

Leibniz's Arithmetization of Aristotle's Syllogistic Focusing on + – Quantification Strategy

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ABSTRACT

This article¹ intends to add a new stroke of the current research to the + – quantification strategy of Leibniz's arithmetic system of Aristotle's syllogism, which was dealt with in 9 papers in April 1679. Leibniz transformed Aristotle's logic into an arithmetic system with prime numbers and its composite numbers which semantics and syntax are constructed with well assigned symbols and characteristic numbers. His goal is to set up the syntax and the semantics of logical square in the algebraic form 'S is P' in which numbers of subjects can be divided by numbers of predicates. In so far as he assigns universal signs and universal numbers to subjects and predicates in 4 propositional forms of the logical square, he could arithmetize Aristotle's syllogism with positive + or negative – quantity. Several scholars including Couturat(1905), L. saw Leibniz's algebraic achievements of formal language as a failure, but since Lukasiewicz, J. I.(1951), Sommers, F.(1982), Sotirov(1999), V. and Glashoff(2002), J. have successfully evaluated Leibniz's algebraic works and his + – quantification strategy of Aristotle's logic. If Leibniz's contradictory axiom 'B is A, and B is not A.' is to be permitted to interpret linguistic interpretation of the syntax theory of logical square, we can get a new perspective about the arithmetization of 4 propositional forms. Leibniz's contradictory axiom will explain that Yin(陰) and Yang(陽) are indivisible, but when they are divided, that they can be differentiated into Yin Yin, Yin Yang, Yang Yin, and Yang Yang which are called 4 Sasang(四象) Elements. Using the axiom of contradiction in the standard of the four proposition types of Aristotle's logical square, it can be interpreted that the Sasang symbols ●●, ●○, ○●, ○○

correspond with Glashoff's C_+ language C_-C_- , C_+C_- , C_-C_+ , C_+C_+ , Sotirov's $s(-p) = 0$, $sp = 0$, $sp \neq 0$, $s(-p) \neq 0$ in the arithmetic operation $= \circ \cdot$, and Sommers' notation $-S - P$, $-S + P$, $+S - P$, $+S + P$, where Leibniz's arithmetization of Aristotle's syllogism shows the same logical structure on the binary language of 00, 01, 10, and 11 each other.

I. INTRODUCTION

In this article I will explore a new possibility of approach to discussions on the axiomatic composition of formal language in Leibniz's works A. VI. 4A, N. 56, 57, 58, 59, 60, 61, 62, 63, 64 in the spring of 1679.² After four years of stay in Paris, Leibniz settled in Hannover in 1678, working for constructing an artificial language that guarantees truth and certainty of statements within the frame based on an arithmetic system with characteristic numbers and universal characters. In the articles above, Leibniz algebraized Aristotle's logic with three types of models, which provided the fundament of development of modern mathematical logic and the base of the facility of applied logic. His transformation of Aristotle's syllogism into modern algebra followed the idea of Cartesian universal mathematics and Hobbes' thinking as reckoning in the world which is to make human thinking perfect.³ His goal is to establish a computable logic as a branch of formal logic through construction of characters and numbers between people and people, people and animal, people and things, and things and things. For this purpose, he constructed syntax and

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²The target of my research is N.56, N.57, N.58, N.59, N.60, N.61, N.63, N.64 in the Academy edition of Leibniz's works, A VI 4A, 195-244. It is linked to <http://www.uni-muenster.de/Leibniz/>. Thanks to the director of the Leibniz Institute and co-editors for allowing the use of Leibniz's works.

³Kneale, W. and Kneale M.(1962), 511.

the semantics of the artificial formal language which could be reduced to arithmetic operations on the logical square. If an artificial language can be correctly constructed with symbolic signs and numbers, it can realize deductive logical formalism within the arithmetic system. The computer he designed is also based on four arithmetic operations, and is closely related to the original draft of artificial languages used in today's worldwide web language in information and communication technology fields, as well as in the field of cryptography.

Couturat, L.(1901) saw Leibniz's attempts to arithmetize the Aristotle syllogism as a failure.⁴ Lewis C. S.(1918) and Jørgensen J.(1931) could not evaluate his achievement of the works in 1687 because of various mistakes in his previous manuscripts. Among the research conditions in which the authenticity of an edition of Leibniz's works was insufficient, the Polish logician Lukasiewicz, J. I.(1951) overcame the view of previous Couturat's opinion.⁵ He assessed that Leibniz implemented the inference of deduction, the law of contradiction, and subalternation rule, in which is completed the arithmetization of Aristotle's syllogism in N. 64 as the most perfect form. As a recent study, Marshall, D.(1977), Thiel, C.(1980), Lenzen, W.(1999), Zalta, E. N.(2007), and Malink, M. & Vasudevan A.(2016), etc. have researched the semantics and the syntactic system of Leibniz's 1686-90's works with focusing on the modern set theory in a literary approach through critical editing of Leibniz's complete works.⁶ Although they showed an axiomatic consistency, their starting points are due to the achievement of intensive transformation of logic through the algebraic operation attempted in the above works. Sotirov, V.(1999) and Glashoff, K.(2002) evaluated positive + and negative - quantification of Aristotle's syllogism through Leibniz's algebraic strategy as an axiomprovable theory, and Sommers F.(1982) understood it based on natural language.⁷ I will present a new approach to + - quantification strategy of the logical square which Leibniz intended in arithmetization of Aristotle's syllogistic, in so far as the axiomatic

function of + - is to be interpreted correspondingly to the symbolic construction of I-Ching logic.

II. HUMAN THINKING ALPHABET AND LOGICAL TRUTH

Leibniz sees logic as a tool for discovery (ars inveniendi) and judgment (ars iudicandi) in science and philosophy of 17th century. For him, logic plays an important role in knowledge production through discovery, and facilitates the expression of knowledge through judgment, for example, in law or probability. So, he thought that it was necessary to perform the function of logic for his contemporary science systematically classifying, compiling in a new encyclopedic system. For example, in astronomy, any logical inference about the lunar eclipse could not be well performed with existing methods of peripatetic syllogistic system assumed as the symbol 'A is a lunar eclipse, B is obscured by the earth, and C is the moon.'

1. major (B is A), minor (C is B), conclusion (C is A).

In the case of syllogistic inference with A, B, and C, the semantic composition of the linguistic image between the geocentric and heliocentric systems is inevitably different.⁸ So, Leibniz uses often to apply in the phase of paradigm shift of the heliocentric world view the principle of substitution (salva veritate) that pioneered Frege's principle to the syntax of the repetitive statement system as the logical object on the rising and setting of Venus.⁹ Following Hobbes's idea who thinks human thinking only as reckoning which goes on mechanical programming of adding or subtracting, he introduces R. Lull's the idea of Ars Magna who anticipated his plan that everything can be discovered and judged by a comparison of letters of the alphabets and an analysis of the words made from them.¹⁰ Leibniz presupposes the existence of original human thinking alphabets that cannot be defined in

⁸Galilei, G.(1610), Sidereus Nuncius, Galilei used a telescope to draw shapes of the moons of Jupiter that could not occur in the geocentric theory, and then derived the basis for supporting the heliocentric theory through a new interpretation of the symbolic meaning about the various figures. Frege completed a syntactic system that was formally valid by applying the principle of substitution for mentions and indications on the same ontological basis for the names called at dawn and at night for Venus. Burkhardt, H.(1980), 258.

⁹Discours de metaphysique § 27.

¹⁰Welch, J. R.(1990), 75.

⁴ Couturat, L.(1901), 77-82.

⁵Lukasiewicz, J.(1950), 126.

⁶Marshall, D.(1986), Lenzen, W.(2004, 2005), Zalta, E. N.(2016), Malink, M. & Vasudevan A.(2016).

⁷ Glashoff, K.(2002-1), 5. Glashoff, K.(2002-2), 6. Sotirov, V.(1999), 388. Sommers, F.(1993). 169-82.

concept and can no longer be verbal analysis.¹¹ For example, if a, b, c, and d terms are assumed to be for human thinking alphabets primitive concepts, a formal calculus of them goes on through analysis and synthesis with addition, multiplication, division, and subtraction of such concepts. For discovering species concept of y among them, it can be started by assuming the highest class with universal characters or symbols. The analysis goes on through reductive resolution: $ab = l$, $ac = m$, $ad = n$, $bc = p$, $cd = r$, and $abc = s$, $bad = q$, $abd = v$, $acd = w$, $bcd = x$.¹²

2. (a, b, c, d)

3. ($ab = l$), ($ac = m$), ($ad = n$), ($bc = p$), ($cd = r$)

3. ($ab = l$), ($ac = m$), ($ad = n$), ($bc = p$),)

4. ($abc = s$), ($bad = q$), ($abd = v$), ($acd = w$), ($bad = x$)

Then, each individual's thought can be expressed in a judgment form of the 'S is P' that reckons process ascending or descending from the highest genus concept to the lowest approximate species concept. So, all human thinking alphabets go up on decomposition of concepts between the subject and the predicate from the species difference to the recent genus. So, human thinking performs only reckoning mechanical programming of adding or subtracting, where the reverse order of analysis is synthesis. The synthesis and decomposition go on either upwards or downwards, where higher complex concepts are synthesized with low simple concepts. And small numbers are multiplied so that the large number could be divided into small numbers.

In this way, a formal language plays a pivotal role in the development of the quantification theory of modern logic, where he worked at logical truths only with definition and identity.¹³ The basis of

¹¹ A VI 4A, N. De alphabeto cogitationum humanarum, 279. Alphabetum cogitationum humanarum est catalogus notionum primitivarum, seu earum quas nullis definitionibus clariores reddere possumus. Kauppi, R.(1960), 39-40. Burkhardt, H.(1980), 94-96. Mates, B. (1986). 48-54, 58. Swoyer, C.(1994). 4.

¹² A VI 4A, N. 129. De synthesisi et analysi universali seu arte inveiendi et iudicandi. 538-9.

¹³ N. 47 starts with the definition and proves logical truth with identity. Assign 2, 3, 5, and 24 to a, b, c, and d, but define (a) a as b c d, making (b) $24 = 2.3.5$. The conceptual synthesis of b, c, d occur in $bc = l$, $bd = m$, $cd = n$, 1, which numbers are $2.3=6$, $2.4=8$, and $3.4=12$.

modern mathematical logic and computational logic lies in 1679's works, in so far as he transformed Aristotle's syllogism into arithmetic algebra through adding, subtracting, multiplying, and dividing operations.¹⁴

III. + -ALGEBRA ON THE LOGICAL SQUARE

Aristotle founded the syllogism system in Analytica Priora through the four types of propositions which are positive universal UA, particular PA and negative universal UN, particular PN. They maintain a formal system in accord with their quantity and quality on the logical square. Leibniz sets up 'S is P' as a standard calculus form, where the subject is S, the predicate is P, and a, i stand for positive universal or individual quantity, and e, o for negative universal or individual quantity. The copula 'is' is laid between the subject term and the predicate term, where it performs four arithmetic and logical operations on the logical square. The copula functions primarily either + or - as a sign of the quality of propositions. Secondly it indicates the difference or identity between adding and subtracting in the calculation of propositions.¹⁵ For the transformation of the four types of propositions into a formal deductive reasoning system, Leibniz constructed an arithmetic form $\frac{S}{P}$ in which the

And then, $a = bcd$, $a = ld$, $a = mc$, $a = nb$, so $24 = 6.4$, $24 = 8.3$, $24 = 12.2$. Leibniz proves the law of identity as follows. Provide (C) $a = a$. If $a = b$, then $a = b$. (C) $a = a$. If $a = b$, then $a = b$ is certain. (C)-1. If $a = b$ is true, then $a = b$ is certain. (C)-2. If $a = b$ is certain, then $a = b$ is true. (C)-3. If $a = b$ is certain, then $a = \text{non } b$ is false. (C)-4. If $a = \text{non } b$ is false, then $a = b$ is certain. (C)-5. If $a = \text{non } b$ is false, then $a = b$ is certainly true. (C)-6. If $a = \text{non } b$ is true, then $a = b$ is certainly true. (C)-7. If $a = \text{non } b$ is certain, then $a = \text{non } b$ is true. In conclusion, if the subject and the predicate are the same, the true proposition is $a = a$. So, logical truth is proven with the definition of identity through an algebraic operation.

¹⁴ Peckhaus, V.(2004), 4-9. Peckhaus, V. divides the logic tradition of beginning 20 century Frege to Goedel into the logic of ratiocinator and lingua characteristic, the former as computational logic and the latter as symbolic logic. He points out that both come from Leibniz, and seems to have influenced Frege's and Schroeder's theory of quantification.

¹⁵ Parkinson, G. H. R.(1966), 3.

subject number divides the predicate number and drops it without a quotient. For a constructive logarithm of this arithmetic transformation, he allocates characters and numbers to subjects and predicates in the form SaP, SeP, SiP, SoP, where a, e, i, o indicates quantification of all, some, any, no. The UAs presented by $vS = rP$ through introducing indefinite terms v and r . The PA gets the logical disjunctive form $\frac{S}{P} \vee \frac{P}{S}$ in which the subject is divided by the predicate, or the predicate is divided by the subject. Since the subject and the predicate in 'S is P' are interchangeable in its position, 'some' is expressed as variable in $rS = vP$ or $vP = rS$.¹⁶

The PN is expressed in $vS \neq rP$ or $\neg(\frac{S}{P})$, where the logical form is $\neg \frac{S}{P} \vee \neg \frac{P}{S}$. The UN is expressed as $rS \neq vP \wedge vP \neq rS$, where its logical form is $\neg \frac{S}{P} \wedge \neg \frac{P}{S}$.

5. UA: $\frac{S}{P}, (vS = rP)$

6. UN: $\neg(\frac{S}{P}), (rS \neq vP) \vee (vS \neq rP)$

7. PA: $\frac{S}{P} \vee \frac{P}{S}, (rS = vP) \vee (vS = rP)$

8. PN: $\neg \frac{S}{P} \vee \neg \frac{P}{S}, \neg(rS \neq vP) \vee \neg(vS \neq rP)$

5, 6, 7, and 8 are arithmetic conditions that satisfy the formal deductive reasoning system on the logical square of Aristotle's syllogism. Leibniz accepts proper names in the form of propositions, while Aristotle did not use proper names as subjects in the syllogism, quantifies singular propositions into general propositions. For example, if Paul is put in S in 'S is P', the subject performs the function of describing the properties of a predicate or affirming or negating a concept or proposition, and one Paul is quantified as all Paul. So, all propositions entering a logical square are algebraically quantified and deconstructed in the calculus form 'S is P'. If the object language 'human is a rational animal' is put into the 'S is P', then it can be transformed into an independent arithmetic proposition. His big idea is to construct a prime number system in the SaP, the SiP, the SeP, the SoP which are quantified and deconstructed algebraically on the logical square. When this plan is realized, so logic can develop into an independent computational system on the logical square. For such computable construction, Leibniz uses the basic property of

arithmetic that every positive integer can be factorized exactly as the product of one or more primes and that all positive integers have one unique prime factorization. The object language like 'Human is a rational animal' can be calculated in the 'S is P' form, where S as a species called 'human' and P as a 'rational animal'. The copula as '=' determines the quality of a given proposition. Then, if the number 2 and character a are assigned for 'animal' and the number 3 and character r for 'rational', we get $6 = 2 \cdot 3$ and $h = ar$.¹⁷ In this case of $2 \cdot 3 = 6$, 6 is decomposed by 2 or 3, and is synthesized by multiplying by 2 and 3.

9. $h = ar, 6 = 2 \times 3$

Here, the high concept assigned to 6 is divided by the lower concept of 2 or 3, and the lower concept of 2 and 3 multiplies each other so that the higher concept of 6 is raised. The composite number of 'S is P' is implanted to a calculation system that adds or decreases the composite concepts, in so far as the number of subject concepts is divided by the number of predicate concepts, and the number of predicate concepts multiplies each other. The problem is the quantification of 'all' and the logarithm of the indefinite quantization range 'some' in the addition and subtraction operation. Leibniz has introduced $+$ and $-$ sign to show the process of quantification on the logical square in N. 60. The $+$ and $-$ sign can't be interpreted here as well as today's understanding of positive and negative number systems. The insertion of $+$ and $-$ sign in front of characters and numbers can be interpreted as explicit indicating divisibility or rationalization between the subject and the predicate in the range of integers.¹⁸ Glashoff, K. interpreted $+$ and $-$ sign on a pair of natural numbers as a conjunction of the subject and the predicate as terms,¹⁹ Sommers, F. understands it from the point of view of natural language as positive or negative expressions²⁰, and Sotirov, V. evaluated that the arithmetization strategy according to the basic theorem of arithmetic was successful.²¹ When the sign $+$ and $-$ is assigned to the subject concept and predicate concept, the whole proposition can be calculated as $+$ and $-$ system on the logical square. Even Leibniz worked intensive to avoid contradictory expressions in $+$ and $-$ system as logical truth, his efforts are not so much supported in

¹⁶Sotirov, V. (1999). 389. Sotirov, G. thinks that Leibniz's number of universal characters can be reasonably constructed for the UA and the PA and the number of S and P at the upper limit in $S = \frac{1}{2}n(n+1)$.

¹⁷ Burkhardt, H. (1980), 123.

¹⁸ Lenzen, W. (2004), 31-4.

¹⁹ Glashoff, K. (2002-2), 6.

²⁰Sommers, F. (1982, 1990, 1993).

²¹Sotirov, V. (1999), 389.

regarding to East Asian philosophy tradition. So, I intend in this paper to engrave his implications and consequences of + - operation on the logical square through a new approach to Leibniz's axiom of contradiction in comparing to I-Ching texts.²²

3. 1. First model

Leibniz conceived the arithmetization of syllogism since 1678, but in April 1679 worked intensively on nine papers.²³ They are divided in three models: N. 56, then N. 57, 58, 59, and N. 60, 61, 62, 63 and 64.²⁴ The arithmetical construction of the subject and the predicate goes on that the composite numbers of subject express the sum of the multiplication of prime numbers in accord with concepts of the predicate, insofar as he presupposes a series of prime numbers 2, 3, 5, 7, 11, 13, etc. omitting 1 in the range of the predicate. It is the same theoretical background, the specific difference in recent approximate genus to define, describe and discover in the tradition of philosophy and for learning prime factorization in school mathematics today.²⁵

N. 56 deals with numbers of dividing $S(\lambda)$ by $P(\mu)$ in the pair of (λ, μ) , in accord with the proof of the uniqueness of the basic theorem of arithmetic that every positive integer can be uniquely factorized as a product of primes. The UA is treated as the positive quality of the proposition in a form $\frac{S}{P}$, where the subject is divided by the predicate like $\frac{6}{3} = 2$. The $\frac{S}{P}$ in (λ, μ) means that it is true when the number of numerators is divided by the number of denominators and falls without a remainder.²⁶

The UN is denoted as $\neg\left(\frac{S}{P}\right)$, where the number of subjects cannot be divided by the number of predicates and the number of predicates cannot be divided by the number of subjects. The PA is denoted by $xS = yP$ as $\frac{S}{P} \vee \frac{P}{S}$. These disjunctive form

means that the number of predicates is divided by the number of subjects or the number of subjects is divided by the number of predicates. The PN is presented as $\neg\frac{S}{P} \vee \neg\frac{P}{S}$. This disjunctive form means that the number of the subject cannot be divided by the number of predicates or the number of the predicate cannot be divided by the number of the subject, where the process of dividing shows the reverse order of the process of multiplying.

However, Leibniz transforms the UA in 'Every H is A' according to sub alternation rule into an algebraic form $\frac{H}{A} = r$ or $H = ar$, where H and A stand for man and animal. The PA 'Some A is H' is presented also as $\frac{H}{A} = r$ or $\frac{A}{H} = t$.²⁷ The PA is either $H = rA$ or $A = tH$.²⁸ The UN 'No H is L' is transformed also according to sub alternation rule as to $\frac{H}{L} = \frac{q}{z}$,²⁹ where the PN 'Some A is not L' is presented as $\frac{H}{L} = \frac{q}{z}$ or $\frac{H}{A} = \frac{v}{r}$.³⁰ But, the UN 'No H is L' is transformed again with another characteristic letters into an algebraic form as $\frac{H}{L} = \frac{q}{z}$, and the PN as $\frac{A}{H} = \frac{v}{r}$ or $\frac{H}{L} = \frac{q}{z}$, where L stand here for stone, and v, r, q, and z are variables as a kind of quantifiers.

$$10. S(\lambda) : P(\mu) = \frac{S}{P}$$

$$11. SaP: \frac{H}{A} = r \text{ or } H = rA$$

$$12. SeP: \frac{H}{L} = \frac{q}{z} \text{ or } (zH = qL)$$

$$13. SiP: H = rA \text{ or } (A = tH) \text{ or } \frac{H}{A} = r \text{ or } \frac{A}{H} = t$$

$$14. SoP: \frac{H}{L} = \frac{q}{z} \text{ or } \frac{A}{H} = \frac{v}{r} \vee (rA = vH)$$

The first model is related to arithmetic forms in which the subject relates to the whole and the predicate to the parts, as well as the case is practiced in the tradition of classical logic. Leibniz assumes probably some integer Γ ecthesis ($\xi\kappa\theta\epsilon\sigma\iota\varsigma$) method and interprets the SaP as $H = ar$, the SeP as $vH = \rho B$, the SiP as $v^\lambda H = r^\mu A$,

²²<https://ctext.org/book-of-changes/xi-ci-shang/zh>.

²³ Lenzen, W.(2004), 7-8. Burkhardt, H.(1980), 322.

²⁴ Glashoff, K.(2002-2), 161.

²⁵ Glasshoff(2002-1),4. Eccthesis($\xi\kappa\theta\epsilon\sigma\iota\varsigma$) is a reasoning method used in the proof of syllogism Darapti, Datisi, Disamis, and Bocardo.

²⁶ A VI 4A, 182-93. UA. $\frac{S}{P}$ succedit, id est numerus S dividi exacte potest per numerum P. PA. vel $\frac{S}{P}$ vel $\frac{P}{S}$ succedit. UN. neque $\frac{S}{P}$ neque $\frac{P}{S}$ eccedit. PN. $\frac{S}{P}$ non succedit.

²⁷ $vH = rA$ in PA, where v and r are to multiply each other to λ and μ .

²⁸ A VI 4A, 185. If $r = \frac{m}{n}$, $t = \frac{x}{w}$, then $\frac{H}{A} = \frac{m}{n}$ and $\frac{A}{H} = \frac{x}{w}$.

So $nH = mA$ and $xH = wA$. It means that $mx = nw$ and $\frac{n}{m} = \frac{w}{x}$. If $x \times t = 1$, then $A = H$.

²⁹ Leibniz uses L, H in the UN and A in the PA as constants and q, z, v, r as variables. He accepts the sub alternation from the UA to the PA, and from the UN to the PA.

³⁰ Glashoff, K.(2002), 161.

and the SoPasA = vH. The H = arof the SaPis to interpret as a kind of logarithm of $v^\lambda H = r^\mu$ of the SiP.³¹

- 15. UA: H = rA
- 16. UN: vH = ρB
- 17. PA: $v^\lambda = r^\mu A$
- 18. PN: ρA = vH

Although Leibniz fully recognized his goal of algebraic syllogism in N. 56, deleted a couple of sentences, and restored them to make a progressive work.³² But, at the end, he ended with sketching his own portrait.³³ It seems an expression of dissatisfaction with the problem of the existential import of all proposition in Aristotle's logical system and the method of proving the uniqueness of the basic theorem of arithmetic. While Aristotle used the law of identity to state a special truth condition that relies only on a syllogistic logical form, Leibniz used only definition and identity to formulate logical truth. But, because the first model presupposes a coherent background theory of numbers, Leibniz's next step is needed to show the process from the UA to the PA for a computable logic.

3. 2. The second model

N. 57, N. 56 and N. 59 belong to the second model. In N. 57, Leibniz analyzes 'Sage believes'.³⁴ The syntactic standard form of this sentence is generalized in 'S is P' by in a three-step procedure. The 1. 'the sage believes.' is transformed into 2. 'the sage is the believer'. From 2. comes 3. 'S is P'. In the 'S is P', when human h, animal a, rational r, as algebraic symbols and characteristic numbers 6, 2, 3 are assigned, their appropriate arithmetic and algebraic expressions are $h = ar$ or $6 = 2 \times 3$. When the metal as m and 3, and the heaviest properties as l and 5 are assigned in the form 'S is P', the combination of the two concepts yields gold, its character s. The composite number 15 is

³¹ A VI 4A, 185. $v^\lambda H = r^\mu$ seems to be considered as a kind of logarithm of $H = ar$. This is the case, if $\lambda=0$. $v^\lambda = 1$, then $\mu^0=1$. $v^0 = 1$, where $v^0, v^1, v^2, v^3, etc.$ are expressed as 1, v, vv, vvv, etc. $r^\mu = r$, if $\mu = 1$.

³² A VI 4A, 184. Leibniz deletes the UA for $\frac{H}{A}$ as $H = rA$, the PA for $\frac{H}{A} = r$ or $\frac{A}{H} = t$ as $H = rA$ or $A = tH$, because $rt = 1$, the UA is the same as the PA.

³³ Glashoff, K. (2002), 161.

³⁴ A VI 4A 195. Itaque cum dicitur sapiens credit, terminus erit non credit, sed credens, idem est ac si dixissem sapiens est credens.

expressed as $15 = 3 \times 5$ or $s = ml$. Leibniz seems often to have taken minerals for a logical analysis based on his working experience at the Harz silver mine from 1680 to 1686, where he might gather lots of dates of geology.

N. 58 handles with the expression as $\frac{b}{a} = y$ or $b = ya$, where a, b and y stand for man, animal and indefinite number. It means that y determines the quantity of $\frac{b}{a}$. The UN is $b \neq l$, if $b = \pi l$ or $\pi = \frac{b}{l}$, where h is human and l is stone. Because π may be infinite, the UN 'no man is stone' is presented like $\frac{\alpha\beta\gamma}{\delta\epsilon} = \frac{f}{g}$, where $\alpha\beta\gamma = f$ divides $\delta\epsilon = g$. Here, it means that δ isn't contained in $\alpha\beta\gamma$ and π indicates semantically destruction of stone or non-stone.³⁵ But, Leibniz doesn't consider the expression 'some meteorological phenomena isn't snow' or 'some stone isn't a man' as the PN, and try to eliminate such kind of negative characteristic numbers as non-existence. So, he does not deal negative concepts of incompatible concepts with minus signs, but in case of man and non-man, with root sign like \sqrt{aa} .

In N. 59 are presented arithmetic expressions the UA: $H = rA$, the UN: $yH \neq rB$, the PA: $rA = vH$ and the PN: $H \neq rA$ on the logical square.³⁶

- 19. $s = \frac{n}{m}$, $n = sm$
- 20. UA: $H = rA$
- 21. UN: $yH \neq rB$
- 22. PA: $rA = vH$
- 23. PN: $H \neq rA$

Glashoff, K. interprets the four types of SaP, SeP, SiP and SoP in Leibniz's arithmetic system as a divisibility relation in the set of positive integers of the rational number domain in \mathbb{N}^2 . For him, according to Corcoran's Natural Deduction Theorem, Leibniz's prime number system is a partially ordered set of arbitrary integers for a and b whose greatest common divisor is 1, in so far the UA is interpreted as $\frac{n}{m}$ in (n, m) . Assuming a certain number ξ of Γ in (λ, μ) and (n, m) , the PA is $\frac{\xi}{\lambda} v^{\frac{\xi}{\lambda}}$ that is satisfied with the arithmetic condition $\mu \ln \lambda = m \mu$, when $m = 1$ or $n = 1$. But, it is not being able to show numbers ϕ, γ that guarantees the validity of $\neg UN(\lambda, \mu) := PN(\phi, \gamma)$ from $H \neq rA$ of the

³⁵ A VI 4A, 207. Small letters, s, t, v, w, x, y, etc. stand for prime numbers or uncertain nonprime numbers, while letters a, b, c are integers.

³⁶ A VI 4A, 218.

UN. Although Glashoff, K. could show a numerical composition that compensates for the weakness of arithmetic algebra³⁷, arithmetic expressions couldn't be solved in algebraic symbols, the problem of arithmetic falsehood due to the displacement of the opposite order on the logical square should be viewed as diagrammatic explications before the number.

3. 3. The third model

N. 60, N. 61, N. 62, N. 63, and N. 64 belong to the third model. N. 64 presents the most complete logarithmic form of Aristotle's syllogism, where the Latin, Greek, Hebrew are introduced as algebraic letters to expand the semantics of the logical square. N. 60 presents firstly a model of $S(+s - \sigma): P(+p - \pi)$ ³⁸ with assigning $+ -$ to two integer pairs for the subject and the predicate. However, because of the problems of numerical analysis including negative concepts and propositions³⁹, Leibniz excludes the analysis of the SeP and the SoP in N. 60. The model of $S(+s - \sigma): P(+p - \pi)$ indicates the divisibility relation of the divisors of the predicate with respect to the composite number of the subject, where $+s$ of the subject in the integer range divides $+p$ of the predicate and the $-\sigma$ of the subject divides the $-\pi$ of the predicate.⁴⁰ For example, assigned numerically,

humans (+130 - 3) show the divisibility of the subject with respect to reason (+10 - 7), animal (+13 - 5) of predicate, so it could be expressed in $\frac{+130}{+13+10}$ and $\frac{+35}{-5-7}$. The PA 'some piety (+10 - 10) are unhappy (+14 - 5)' is true because +10 is not dividable by +14, and -3 is not dividable by -5. The PA 'some happy men (+11 - 9) are miserable (+5 - 14)' is also true because +11 is not dividable by +5, and -9 is not dividable by -14. However, the UA is presented as $\frac{s}{p} \wedge \frac{\sigma}{\pi}$, insofar as s is divided by p and σ is divided by π . The arithmetic condition of the PA is $\gcd(s, \pi) = 1, \gcd(p, \sigma) = 1$, where either s doesn't include the divisor of p , or σ doesn't include the divisor of π . So, the PA is $\text{non}(\frac{s}{p})$ or $\text{non}(\frac{\sigma}{\pi})$.

N. 61 deals the UN that has $\gcd(s, \pi) \times \gcd(p, \sigma) > 1$ in $\neg \frac{s}{p} \wedge \frac{\sigma}{\pi}$, where the numerator is $\gcd(s, \pi) > 1$ and the denominator is $\gcd(p, \sigma) > 1$. The UN indicates that the subject s cannot divide the p and the subject σ cannot divide the predicate π . The PN is presented as $\text{non}(\frac{s}{p}) \vee \text{non}(\frac{\sigma}{\pi})$ or $\neg \frac{s}{p} \vee \neg \frac{\sigma}{\pi}$, so it has the divisor between s and π , and p and σ in $S(s, \pi)$ and $P(\sigma, p)$.

N. 62 refers to the rules that lead to sub alternation from the UA to the PA and the obverse relation of the UA for 'all wise men (+20 - 21) are piety (+10 - 3)' with implication of existence. The UA is obverted to 'no non piety person (+3 - 21) is a wise man (+20 - 21)'. Because +3 and -21, -10 and +20 in the UN do not satisfy the divisibility condition of the UA, the UN is true. In N. 63, introducing another numbers for wise men, the UA 'all wise men (+70 - 33) are piety (+10 - 3)' is true because it is $\frac{+70}{+10}$ and $\frac{-33}{-3}$. 'All human beings (+130 - 35) are rational animals (+10 - 7, +13 - 5)' is also true because it is $\frac{+130}{+13}$, and $\frac{-35}{-5}$.

³⁷Glashoff, K. (2002), 4. Glashoff, K proposes in a universal characteristic number $C+$ language set that has $(m, \mu) \rightarrow \frac{m}{\mu}$ as a pair of positive integers from each other in natural numbers m and n .

³⁸Lenzen, W. (2004), 19. I insert "∞" between the subject and the predicate as the same meaning for "=" or "∞". Leibniz uses "=" or "∞" as a same meaning since 1685. For example, $A \infty B$ means that A and B are the same or coincidence ($A \infty B$ significat A et B esse eadem vel coincidentia.).

³⁹Leibniz says that if 'all human beings are not stones' and 'all human beings are stones', both of them is the same judgment for contradictory expression. For example, according to a contradictory axiom 'B is A and B is not A' we can say, 'The PN is false, if the PN is true' or 'The PN is true, if the PN is false.' If we put human as b and stone as l , it is $b = \pi l$. At this time, the existence of the stone is decomposed in $\pi = \frac{b}{l}$.

⁴⁰Hongsungsa, Hongyounghee & Kimchangil (2011), 1-6. Korean mathematician Hong Jeong-Ha (1684-1727) gave a numerical solution for Leibniz's problem. He asks when

and where if two persons A and B walk the same road each day, 85 ri and 65 ri (1 ri is 0.392727km after old Korean measurement unit). According to Leibniz' algebraic program, if A and B are divided by obtaining the greatest common divisor of 85 and 65, then $\frac{85}{5} = 17, \frac{65}{5} = 13$ the relationship between the predicates $P(86, 13), P(65, 17)$ is established. Where (+-1105) is established, come $P(+85, -13)$ and $P(+65, -17)$. So, comes $85 = 6 \times 13 = 65 \times 17 = 1106$ as result. It means that A and B meet together in 1106 ri after the 13th day and 17th day.

Because the +10 of 'Some piety man(+10 - 3) is not a wise man(+70 - 33)' is not divisible by +70 and -3 is not divisible by -33, the PA is true. The PA 'Some piety man(+10 - 3) is not a wise man(+70 - 33)' is true because +10 is not divisible by +70 and -3 is not divisible by -33. The PN 'Some wise men (+70 - 33, +cdh - ef) are not happy(+8 - 11, +g - f)' is true because +70 is not divisible by +8 and -33 is divisible by -11. The PN 'Some wise men are not happy' is true because +70 is not divisible by +8 and -33 is divisible by -11. The UN 'No piety man(+10 - 3, +cd - e) is unhappy(+5 - 14, +l - cm)' is true because +10 and -14 have common divisors. Where Leibniz sets up some happy man(+11 - 9, +n - p), the PA 'Some happy man (+11 - 9, +n - p) are unhappy(+5 - 14, +l - cm),' is true because there is no common divisor between +(11, -14) and (-9,

24. $S(+s - \sigma) : P(+p - \pi)$

25. $UA: \frac{s}{p} \wedge \frac{\sigma}{\pi}$

26. $UN: \neg \frac{s}{p} \wedge \neg \frac{\sigma}{\pi}, \gcd(s, \sigma) > 1, \gcd(p, \pi) > 1$, there is no common divisor for (s, p) and (σ, π)

27. $PA: \neg \frac{s}{p} \vee \neg \frac{\sigma}{\pi}, \gcd(s, \pi) \times \gcd(p, \sigma) > 1, \gcd(s, \pi) = 1, \gcd(p, \sigma) = 1$

28. $PN: \neg \frac{s}{p} \vee \neg \frac{\sigma}{\pi}$, there is common divisor for (s, π) and (p, σ)

29. $UA: S(+20 - 21) : P(+10 - 3)$

30. $UN: S(+3 - 21) : P(+20 - 21)$

31. $UA: S(+130 - 35) : P_1(+10 - 5) \wedge P_2(+13 - 5)$

32. $UN: S(+70 - 33, +cdh - ef) : P(+8 - 11, +g - f)$

33. $UN: S(+10 - 3, +cd - e) : P(+5 - 14, +l - cm)$

34. $PA: S(+11 - 9, +n - p) : P(+5 - 11, +l - cm)$

35. $UA: S(+70 - 33, +cdh - ef) : P(+10 - 3, +cd - e)$

36. $UN: S(+10 - 3, +cd - e) : P(+5 - 14, +l - cm)$

37. $PA: S(+11 - 9, +n - p) : P(+5 - 14, +l - cm)$

N. 64 presents the most complete current algebraic arithmetic form $S\left(\frac{mp}{l}, \frac{ls}{m}\right) : P\left(\frac{\mu\pi}{\lambda}, \frac{\lambda\sigma}{\mu}\right)$ in $s = \frac{mp}{l}$, $\sigma = \frac{\mu\lambda}{\lambda}$, $p = \frac{ls}{m}$, $\pi = \frac{\lambda\sigma}{\mu}$, if the partial ordered sets of a, e, i, o of $S(+s - \sigma) : P(+p - \pi)$ are $ls = mp$ and $\lambda\sigma = \mu\pi$. The UA is true, if $l = 1$ and $\lambda = 1$. The PA is true in $s = mp$ and $\sigma = \mu\pi$, if $\frac{s}{\pi} \vee \frac{\pi}{s}$ in $s = \frac{mp}{l}$, $\pi = \frac{\mu\pi}{\lambda}$, $l > 1$, $\lambda > 1$. The UN is true, if (s, π) and (σ, p) are not each other prime and they have common divisor. The PN is true, if (s, π) and (σ, p) are each other prime and there is no common divisor. The UN 'No H is B' is presented as $\frac{\alpha\beta\gamma}{\delta\varepsilon} = f$ in $\alpha\beta\gamma = f$ and $\delta\varepsilon = g$, where f and g stand for human and stone. The PN is $H(+s - p, ls = mp) : (+p - \pi, +\sigma\lambda = \mu\pi)$. This is true, when $s = \frac{mp}{l}$, $\sigma = \frac{\mu\pi}{\lambda}$, $p = \frac{l}{m}$, $\pi = \frac{\lambda\sigma}{\mu}$, in $\frac{\alpha\beta\gamma}{\delta\varepsilon} = \frac{f}{g}$. Leibniz thinks that humans can think and stones cannot, so, it is

+5). The PA 'Some wise men(+70 - 33, +cdh - ef) are piety(+10 - 3, +cd - e)' is true because there is no common divisor between (+70, -3) and (+33, -10). The subalternation PA of UA 'all wise men (+70 - 33, +cdh - ef) are piety (+10 - 3, +cd - e)' is true because it is $\frac{-33}{-3}$. The UN 'No piety man(+10 - 3, +cd - e) is unhappy(+5 - 14, +l - cm).' Therefore, in PN 'any piety man is not unhappy,' +10 and -14 has a common divisor. These PN is true because -3 is not divisible by -14, and -3 is not divisible by -14. In animals (+13 - 5), rational (+10 - 7), and human(+130 - 15), the composite number of the rational animal concept is 35 by multiplying -5 and -7, but it contradicts -35, so animals and reason are incompatible concepts for humans.

expressed as $\frac{human}{nostone} = \frac{f}{nog}$, where concepts of humans and stone are incompatible.

38. $S\left(\frac{mp}{l}, \frac{ls}{m}\right) : P\left(\frac{\mu\pi}{\lambda}, \frac{\lambda\sigma}{\mu}\right) = \frac{S}{P}$, $(s = \frac{mp}{l}, \sigma = \frac{\mu\pi}{\lambda}, p = \frac{ls}{m}, \pi = \frac{\lambda\sigma}{\mu})$

39. $UA: S\left(\frac{mp}{l}, \frac{ls}{m}\right) : P\left(\frac{\mu\pi}{\lambda}, \frac{\lambda\sigma}{\mu}\right) = \frac{S}{P}$, $(s = \frac{mp}{l}, \sigma = \frac{\mu\pi}{\lambda}, p = \frac{ls}{m}, \pi = \frac{\lambda\sigma}{\mu})$

40. $UN: \frac{\alpha\beta\gamma}{\delta\varepsilon} = \frac{f}{g}$, there is no common divisor between (s, π) and (σ, p)

41. $PA: S\left(\frac{mp}{l}, \frac{ls}{m}\right) : P\left(\frac{\mu\pi}{\lambda}, \frac{\lambda\sigma}{\mu}\right)$, $(l > 1, \lambda > 1)$, there is no common divisor between (s, π) and (σ, p)

42. $PN: \frac{\alpha\beta\gamma}{\delta\varepsilon} = \frac{f}{g}$, there is no common divisor between (s, π) and (σ, p)

43. $PN: S(+s - \sigma, ls = mp) : P(+p - \pi, \sigma\lambda = \mu\pi)$, $\frac{\alpha\beta\gamma}{\delta\varepsilon} = \frac{f}{g}$

Sotirov, V. points out the existence of Leibniz's universal characteristic number $u > 1$ in $(s < u, p < u)$, where S is a divisor of P in $gcd(S \wedge p)$, and λ in $U(\lambda, \mu)$ contains the smallest number that divides μ .⁴¹ Here exists the greatest common divisor between S and P . The UA indicates in above 38 and 39 that $\frac{S}{P}$ is composite numbers of λ and μ , and $s = xp$ is semantically conjunctive intersection of S and P . The UA is $S \subseteq P$, and the PA is $(S \cap P) = \emptyset$, where S and P are not empty. If there is a composite number of Leibniz's universal property number u in $S \subseteq P$, then the composite number of the UA is presented as $s = sp$, the UN as $s = s(\neg p)$, the PA as $s \neq s(\neg p)$, the PN as $s \neq sp$. Boole's algebraic expressions are $UA: s(\neg p) = 0$, $UN: sp = 0$, $PA: sp \neq 0$, $PN: s(\neg p) \neq 0$.

According to the natural deduction model of Cocoran, Glashoff, K. found in 39, 40, 41, and 42 syntactic form of $C_+ \times C_+$, $C_+ \times C_-$, $C_- \times C_+$, $C_- \times C_-$, which are grounded in four proposition forms SaP , SiP , SoP , SeP .⁴² Where the language set C_+ consists of the subject $C_+(\pi, p)$ and the predicate $C_-(\pi, p)$, the truth condition of the arithmetic system of the C_+ language has the same structure of the material implication of the propositional logic, which are false only if the

⁴¹According to the research tradition of Lukasiewicz, J. and his disciples Slupecki on the truth condition of the transition from the SiP to the SaP in Aristotle's syllogistic, Sotirov, V. considers the $\frac{S}{P}$ of the Aristotle syllogism as a recursive, transitive, and antisymmetric partially ordered set. For him, if the greatest common divisor of (S, P) is greater than 1 and the least common multiple is less than u , Leibniz's arithmetic is maintained. Regarding to a term of universal set U , $\neg t$ is the complement set of U in $u > 1$, where 1) every integer is a divisor different from 1 and u , and 2) $\neg t$ is $\frac{a}{a}$ in terms of the integer a .

⁴²Sotirov, V. showed that when the UA has a universal number among integers greater than 1, and the PA has a greatest common divisor with the number of subjects and predicates greater than 1, Leibniz's algebraic is proven.⁴² In Glashoff, K. and Sotirov, V., if there is a multiplier between the subject and the predicate in the UA and the division number in the PA , and both have a common divisor with the number of subjects and predicates greater than 1, Leibniz's arithmetic is valid.

antecedent is true and the latter is false, and is true in any other case.

In the third model, the UA holds if $l = 1$ and $\lambda = 1$. The PN holds if $1 > 1$ or $\lambda > 1$. The UN holds if (s, π) and (σ, p) are each other prime or have no common factor. The PA holds if (s, π) , and (σ, p) are mutually prime or have no common factor.

44. $UA: s = sp$, $UN: s = s(\neg p)$, $PA: s \neq (\neg p)$, $PN: s \neq sp$

45. $UA: s(\neg p) = 0$, $UN: sp = 0$, $PA: sp \neq 0$, $PN: s(\neg p) \neq 0$

46. $C_+ \times C_+$, $C_+ \times C_-$, $C_- \times C_+$, $C_- \times C_-$

IV. SOMMERS' PROPOSAL

Sommers, F. sees that all concepts have a positive + or negative - function, and in particular, natural language reveals very well the two functions of copula.⁴³ All-natural language expressions are loaded negative or positive. They are computed through logical terms. Even in everyday life language, logical linking words compute linguistic judgments as + - characteristics. The words 'is', 'some', 'both', 'and', 'what', 'then' belong to + operations, while 'any', 'all', 'no', 'are not', and 'if' are - operations. Even a child who doesn't learn logic understand 'all dogs are meek' as negative because boys and girls read 'all' as - function and 'be docile' as + function in terms of computation. An original intuition for quantified expression as 'everyone' does not see 'everyone' as 'everyone'. All this arithmetic function is already hidden in natural language. The natural expressions have nothing to do with any concept of all quantity and individual quantity in sense of Aristotle's logic. All quantifier of Aristotle's logic is a negative expression regardless of the implications of existence, while existential quantification is related to a positive expression. The natural language performs the same computational relationship as artificial language in everyday life. So, Sommers' viewpoint of natural language is useful to interpret Leibniz's + - arithmetic system. According to his proposal, if we can interpret that 'all S is P' is $\frac{S}{P}$ and 'all S is not P' is $(\frac{S}{P})^{-1}$ in natural language

⁴³Sommers, F. (1993), 169-82. Englerbretsen, G. (2016), 269-91.

expression.⁴⁴ The *UA* is expressed as $-S + P$, the *UN* as $-S - P$, and the *PA* as $+S + P$. And the *PN* can be represented by $+S - P$. In general, if the *UA* is expressed as $S \supset P$, the *UN* as $\neg(S \supset P)$, the *PA* as $S \vee P$, and the *PN* as $\neg S \wedge P$. It corresponds to Sommers' notations, $-S+P$, $+S+P$, $+S-P$, $-S-P$. These syntax reflect logical symbols $S \supset P$, $\neg(S \supset P)$, $S \vee P$, and $\neg S \wedge P$ on the logical square.

47. *UA*: $-S + P$, *UN*: $-S - P$, *PA*: $+S + P$, *PN*: $+S - P$

48. *UA*: $S \rightarrow P$, *UN*: $\neg(S \rightarrow P)$, *PA*: $S \vee P$, *PN*: $\neg S \wedge \neg P$

⁴⁴ The syllogistic algorithm is $\frac{P}{M} \cdot (\frac{S}{M})^{-1} = (\frac{S}{P})^{-1}$ valid because of $(\frac{S}{P})^{-1} = \frac{P}{S}$, when B^{-1} is presupposed in 'All A is B'. So, there is no difference between the *UA* and the *PA*.

V. A NEW SYNTAX OF CONTRADICTION AXIOM

In N. 59, Leibniz applies the contradiction axiom of 'A is B' and 'A is not B' to the logical squares and formulates them as meta-truth predicates.

1. If it is true that B is A, then it is false that B is non-A: $(BA) := \neg(B \neg A)$
 2. If it is true that B is non A, then is false that B is A: $(B \neg A) := \neg(BA)$
 3. If it is false that B is A, then it is true that B is non A: $\neg(BA) := (B \neg A)$
 4. If it is false that B is non-A is, then it is true that B is A: $\neg(B \neg A) := (BA)$
- The (BA) , the $(B \neg A)$, the $\neg(BA)$ and the $\neg(B \neg A)$ are rearranged on the logical square as the same form the $\neg(B \neg A)$, the $\neg(BA)$, the $(B \neg A)$ and the (BA) .
49. $UA: \neg(BA) \rightarrow \neg(B \neg A)$
 50. $UN: (B \neg A) \rightarrow \neg(BA)$
 51. $PA: \neg(BA) \rightarrow (B \neg A)$
 52. $PN: \neg(B \neg A) \rightarrow (BA)$

Leibniz follows Aristotle that the UA and the UN cannot be both true and false at the same time. According to the opposition theory, the PA and the PN can be mutually false, but both can be also true. The 5-8, 11-18, 20-23, 25-28, 39-42, and 44-52 are relationally established according to Aristotle's opposite contradiction theory. Since upper universal propositions on the logical square maintain contradictory oppositions under particular propositions, it is $UA \leftrightarrow PN$, $UN \leftrightarrow PA$. The sub-random change goes down from the universal quantity to the individual quantity, the PA comes from the UA , the PN comes from the UN , so $UA \rightarrow PA$ and $UN \rightarrow PN$. Aristotle explains according to the theory of opposition theory that the UA and the PN , and the UN and the PA cannot be true or false at the same time. It is $UA = \neg PN$, $UN = \neg PA$.

53. $UA \leftrightarrow PN, UN \leftrightarrow PA$
54. $UA \rightarrow PA, UN \rightarrow PN$
55. $UA = \neg PN, UN = \neg PA$

In order to give this process validity, in the analysis of the categorical proposition of first-order logic, existential import is required that the subject term is not empty.⁴⁵ Leibniz's contradictory axiom

⁴⁵However, if the UA cannot characterize the number of negative concepts and the PA cannot handle the negation of the UN , then inference cannot be handled. This is the problem of the fundamental linguistic image (像) that was melted into everyday language before the symbols and

$B = A \wedge B \neq A$ ⁴⁶ explicates a dividable process of union between the subject and the predicate. In the Grand Terminus chapter of I-Ching, Taegeuk (太極) begins with two contradictory logical values, Yin and Yang, which is not divided. Its progress called Yangaeysasang (兩儀生四象). In Leibniz's contradictory axiom, $B = A \wedge B \neq A$ can be paraphrased as 'Yin is Yang and Yang is not Yin.'. The pole of Yin and Yang is exchanged, when Yang turns into negative - and Yin turns into positive +, insofar as Yang contains Yin and Yang does not contain Ying. When the meta-statement for B and A is transferred to Yin and Yang language and their linguistic image $\bullet\circ$, their algebraic symbolic formation could be expressed with Yin and Yin, Yin and Yang, Yang and Yin, Yang and Yang which is called Sasang. The Sasang $\bullet\bullet$, $\bullet\circ$, $\circ\bullet$, $\circ\circ$ show their own universal progress as symbolic forms, before they are revealed as a number. The root of their binary number lies in 00, 01, 10, 11. They reflect the same structure on the logical square, when the four arithmetic operations are converted into the binary symbolic expression \square and \square , where $+ -$ signs are replaced with the signs of 0 and 1. The Sasang is replaced with 00, 01, 10, 11, where 00, 01, 10, 11 are in accord with Sommers' $-S - P$, $-S + P$, $+S - P$, $+S + P$. The Sasang gives birth to 8 hexagrams (四象生八卦), which are generated by 16 truth tables in modern logic, where the truth table for p and q is established with four truth predicates (True, True), (True, False), (False, True), (False, False). This is the same structure for the Sasang.

56. $(\bullet\circ), (0, 1)$
57. $(\bullet\bullet, \bullet\circ, \circ\bullet, \circ\circ), (00, 01, 10, 11)$

From this truth predicates can be followed 16 truth grounds as the truth function of elementary propositions, where Wittgenstein formulated it in 5.1 of *Tractatus Logico-Philosophicus*. If Aristotle's syllogism will be replaced by Leibniz's four arithmetic operations, it is easy to see, that the frame of thoughts characterized by the truth table of p and q rooted in binary arithmetic. Here, a binary system in which no other numbers other than 0 and 1 appear can be expressed in Leibniz's logical square, where 0 and 1 are another expression of the $+ -$ computation

numbers of universal characteristics, and it seems to belong to metaphysics before the domain of natural language rather than in computational language.

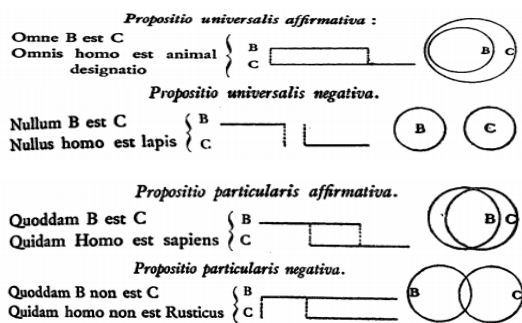
⁴⁶A VI 4A, 217 Termini contradictorii sunt, quorum unus est positivus alter negativus hujus positivi, ut homo et non homo.

reasoning that Leibniz conceived. Leibniz's another work in which he proved the logical square of the syllogism system with the Venn diagram shows also an analogy with the Sasang system.⁴⁷

VI. CONCLUSION

Leibniz's Arithmetization of Aristotle syllogism brings the birth of the computational logic which begins and ends with 0 and 1. This idea starts in Leibniz's research for the artificial language in the spring of 1678, which computes the universal characteristic number of the concept of the subject and the predicate according to three models through algebraically + - transformation on the logical square. Glashoff, K. and Sotirov, V. assessed that Leibniz's + - arithmetic work was successful, and Sommers, F. showed that expressions of logical calculation were possible even in natural language. Four propositional forms are expressed through arithmetic operations of addition, subtraction, multiplication, and division on the logical square. Leibniz applied the contradictory axiom to verify arithmetic platform of Aristotle's logical square. This

⁴⁷Couturat, L (1901), 292-298. Leibniz as Venn diagram's forerunner expressed the concepts of B and C as line segments and lowered the slash to indicate the overlapping portion as the quantity of propositions. The order of UA, PN, PA, UN godown from the top right on the logical square to the bottom, then going to the bottom right and going up to the top. It corresponds to the order ●○, ○●, ○○, ●●.



is absent in Aristotle and can be interpreted analogously in the I-Ching logic system. I pointed out that the Yin-Yang idea can contribute to forming the basis of the truth table of propositional logic on behalf of the basis of the logical square. Modern Computer based on four arithmetic operations through numbers and symbols in the alphabet of human thinking. The Yin and the Yang process of Taegeuk and propositional logical development of eight trigrams open up a new possibility of interpretation on Leibniz's + - quantification strategy on the logical square.

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 - N. 57 Elementa calculi.
 - N. 58 Calculi universalis elementa.
 - N. 59 Calculi universalis investigationes.
 - N. 60 Calculi consequentia.
 - N. 61 Modus examinandi consequentias per numeros.
 - N. 62 Regulae bonitate consequentiarum formisque et modis syllogismorum categoricorum judicari potest, per numeros.
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