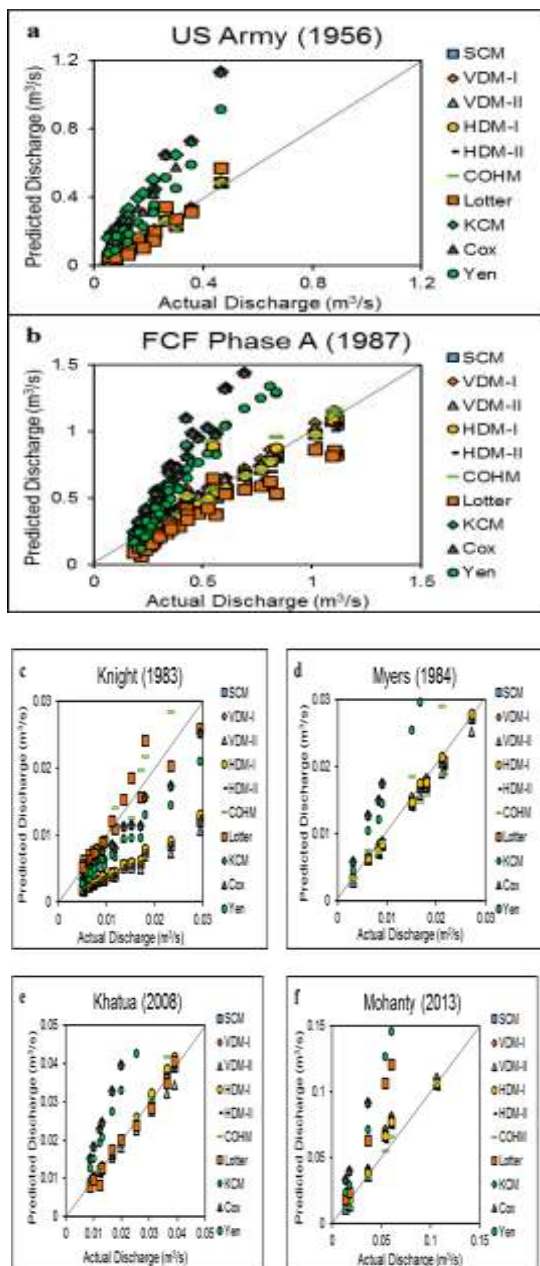
In order to examine the methods and check the suitability of subsection divisions, a large number of laboratory and field data were collected and applied. The data set resulting from the US-Army (1956) observation on compound channel section has been widely used to obtain the flow parameter in any section. Two series of data are considered in this paper indicating the validation of the data set on the experimental flume. The laboratory data also includes the UK Flood Channel Facility (FCF) (1987); a large scale national facility for undertaking experimental investigations of overbank flows in rivers. The FCF has been used for overbank flow studies on straight, meandering and free formed channels with floodplains, with either rigid or mobile boundaries. It serves as the high quality benchmark data which is used to validate all the research work performed by many researchers. 51 groups of data have been considered in this paper with symmetrical cross section. Experimental findings from Knight (1983) are considered consisting of one rectangular main channel and two symmetrical prone floodplains. Three such data series having different floodplain width are presented demonstrating its effect on the discharge capacity of the compound channel with other variables remaining constant. Straight smooth channel boundary data are taken from Myer (1984) relating to asymmetrical as well as symmetrical compound cross sections from which only the symmetric data set is considered. Data sets from similar experimentations conducted at SIRDA Institute of Technology, Sundernagar by Khatua (2008) and Mohanty (2013) have also been considered in the study.

**Table 1:** Parameters for Experimental Data sets used

DATA SERIES		Slope		Manning's n		h (in m)	b (in m)	B (in m)	Bed Slope S <sub>0</sub>	$\beta = \frac{h}{H}$	Discharge, Q
		S <sub>1</sub>	S <sub>2</sub>	n <sub>mc</sub>	n <sub>fp</sub>						
US Army (1956)	I (9)	0	0.5	0.012	0.035	0.1524	0.3048	9.144	0.001	0.1667-0.375	0.0504-0.4648
	XI (9)	0	0.5	0.012	0.035	0.1524	0.6096	4.8768	0.001	0.1667-0.375	0.0775-0.3539
FCF Phase- (1987)	1 (8)	0	1	0.01	0.01	0.15	1.5	10	0.001027	0.0565-0.4002	0.2082-1.0145
	A2 (10)	1	1	0.01	0.01	0.15	1.5	6.3	0.001027	0.0141-0.4790	0.2143-1.1142
	3 (10)	1	1	0.01	0.01	0.15	1.5	3.3	0.001027	0.0506-0.5002	0.2251-0.8349
	7 (7)	1	1	0.01415	0.01415	0.15	1.5	6.3	0.001027	0.0378-0.5041	0.216-0.5434
	8 (8)	1	0	0.01	0.01	0.15	1.5	6.0	0.001027	0.0504-0.4995	0.1858-1.1034
	10 (8)	1	2	0.01	0.01	0.15	1.5	6.6	0.001027	0.0508-0.4637	0.2368-1.0939
Knight (1983)	2 (6)	0	0	0.01	0.01	0.076	0.076	0.152	0.000966	0.1079-0.4926	0.0052-0.0171
	3 (6)	0	0	0.01	0.01	0.076	0.076	0.228	0.000966	0.1314-0.4906	0.005-0.0234
	4 (6)	0	0	0.01	0.01	0.076	0.076	0.305	0.000966	0.1058-0.5058	0.0049-0.0294
Myers (1984) (10)	0	0	0.01	0.01	0.08	0.16	0.75	0.00093	0.0980-0.4764	0.0083-0.0272	
Khatua (2008) (10)	0	0	0.01096	0.01096	0.12	0.12	0.44	0.0019	0.1189-0.4614	0.0087-0.039	
Mohanty (2013) (6)	0	1	0.01	0.01	0.065	0.33	3.95	0.0011	0.1095-0.4347	0.0135-0.1061	

#### IV. RESULTS AND ANALYSIS

The data sets are used to plot graph in-between the actual discharge and predicted discharge values. Data are sorted according to their year of performance and taken separately but as a whole. The predicted discharge is calculated by using various methods as described earlier and compared with the actual value to obtain a best suitable method for future prediction in various water resources engineering application.

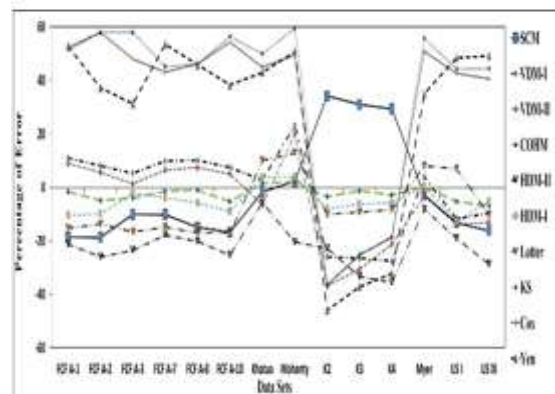


**Figure 2 (a-f):** Scatter Plots of Actual v/s Predicted Discharge for different data sets

US Army data sets as shown in fig 2a

having 2 groups in which COHM HDM-I and Lotter are the most fitting methods as compared than others According to the fig 2b, it shows the data series FCF Phase A in which three methods are giving satisfactory results such as COHM, HDM-I and Lotter. FCF data sets are having more accuracy in finding the most appropriate methods as it involves more data points. In fig 2c the data sets of Knight et. al (1983) having 3 groups of data are plotted to get the scatter which shows the Lotter and COHM method shows best fit among all. However, Myers Series have proved that HDM-I and Lotter method to be the best suited methods having less error. The two data sets performed at SIRDA Institute of Technology, Sundernagar are giving a significant results in which the data carried out by Khatua shows Lotter, HDM-I and COHM gives commendable results whereas Mohanty data sets shows COHM to provide better results.

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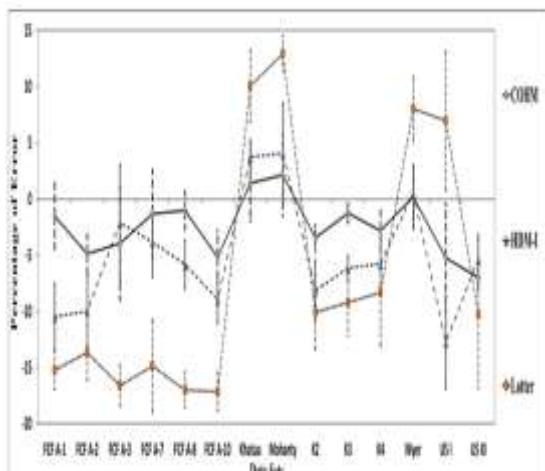


**Figure 3:** Percentage of Error for different Data Series

Hence the results are not quite conclusive when taking scatter plots of different data sets as a whole. Each data set consists of different series of channel; hence each such series needs to be analyzed individually. Therefore, the percentage error in each such channel is calculated individually for all the studied models and represented in fig 3. It is quite evident that some of the channel division methods over predict the discharge capacity and hence need to be avoided. For the case of other channel division methods, in accordance to the above scatter plots in fig 2, the results obtained by these methods seem to be quite similar. Hence by detailed study it is observed that HDM-I provides with the best estimation among the channel division methods. From fig. 3 it is observed that the Lotter methods gives acceptable results with respect to the other prediction methods by using composite roughness. The Coherence

method on the other hand seem to predict the discharge capacity quite preferably for almost all the data series. The three methods thus commenced need to be further analyzed.

Figure 4 contains the percentage of error by the three best methods along with the standard deviation for each series. The standard deviation values provide a more holistic idea about the mean error and thus helps in identifying a better quality method. It is observed that the Coherence Method provides the least percentage of error with an acceptable range of standard deviation in its value.



**Figure 4:** Percentage of Error with Standard Deviation for different Data Series

## VI. CONCLUSIONS

The conclusions from the paper are that

1. Horizontal division method I (HDM-I) is observed to provide with acceptable discharge predicted with respect to the other channel division methods
2. From the different methods of discharge prediction by composite roughness method, Lotter method seems to be promising with respect to the other available methods
3. Coherence method is observed to provide with best acceptable results in predicting the discharge capacity with an acceptable range of standard deviation.
4. Although the above methods show promising results in predicting discharge capacity in straight compound channels, it is important to mention that other compound channel methods need to be analyzed with a wider range of data sets in order to find an acceptable method.

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