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Iterative Linearization via Collocation Method for Solving Non-Linear Differential Equations

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ABSTRACT: This paper introduces an effective numerical approach for addressing non-linear differential equations by iterative linearization utilizing the collocation technique. The suggested method integrates the benefits of iterative linearization with collocation to precisely approximate solutions. Numerical experiments indicate the method's efficiency and robustness in addressing numerous non-linear problems, exhibiting higher accuracy and convergence rates compared to existing methods.

Keywords: [Non-linear differential equations, Iterative linearization, Collocation method, Numerical solution.]

I. INTRODUCTION

Over the years, researchers have made various attempts to represent real-life problems, leading to the development of non-linear differential equations. Non-linear differential equation is a topic that comes up regularly in applied mathematics, science, and engineering. It has a key role in simulating many events in physics, engineering, and other fields [1, 2].However, solving these equations analytically is often challenging. Traditional numerical methods, such as Runge-Kutta and finite difference, may struggle with accuracy, stability, and computational efficiency [3]. To address these limitations, researchers have explored alternative approaches, including collocation methods [4], shooting method, Galerkin residual method, Galerkin method with Hermite polynomials, homotopy perturbation method, weighted residual method [5], homotopy analysis method, and perturbation techniques, among others [6]. Collocation methods have also been widely used to solve non-linear differential equations by the authors in [7, 8]. These methods involve approximating the solution using a set of basis functions and collocating the residual at specific points. Researchers in [9] have demonstrated the effectiveness of collocation methods in solving non-linear boundary value problems (BVPs) and initial value problems (IVPs). [10], implemented an efficient numerical method for solving higher-order linear and non-linear two-point boundary value problems, which was based on the weighted residual via the partition method. With that method, the two-point Taylor polynomial was used as a trial function to obtain the residual function. Linearization approach is another technique employed by researchers to solve non-linear differential equations. In this approach, the nonlinear functions available in a given differential equation will be linearized in order to simplify the differential equation into its linear form. This procedure will not account for all variables that is present in the original differential equation, as some important variables would have been lost through the process of linearization which would have consequently affected the accuracy of the result. Some of the researchers that have employed this procedure can be found in the work of [11, 12, 13].

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In order to overcome this drawback associated with the linearization approach, an enhanced form of the linearization method called iterative linearization method is proposed by using the result of the linearization as initial guess to solving the nonlinear differential equation iteratively until a desired solution is achieved.

Therefore, the main subject of this paper is the establishment of an efficient and accurate numerical approach for solving non-linear differential equations. Specifically, this research intends to investigate the effectiveness of iterative linearization via collocation method in solving non-linear differential equations, evaluate the accuracy and efficiency of the proposed method, and compare the findings with the results in the previous literature. The outcomes of this research will have implications for improving the accuracy and efficiency of numerical solutions for non-linear differential equations, enhancing modeling and simulation capabilities in various fields, and providing a reliable and robust numerical method for solving complex problems. The significance of this study lies in its ability to address the limitations presented by conventional numerical methods, provide a more accurate and efficient solution technique for non-linear differential equations, and contribute to the advancement of numerical analysis and computational science by exploring the iterative linearization via collocation method.

II. CONCEPT OF ITERATIVE LINEARIZATION TECHNIQUE (ILT)

Consider a non-linear differential equation of the form

$$Lu + Ru + Nu = h(x) \qquad (1)$$

Where L is the highest order derivative that is linear, R is the remaining term whose derivative is less than L, N is the non-linear term and $h(x)$ is the source term. The boundary conditions associated with (1) is given by,

$$B(\mu_i) = B_i, \qquad (2)$$

The following steps are considered while solving (1) using ILT:

(i) Linearize equation (1), using Taylor series expansion, that is

$$Lu + Ru + Nu\big|_{linear} = g(x) \qquad (3)$$

(ii) Determine the initial approximation u_o by solving equation (3) using collocation method,

$$Lu_0 + Ru_0 + Nu_0 = g(x) \qquad (4)$$

The process of collocation follows the following steps

(a) a trial function of the form

$$u_0 = v_0 + \sum_{j=1}^{n} c_j v_j \qquad (5)$$

is assumed. Where v_0 is made to satisfy the given boundary conditions and v_j satisfies the homogeneous boundary conditions.

(b) Substitute equation (5) into (4) gives the Residual which are evaluated within the domain of the problem. This procedure gives system of algebraic equations which are solved simultaneously to obtain constants c_i.

(c) on substituting the constants into the assumed function, this gives the solution to the initial approximation which will be used in the subsequent iteration.

(iii) The subsequent iterations is obtained using collocation method in the iterative sequence

$$Lu_n + Ru_n + Nu_{n-1} - h(x) = 0 \qquad (6)$$

III. NUMERICAL EXPERIMENTS

Illustration 1: Consider a nonlinear boundary value problem given as follows:

$$\frac{d^2u}{dx^2} + 3u^{\frac{1}{3}} - 3x^2u^{-\frac{1}{3}} = 0, \qquad (7)$$

with the boundary conditions;

$$u(0) = 2\sqrt{2}, \quad u(1) = 1. \qquad (8)$$

Where the exact solution is obtained as

$$u(x) = \left(2 - x^2\right)^{\frac{3}{2}} \qquad (9)$$

Linearizing (7) involves multiplying (14) by u that is

$$u\frac{d^2u}{dx^2} + 3u^{\frac{4}{3}} - 3x^2u^{\frac{2}{3}} = 0 \qquad (10)$$

by replacing $u^{\frac{2}{3}}$ in (10) with the linear function u gives

$$u\frac{d^2u}{dx^2} + 3u^2 - 3x^2u = 0 \qquad (11)$$

Dividing (11) through by u gives

$$\frac{d^2u_0}{dx^2} + 3u_0 - 3x^2 = 0 \qquad (12)$$

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Equation (12) is in its linear form, so solving the linearized equation in (12) following the procedure discussed in section 2, considering a trial function.

$$u_0 = v_0 + \sum_{j=i}^{n} c_j v_j \quad (13)$$

For n=2, the assumed function becomes

$$u_0 = 2\sqrt{2} + \left(1 - 2\sqrt{2}\right)x + c_1\left(x - x^2\right) + c_2\left(x^2 - x^3\right) \quad (14)$$

Substituting (14) into the linearized equation in (12) to obtain the residual as

$$u_{0R} = 2c_1 + c_2(-6x + 2) + 8.485281372 \ -5.485281372 \ x + 3c_1\left(x - x^2\right) + 3c_2\left(x^2 - x^3\right) - 3x^2 \quad (15)$$

Collocating (15) at $\frac{1}{4}$ and $\frac{3}{4}$ results to system of equations which are:

$$\frac{23}{16}c_1 + \frac{15}{64}c_2 + 6.926461029 = 0, \quad (16)$$

$$-\frac{23}{16}c_1 + \frac{41}{64}c_2 + 2.683820343 = 0 \quad (17)$$

Solving (16) and (17) simultaneously to obtain

$$c_1 = 4.122962322, \qquad c_2 = -1.560511516.$$

Substitutingthe values of c_1 and c_2 on (13) to obtain the initial approximation u_0 as

$$u_0 = 2.828427124 + 2.294535198x - 5.683473838x^2 + 1.560511516x^3, \quad (18)$$

This is the initial approximation, in order to obtain the next approximation,
Substitute (18) into the original non-linear BVP in (7) to obtain the iterative sequence

$$\frac{d^2u_1}{dx^2} + 3u_0^{\frac{1}{3}} - 3x^2 u_0^{-\frac{1}{3}} = 0, \quad (19)$$

with

$$u_{1R} = 2c_1 + c_2(2-6x) + (2.828427124 + 2.294535198x - 5.683473838x^2 + 1.5605511516x^3)^{\frac{1}{3}} -$$

$$3x^2(2.828427124 + 2.94533198x - 5.683473838x^2 + 1.560511516x^3)^{-\frac{1}{3}}.$$

(20)

Following the same procedure as done for the initial approximation, then the constants are

$$c_1 = 1.967347576$$

$$c_2 = -0.5940720030$$

And

$$u_1 = 2.828427124 + 0.138920452x - 2.561419579x^2 + 0.5940720030x^3, \quad (21)$$

The other iterative solutions are obtained using the iterative sequence

$$\frac{d^2u_n}{dx^2} + 3u_{n-1}^{\frac{1}{3}} - 3x^2 u_{n-1}^{-\frac{1}{3}} = 0, \qquad n = 2,3,... \quad (22)$$

Following the same procedure as done for u_1, then,

$$u_2 = 2.828427124 \ + 0.042769629 \ x - 2.48509481 \ x^2 + 0.6138980644 \ x^3. \quad (23)$$

$$u_3 = 2.828427124 + 0.036895708x - 2.482546870x^2 + 0.6172240384x^3 \quad (24)$$
$$u_4 = 2.828427124 + 0.036489592x - 2.482424123x^2 + 0.617507407x^3. \quad (25)$$
$$u_5 = 2.828427124 + 0.036460343 - 2.482416464x^2 + 0.6175289948x^3. \quad (26)$$

When $n = 3$, the assumed function is ,

$$u = 2\sqrt{2} + (1 - 2\sqrt{2})x + c_1(x - x^2) + c_2(x^2 - x^3) + c_3(x^3 - x^4). \quad (27)$$

And the iterative solutions were calculated to be,

$$u_0 = 2.828427124 + 2.112491549x - 4.24260686x^2 - 1.058643865x^3 + 1.36036587x^4. \quad (28)$$
$$u_1 = 2.828427124 - 0.10635234x - 2.121320343x^2 - 0.177916474x^3 + 0.3643326320x^4 \quad (29)$$
$$u_2 = 2.828427124 + 0.020533992x - 2.121320343x^2 - 0.020796606x^3 + 0.293155879x^4. \quad (30)$$
$$u_3 = 2.828427124 + 0.014541197x - 2.121320343x^2 - 0.0131442993x^3 + 0.2914963213x^4. \quad (31)$$
$$u_4 = 2.828427124 + 0.014075770x - 2.121320343x^2 - 0.0126268421x^3 + 0.2914442907x^4 \quad (32)$$
$$u_5 = 2.828427124 + 0.014038566x - 2.121320343x^2 - 0.0125870546x^3 + 0.2914417079x^4 \quad (33)$$

In a case of four constants $(n = 4)$, the trial function gives

$$u = 2\sqrt{2} + (1 - 2\sqrt{2})x + c_1(x - x^2) + c_2(x^2 - x^3) + c_3(x^3 - x^4) + c_4(x^4 - x^5) \quad (34)$$

Repeating the same procedure to obtain,

$$u_0 = 2.82847124 + 2.153756941x - 4.24205686x^2 - 1.139140023x^3 + 1.542297070x^4 - 0.142700426x^5 \quad (35)$$
$$u_1 = 2.82847124 + 0.09786787x - 2.121320343x^2 - 0.151214819x^3 + -0.2754422515x^4 + 0.0707979162x^5 \quad (36)$$
$$u_2 = 2.82847124 + 0.007293089x - 2.121320343x^2 + 0.0104944618x^3 + +0.2011669485x^4 + 0.0739387200x^5 \quad (37)$$
$$u_3 = 2.828427124 + 0.001201388x - 2.121320343x^2 + 0.0177364931x^3 + +0.2019307364x^4 + 0.0720246018x^5 \quad (38)$$
$$u_4 = 2.828427124 + 0.000759284x - 2.121320343x^2 + 0.0182029786x^3 + +0.2020637194x^4 + 0.0718672372x^5 \quad (39)$$
$$u_5 = 2.828427124 + 0.000726551x - 2.121320343x^2 + 0.0182364999x^3 + +0.202074616x^4 + 0.071855552x^5 \quad (40)$$

The procedures is repeated for five constants as well.

Illustration 2: Consider the nonlinear differential equation [14],

$$y'' = \cos(y)\sin(y') + 2y + \cos(1 - x^2)(2x) - 2(x^2 - 1) + 2, x \in [-1,1] \quad (41)$$

Subjected to the boundary conditions

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$$y(-1)=0, y(1)=0. \quad (42)$$

Whose exact soluton is

$$y(x)=x^2-1. \quad (43)$$

Employing Taylor's series to linearize the nonlinear term in (41) gives

$$\cos(y)\sin(y') \approx -y'+... \quad (44)$$

Substituting (44) into (41) to obtain

$$y_0''+y_0'-2y_0-\cos\left(1-x^2\right)\sin(2x)+2\left(x^2-1\right)-2=0 \quad (45)$$

Using collocation method to solve the linearised differential equation in (45)

When $n=2$

$$y_0=c_1\left(x+1-0.5(x+1)^2\right)+c_2\left((x+1)^2-0.5(x+1)^3\right) \quad (46)$$

Applying (46) in (45) gives the residual

$$y_{0R}=y_0''+y_0'-2y_0-\cos\left(1-x^2\right)\sin(2x)+2\left(x^2-1\right)-2$$

$$y_{0R}=-c_1+c_2(-1-3x)-c_1x+c_2\left(2x+2-1.5(x+1)^2\right)-2c_1\left(x+1-0.5(x+1)^2\right)- 2c_2\left((x+1)^2-0.5(x+1)^3\right)-\cos\left(x^2-1\right)\sin(2x)+2x^2-4. \quad (47)$$

Collocating (47) at points $\frac{1}{4}$ and $\frac{3}{4}$ and solving the resulting systems of equation gives

$$c_1=-2.106213056, \quad c_2=0.1622109263 \quad (48)$$

Substituting (48) into (46) to obtain

$$y_0=-2.106213056-2.106213056x+(x+1)^2-0.08110546315(x+1)^3 \quad (49)$$

The iterative sequence for the subsequent iteration becomes,

$$y_n''=\cos\left(y_{n-1}\right)\sin\left(y_{n-1}'\right)+2y_n+\cos\left(1-x^2\right)(2x)-2\left(x^2-1\right)+2 \quad (50)$$

That is,

$$y_1=-1.95186898-1.953186898x+0.9610769693(x+1)^2+0.00775823985(x+1)^3 \quad (51)$$

$$y_2=-2.003265418-2.003265418x+1.002925084(x+1)^2-0.0006461874(x+1)^3. \quad (52)$$

$$y_3=-1.999720941-1.999720941x+0.9997538612(x+1)^2+0.00005330465(x+1)^3 \quad (53)$$

$$y_4=-2.000022854-2.000022854x+1.000020258(x+1)^2-0.00000441525(x+1)^3 \quad (54)$$

$$y_5=-1.999998100-1.999998100x+1.9999983191(x+1)^2+3.6545\times10^{-7}(x+1)^3. \quad (55)$$

The procedure is repeated for n=3,4 and 5 as well and the results are computed.
In each case the error is calculated as,

$$E=\left|U_{exact}-U_{computed}\right| \quad (56)$$

IV. RESULTS AND DISCUSSION

Table 1: Errors in illustration one when considering two constants for the trial function

X	Eu_0	Eu_1	Eu_2	Eu_3	Eu_4	Eu_5
0	2×10^{-9}	2×10^{-9}	2×10^{-9}	2×10^{-9}	2×10^{-9}	2×10^{-9}
0.1	0.195365958	0.010058585	0.001226577	0.000667990	0.000628889	0.000626062
0.2	0.328479302	0.014507007	0.001511558	0.002557817	0.002631864	0.002637233
0.3	0.407736332	0.015942925	0.005497789	0.006940850	0.007043986	0.007051489
0.4	0.440860690	0.016291344	0.008688155	0.010417190	0.010541861	0.010550953
0.5	0.434857802	0.016759055	0.009756908	0.011641135	0.011778085	0.011788096
0.6	0.395943401	0.017763153	0.008167998	0.010056679	0.010194952	0.010205080
0.7	0.329435966	0.018823477	0.004282626	0.006005067	0.006132007	0.006141323
0.8	0.239595006	0.018400904	0.000479037	0.000886515	0.000987764	0.000995209
0.9	0.129371148	0.013647422	0.003387937	0.002589880	0.002530377	0.002525996
1.0	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000	0.000000000
MAE	0.263785964	0.016880018	0.003912525	0.004704829	0.0047699806	0.004774676

Table 2: Errors in illustration one when considering three constants for the trial function

X	Eu_0	Eu_1	Eu_2	Eu_3	Eu_4	Eu_5
0	2×10^{-9}	2×10^{-9}	2×10^{-9}	2×10^{-9}	2×10^{-9}	2×10^{-9}
0.1	0.189086805	0.01046732	0.00203537	0.00144358	0.00139755	0.001393875
0.2	0.33092724	0.02000535	0.00398378	0.00284378	0.00275476	0.002747634
0.3	0.423099990	0.027892166	0.005809028	0.004204359	0.004078281	0.004068174
0.4	0.465775957	0.033607577	0.007505837	0.005555983	0.005401597	0.005389196
0.5	0.461672710	0.036787588	0.009054298	0.006910719	0.006739436	0.006725646
0.6	0.415983002	0.037152906	0.010348378	0.008190525	0.008016297	0.008002234
0.7	0.336267207	0.034401384	0.011088364	0.009119700	0.008958896	0.008945880
0.8	0.232291613	0.028046298	0.010619149	0.009063158	0.008934442	0.008923993
0.9	0.115778497	0.017166446	0.007680316	0.006776525	0.006700730	0.006694557
1.0	0.000000000	4×10^{-10}	0.000000000	0.000000000	4×10^{-10}	0.000000000
MAE	0.270082749	0.02232002115	0.006193139545	0.004918940036	0.004816545127	0.004808290041

Table 3: Errors of illustration one when considering four constants for the trial function

X	Eu_0	Eu_1	Eu_2	Eu_3	Eu_4	Eu_5
0	2×10^{-9}	2×10^{-9}	2×10^{-9}	2×10^{-9}	2×10^{-9}	2×10^{-9}
0.1	0.193149614	0.009637285	0.000734120	0.000132249	0.000088517	0.000085278
0.2	0.338781776	0.018401528	0.001462411	0.000302617	0.000218091	0.000211825
0.3	0.434433090	0.025516458	0.002116176	0.000485736	0.000366394	0.000357539
0.4	0.480326546	0.030364126	0.002614321	0.000641083	0.000495890	0.000485100
0.5	0.479154698	0.032524297	0.002906509	0.000753834	0.000594485	0.000582626
0.6	0.435836965	0.031787936	0.002990423	0.000849826	0.000690320	0.000678427
0.7	0.357240819	0.028134607	0.002892917	0.000974424	0.000830437	0.000819673
0.8	0.251848834	0.021655704	0.002597045	0.001117234	0.001005295	0.000996907
0.9	0.129337531	0.012389509	0.001880859	0.001048621	0.000985124	0.000980351
1.0	0.000000000	3×10^{-10}	0.000000000	0.000000000	0.000000000	1×10^{-10}
MAE	0.2818281705	0.01912831385	0.001835889364	0.0005732387223	0.000479505	0.0004725207364

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Table 4: Errors of illustration one when considering five constants in the trial function.

X	Eu_0	Eu_1	Eu_2	Eu_3	Eu_4	Eu_5
0	2×10^{-9}	2×10^{-9}	2×10^{-9}	2×10^{-9}	2×10^{-9}	2×10^{-9}
0.1	0.193290315	0.009610515	0.000681664	0.000085105	0.000042560	0.000039478
0.2	0.339175763	0.018190673	0.001255371	0.000111004	0.000029213	0.000023289
0.3	0.435123312	0.025045941	0.001707619	0.000104886	0.000010069	0.000018403
0.4	0.481253030	0.029683728	0.002041418	0.000106288	0.000033150	0.000043272
0.5	0.480226522	0.031714702	0.002228849	0.000121582	0.000031071	0.000042166
0.6	0.437025435	0.030838539	0.002210351	0.000121773	0.000030382	0.000041458
0.7	0.358609786	0.026903949	0.001930614	0.000073403	0.000062643	0.000072561
0.8	0.253438092	0.020024901	0.001392966	0.000015468	0.000119145	0.000126713
0.9	0.130815227	0.010718999	0.000698228	0.000068281	0.000124911	0.000129049
1.0	1×10^{-10}	1×10^{-9}	9×10^{-10}	0	9×10^{-10}	0
MAE	0.282632334	0.01843017727	0.001286098445	0.00007343563636	0.000043922445	0.000048762818

Table 5: Errors of illustration 2 when considering two constants for the trial function

X	Ey_0	Ey_1	Ey_2	Ey_3	Ey_4	Ey_5
0	0.0279989348	0.01564831115	0.00098652140	0.00008622485	0.0000070112	5.8455×10^{-7}
0.1	0.0357483855	0.01472376224	0.00091268342	0.00008008549	0.0000065036	5.4241×10^{-7}
0.2	0.0424512267	0.01353279646	0.00082299283	0.00007254144	0.0000058825	4.9150×10^{-7}
0.3	0.0476208215	0.0121219640	0.00072132472	0.00006391232	0.0000051743	4.3189×10^{-7}
0.4	0.0507705411	0.0105378132	0.0006115582	0.0000545190	0.0000044054	3.678×10^{-7}
0.5	0.0514137499	0.0088268935	0.0004975705	0.0000446792	0.0000036025	3.014×10^{-7}
0.6	0.0490638149	0.0070357544	0.0003832376	0.0000347148	0.0000027909	2.339×10^{-7}
0.7	0.0432341065	0.0052109464	0.0002724367	0.0000249447	0.0000019981	1.675×10^{-7}
0.8	0.0334379889	0.0033990198	0.0001690449	0.0000156887	0.0000012507	1.053×10^{-7}
0.9	0.0191888313	0.0016465201	0.0000769404	0.0000072676	5.762×10^{-7}	4.86×10^{-8}
1.0	0.0000000000	1.2×10^{-9}	2×10^{-10}	1×10^{-9}	2×10^{-9}	4×10^{-10}
MAE	3.6448×10^{-2}	8.42579×10^{-3}	5.45431×10^{-3}	4.40393×10^{-5}	3.5634×10^{-6}	4.2005×10^{-7}

Table 6: Errors of illustration 2 when considering three constants for the trial function

X	Ey_0	Ey_1	Ey_2	Ey_3	Ey_4	Ey_5
0	0.0480526491	8.8189×10^{-3}	8.7505×10^{-4}	0.000242401500	1.86504×10^{-5}	1.87215×10^{-6}
0.1	0.0519308805	0.0085302805	0.0007325906	0.000218461256	0.00001789951	0.00000153650
0.2	0.0550948176	0.0080356263	0.0006002095	0.00019256310	0.00001668154	0.00000122902
0.3	0.0572004614	0.0073536453	0.0004829701	0.00016576234	0.00001504703	9.6302×10^{-7}
0.4	0.0578399458	0.0065082357	0.0003834988	0.0001389162	0.0000130698	7.454×10^{-7}
0.5	0.0565415410	0.0055285179	0.0003019846	0.0001126855	0.0000108358	5.740×10^{-7}
0.6	0.0527696516	0.0044488314	0.0002361833	0.0000875325	0.0000084485	4.428×10^{-7}
0.7	0.0459248123	0.0033087420	0.0001814172	0.0000637216	0.0000060274	3.410×10^{-7}
0.8	0.0353436981	0.0021530316	0.0001305692	0.0000413176	0.0000037076	2.504×10^{-7}
0.9	0.0202991134	0.0010317092	0.0000740918	0.0000201870	0.0000016439	1.453×10^{-7}
1.0	8×10^{-10}	0	0	0	2×10^{-9}	4×10^{-10}
MAE	0.4809975717	0.0050652320	0.0003635066	0.000116686236	0.00001018304	0.00000736362

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Table 7: Errors of illustration two when considering four constants in the trial function.

X	Ey_0	Ey_1	Ey_2	Ey_3	Ey_4	Ey_5
0	0.0650889049	0.03203377404	0.01532188088	0.003543503480	0.0004983221190	0.00002289017242
0.1	0.0397194116	0.02492741751	0.01357050224	0.003170564116	0.00045353624	0.000022948379
0.2	0.0179939533	0.0190835835	0.01179691593	0.00278725819	0.00040484676	0.00002218630
0.3	0.0002786378	0.0144603067	0.0100543569	0.00240227173	0.00035338912	0.00002062165
0.4	0.0133088358	0.0108968626	0.0083770586	0.0020209776	0.0003000537	0.0000183197
0.5	0.0227688008	0.0081621463	0.0067833947	0.0016466565	0.0002456554	0.0000154022
0.6	0.028089221	0.0060030530	0.0052790359	0.0012816956	0.0001910777	0.0000120450
0.7	0.029115302	0.0041928538	0.0038601041	0.0009288075	0.0001374378	0.0000084855
0.8	0.025419098	0.0025795798	0.0025163193	0.0005922407	0.0000862346	2.504×10^{-7}
0.9	0.016169142	0.0011344011	0.0012341587	0.0002789876	0.0000395088	0.0000020512
1.0	6×10^{-9}	1×10^{-9}	0	1×10^{-9}	0	2×10^{-10}
MAE	0.0234501193	0.01122490721	0.00718218495	0.001695724001	0.0002464149381	0.00001320006377

Table 8: Errors of illustration two when considering five constants in the trial function.

X	Ey_0	Ey_1	Ey_2	Ey_3	Ey_4	Ey_5
0	0.00050539765	0.01596552784	0.001203569668	0.0002120879828	0.00004028501257	0.000006677280602
0.1	0.01331514757	0.01527564978	0.001091588955	0.00018849425	0.000036310111	0.00000601950
0.2	0.02423136158	0.01419719214	0.00097749637	0.00016425518	0.00003207956	0.00000531950
0.3	0.0328572357	0.0127866846	0.00085990560	0.00014019245	0.00002772949	0.00000459873
0.4	0.0389091263	0.0111302397	0.0007362738	0.0001168591	0.0000233544	0.0000038747
0.5	0.0421504545	0.0093312047	0.0006043684	0.0000945805	0.0000190210	0.0000031551
0.6	0.0423118779	0.0074908425	0.0004639738	0.0000734902	0.0000147763	0.0000024500
0.7	0.0389977103	0.0056820271	0.0003188415	0.0000535821	0.0000106702	0.0000017676
0.8	0.0315786208	0.0039159570	0.0001788924	0.0000347667	0.0000067634	0.0000011183
0.9	0.0190705664	0.0021019103	0.0000626637	0.0000169300	0.0000031540	5.202×10^{-7}
1.0	0	0	1.9×10^{-9}	8×10^{-10}	2×10^{-10}	9×10^{-10}
MAE	0.02581159097	0.00889792857	0.000590688735	0.0000995772057	0.00002141436735	0.000003227437327

Tables 1-4 show the errors in illustration one considering two contants, three constants, four constant and five constants for the trial function, the mean absolute error (MAE) at each iterate were also calculated. It was observed that as the iteration increases, the error decreases from u_0 to u_2 but later increases from u_3 to u_5, this happened due to the choice of collocating nodes that was used, but it was observed that as the number of constants used in the trial function increases, the error reduces. That is as the iteration increases the error obtained when three contants were considered were found to be lesser than that of two constants, error obtained when four constants were considered were found to be lesser than that of two and three constants.
The errors obtained where two constants, three constants, four constants and five constants were considered for the trial function in illustration two are shown in Tables 5-8 which exhibited the same behaviour as that of illustration one.

V. CONCLUSION

This study employed iterative linearization via collocation technique to solve nonlinear boundary value problems. The nonlinear term was transformed into a linear algebraic form using Taylor's series expansion, the resulting linear differential equations were solved using the collocation method to obtain the initial approximation. Subsequent solution were obtained iteratively by employing the original nonlinear problem. The effects of varying the number of constants in the trial function wereinvestigated, revealing decrease in error with increasing constants in illustrations considered.

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