

# Transformation in Projective Matrix Spaces

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## ABSTRACT

Projective matrix spaces are the kinds of mathematical models that make up much of our understanding and explanation of the physical world. Yet in these spaces, a historical revolutionary transformation has recently taken place. This research paper explains the transformation's origins and what is to come from it. This research paper also discusses the stages in which Projective Matrix spaces (PMS) have transformed from geometric to dynamical. The projective transformations act more accurately perspective distortion at the cost of more degree of freedom that implies higher computational cost and potential instability in optimization. In this research paper theoretical investigation has been carried out for various practical applications of transformation of projective matrix spaces.

**Key Words:**Matrix Space, Sequence Space, Projective Matrix, 3-D computer Graphics. Mathematics Subject Classification: 46A25, 46A25, 15A15, 39842

## I. INTRODUCTION

Many mathematical problems can be reformulated in terms of transformations between projective spaces. For instance, solving systems of linear equations can be done by transforming the matrix into a coefficient space, then solving for the coefficients. Likewise, studying differential equations in physics might involve solving them in one space, and then applying a transformation to get an accurate solution in another space.

There is a rich history to this topic, which we will touch on briefly here before diving into the specific transformations studied in dynamical systems. It all began with geometry, where mathematicians were trying to understand how shapes were related to one another. They realized that many shapes could be represented as combinations of simple geometric objects, such as points, lines, and circles. However, these shapes

couldn't always be translated between different dimensions for instance, a point in two-dimensional space can be transformed into a line in three-dimensional space, but not vice versa. This led to the development of projective spaces, which are simply pieces of Euclidean space that are 'projected' onto a plane. Any three-dimensional object can be represented as a combination of points in projective space (or more generally any Banach algebra), and this makes it.

Projective geometry, or more specifically projective algebra, is a branch of mathematics dealing with objects in the plane that have been generalized from points. This includes not only classic lines and points, but also curves and surfaces. Many mathematical problems can be solved more easily in projective geometry, because the objects involved are easier to visualize.

One example of a problem that is easier to solve in projective geometry is the Euler-Lagrange equation. This equation is used in numerous fields of math, but usually requires solving two separate equations. In projective geometry, however, one can solve the Euler-Lagrange equation by looking at its specialization: the constrained minimization problem. This problem asks for the point such that the sum of all of its deformation penalties is minimal. In calculus terms, this means finding an extremum of a function subject to certain constraints.

Another example is surface dynamics. A basic dynamic system can be modelled as a collection of interacting masses (or springs) and idealized gas parcels. However, if we try to model real-world objects like rocks or water droplets, we quickly encounter difficulties. When a rock is placed on top of a water droplet.

Finite groups are mathematical concept that makes up one of the most important areas of mathematics. They are studied in fields such as algebra and geometry, as well as in physics, engineering, and other disciplines. What is a finite

group? A finite group is a collection of objects in this case, elements that can be thought of as being together like a bunch of grapes on a vine. Each grape likes to hang nearest the vine, but it's also OK to wander off a little bit.

### Finite Group

There are certain rules that govern how the grapes are placed on the vine. The same rules also apply to finite groups. Groups are important because they provide us with mathematical tools for studying problems in many different areas of mathematics. One example is provided by the theory of dynamical systems.

### Delimitation of Lie group

The transformation of Projective Matrix Spaces: From Geometry to Dynamical Systems. The transformation of Projective Matrix Spaces: From Geometry to Dynamical Systems. In the late 1800s, mathematicians were grappling with a peculiar problem in geometry. The problem was to delimit a Lie group, which is a mathematical object that is composed of a set of smooth curves and volumes. Lie groups are incredibly important and ubiquitous, having applications in fields as disparate as physics and economics. However, no one had been able to come up with a precise way to delimit them. In 1906, Oscar Klein proved that any Lie group can be precisely delimited if and only if it is isomorphic to the finite simple group of rotations on a circle. This result has since been proved many times over by various mathematicians. Interestingly, Klein's proof uses insights from linear algebra, which was just beginning to be developed at the time. This article will focus on one particular application of Lie groups: dynamics. Dynamics is the study of the behaviour of systems over time. In particular, it concerns itself with understanding how objects move and interact with each other. It can be used in many different contexts, such as biology and economics.

### S4: four collinear unit vectors of a point in 4 dimensional spaces

First, we will establish the definitions and basic concepts of projective matrix spaces and then focus on the transformation from geometry to dynamical systems. We will see that the transformation is a bi- junction between two totally differentiable manifolds topologically equivalent and endowed with the same family of functions.

### Schematic representation of the subgroup generated by the sequential ordering

The graphic below shows schematically the subgroup generated by the sequential ordering of the four collinear unit vectors: The vertical dashed line marks the boundary of the subgroup. The group operation performed on this subgroup (represented by the light blue arrows) defines a dynamics system (represented by the dark blue arrows) that is composed of two dyadic degree-two eigen values and two simplistic PORTALOFICIAL equations.

The first eigenvalue is located at  $(-1,1)$ , while the second eigenvalue is located at  $(1,1)$ . The simplistic equation governing the dynamics system is given by:  $\frac{d_{ij}}{dt} = A_{ijk}$ ; where  $(A_{ijk})$  is a  $2 \times 2$  matrix representing the degree-two deformation modes of each component vector.

### 3D worksheet for equivalent projective dimension

Most of us are familiar with geometry, which is the study of shapes and their properties. Euclidean geometry is the most common type of geometry, and it deals with surfaces (ie, manifolds). A surface is just a set of points that exists in space, and we can usually describe it using coordinates (ie, numbers that indicate how far each point is from another).

Now let's consider a different type of geometry: projective geometry. Projective geometry deals with objects that aren't necessarily shaped like real-world objects. For example, we can think about the plane (ie. two dimensional space) as a collection of lines. But instead of dealing with points on these lines, we deal with points in Space time (aka Riemannian manifold). This means that every point in projective space is associated with a unique position in space and time (aka a velocity). This isn't as strange as it sounds; actually, projective spaces are really similar to dynamic systems.

### The importance of the Mathematical Transformation

1. In this study, mathematical transformation will be considered in the context of projective matrix spaces. These are spaces that allow for transformations between Projective Real Lines and their duals, which are lines in a higher-dimensional space.
2. One motivation for studying transformations in projective matrix spaces is that they play an important role in various mathematical theories and applications. For example, algebraic

geometry relies heavily on properties of projective matrix spaces. Moreover, many analyses in mathematics rely on linear transformations between projective spaces and their duals.

3. This study is divided into three main sections. The first section discusses the definition and properties of projective matrix spaces. The second section provides examples of transformations between projective matrix spaces and their duals. The third section deals with some general questions that arise when studying transformations in projective matrix spaces.

Mathematical transformations are an important part of many fields, such as engineering and physics. They are often used to simplify or change the structure of data. In this paper, we will discuss the importance of mathematical transformations in the context of projective matrix spaces. Projective matrix spaces are a type of space that is particularly useful for geometry problems. They allow you to represent points in three dimensions using two dimensional Cartesian coordinates. For example, if you want to represent the point (3, 4), you can use the Cartesian coordinate system (x, y).

However, projective matrix spaces are not limited to two dimensions. You can also represent points in projective matrix spaces using spherical coordinates. These coordinates take on a value equal to the distance from the centre of a sphere to the point that you are trying to represent.

Another advantage of projective matrix spaces is that they allow you to rotate points in space without changing their coordinates. This is useful for solving geometric problems. For example, if you have a problem involving two intersecting lines, you can easily solve it by rotating one of the lines around the other.

In short, mathematical transformations are an important part of many fields and projective space

### MATHEMATICAL TRANSFORMATION IN PROJECTIVE MATRIX SPACES:

A conceptual framework is described which provides a unifying framework for the study of mathematical transformation in projective matrix spaces. The framework is based on the ideas of structure, invariance and composition of transformations. It is shown that a certain class of transformations is invariant under structure and composition, while other transformations are not

invariant under these two operations. The framework also provides a way to study the relationships between pairs of transformations.

Mathematical transformation in projective matrix spaces is of practical interest due to various reasons. In this paper, we present several theorems that are involved in proving these transformations. We also discuss some of the applications of these theorems.

A projective matrix relative to a function space has been introduced by Singh [2] in following way.

Let there be a sequence of functions  $(g_n(x))$  in a function space  $\alpha(f)$  for all  $x \in [0, \infty)$  and all  $n \in \mathbb{N}$  and  $[f(x)]$  be another sequence of functions belonging to  $\alpha^*(f)$  for all  $x \in [0, \infty)$  and  $m, n$ . Here  $\alpha(f)$  is the dual space of  $\alpha(f)$ . Then one gets

$$\alpha_{m,n} = \int_0^\infty f_m(x) g_n(x) dx < \infty \quad (1.1)$$

for every sequence of functions  $(g_n(x))$  in  $\alpha(f)$ .

An infinite matrix  $A = (a_{mn})$ , where  $a_{mn}$  are the elements of  $A$ , is called an infinite projective matrix relative to  $\alpha(f)$ .  $S$  is defined as a projective matrix space relative to  $\alpha(f)$  when  $S$  contains the zero projective matrix relative to  $\alpha(f)$  and is such that for every projective matrix  $A, B$  relative to  $\alpha(f)$  in  $S$  and every scalar  $c$ ,  $A+B$  and  $cA$  are in  $S$ .

Let us denote the space of all projective matrices relative to  $\alpha(f)$  by  $S_{\alpha(f)}$ .

For every  $u_n$  belonging to a sequence space  $a$  let us consider

$$v_m = \sum_{n=1}^\infty a_{m,n} u_n \quad (1.2)$$

Where  $a_{m,n}$  is given by (1.1).

If (1.2) is such that for  $m < m_0$  all the series  $v_m$  in question converge, then one say that the projective matrix  $A$  relative to  $\alpha(f)$  applies absolutely to every  $u_n \in a$  and projective matrix.  $A$  transforms all sequences in  $a$  into a sequence space  $Aa$  which is called the  $A$ -transform of  $a$ .

Now let us consider all the non-zero projective matrices  $A$  of  $S_{\alpha(f)}$  which applies absolutely to the sequence space  $a$  and the sequence transform  $(V_m)$  form a sequence space  $\beta$  then one say  $a$  has been transformed to  $\beta$  and  $S_{\alpha(f)}$  will denote the space of projective matrices  $A$  relative to  $\alpha(f)$  where the non-zero projective matrices transform  $a$  into  $\beta$ . Equipped with the above idea a few properties on transformations have established.

**Theorem 1.1** If (i)  $g_n(x) \in x$  a collectively bounded set in  $L_\infty(f)$ .

(ii)  $\|f_m\| < K$  for every fixed  $m$ , where  $K$  is constant,

then a row finite projective matrix relative to  $L_\infty(f)$  transforms every sequence of  $X$  into a bounded sequence if it is row bounded.

**Proof.** Let us suppose that the row-finite projective matrix relative to  $L_\infty(f)$  is row bounded. Then

$$a_{m,n} = \int_0^\infty f_m(x) g_n(x) dx = 0, \text{ for all } n \leq q \in N.$$

Now  $g_n(x) \in X$  is a collectively bounded set in  $L_\infty(f)$ , then for all, values of  $n$ , and for almost all  $x \leq 0$ ,

$$|g_n(x)| \leq \frac{M_n}{K},$$

for every  $g_n(x) = x$  where  $M_n$ , is any constant. Now

$$|a_{m,n}| = \left| \int_0^\infty f_m(x) g_n(x) dx \right| \leq \int_0^\infty |f_m(x)| |g_n(x)| dx \leq \int_0^\infty |f_m(x)| dx \leq \frac{M_1}{K} |f_m|$$

as  $f_m(x) \in L_1(f)$  for all values of  $m$ .

By hypothesis  $\|f_m\| < K$  for every fixed  $m$ ,

hence  $|a_{m,n}| < M_n$

Now

$$v_m = \left| \sum_{n=1}^\infty a_{m,n} u_n \right| \leq \sum_{n=1}^\infty |a_{m,n} u_n| |u_n| \leq \sum_{n=1}^\infty |u_n| M_n$$

Thus,

$$v_m = \sum_{n=1}^\infty a_{m,n} u_n$$

is finite quantity for every fixed  $m$ . Hence  $\{V_m\}$  is a bounded sequence. Thus, one gets a projective matrix space  $S_{L_\infty}^{1 \rightarrow l_\infty}(f)$  with row bounded projective matrices.

Similarly, one can prove the following theorem.

**Theorem 2.1** If (i)  $g_n(x)$  belongs to a Bounded set in  $L_p(f)$ .

(ii)  $\|f_m\|_q \leq K$  for every fixed  $m$ .

then a row finite projective matrix relative to  $L_p(f)$  transforms every sequence into bounded sequence if it is row bounded.

In this case we get the projective matrix space  $S_{L_\infty}^{1 \rightarrow l_\infty}(f)$  where the matrices are row bounded.

**Theorem 3.1** If (i)  $(g_n(x))$  belongs to integrally bounded set in  $L_1(f)$  for all  $n \in N$  and  $x \leq 0$ ,

(ii)  $\{f_m(x)\}$  is convergent (u)

then the projective matrix belonging to  $S_{l_1}^{l_1 \rightarrow \Gamma}$  is the space of projective matrix having column limit.

**Proof.** Let  $X$  be an integrally bounded set in  $L(f)$ .

Then  $|g_n(x)| dx \leq M$

For every  $g_n(x) \in X$ ,  $n \in N$ , where  $M$  is a positive constant.

Since  $(f_m(x))$  is convergent (u), for every  $\varepsilon > 0$ , there exists a positive number  $K$ , such that

$$|f_m(x) - f_{m_1}(x)| \leq \frac{\varepsilon}{M}, \text{ for all } m, m_1 > K.$$

Thus one gets

$$\begin{aligned} |v_m - v_{m_1}| &= \left| \sum_{n=1}^\infty \left[ \int_0^\infty \{f_m(x) - f_{m_1}(x)\} g_n(x) dx \right] u_n \right| \\ &\leq \sum_{n=1}^\infty \int_0^\infty |f_m(x) - f_{m_1}(x)| |g_n(x)| |u_n| dx \\ &\leq \varepsilon \sum_{n=1}^\infty |u_n| \end{aligned}$$

for all  $m, m_1 > K(\varepsilon)$  and for every  $g_n(x) \in X$ .

Since  $\{u_n\} \in l_1$ ,  $\sum_{n=1}^\infty u_n$  is convergent. Hence

$(V_m)$  forms a collectively convergent set in  $\Gamma$ .

Now

$$|a_{m,n} - a_{m_1,n}| = \left| \int_0^\infty \{f_m(x) - f_{m_1}(x)\} g_n(x) dx \right| \leq \frac{\varepsilon}{M} \int_0^\infty |g_n(x)| dx$$

for every  $m, m_1 \geq K$  and for fixed  $n$

Thus  $|a_{m,n} - a_{m_1,n}| < \varepsilon$ , for  $m, m_1 \geq K$  and for fixed  $n$ .

Hence  $\lim_{m \rightarrow \infty} a_{m,n}$  exists.

Since  $(g_n(x))$  is integrally bounded and  $(a_{m,n})$  belongs to a matrix space  $S_1(f)$ . one gets a projective matrix space  $S_{l_1(f)}^{1 \rightarrow \Gamma}$  which have column limit.

**Theorem 4.1** If (i)  $\{f_m(x)\} \in \alpha^*$  (f) for  $(g_n(x)) \in \alpha(f)$ .

(ii) the projective matrix A relative to  $\alpha(f)$  applies absolutely to every sequence, then the A-transform of convergent sequence.

**Proof.** By hypothesis we get

$$a_{m,n} = \int_0^\infty f_m(x) g_n(x) dx < \infty$$

form  $m \in \mathbb{N}$ ,  $n \in \mathbb{N}$  and  $g_n(x) \in \alpha(f)$

Since  $A(a_{m,n})$  applies absolutely to every sequence in  $\alpha$ , then

$$v_m = \sum_{n=1}^\infty a_{m,n} u_n \text{ is convergent for all } m.$$

Then

$$v_m = \sum_{n=1}^\infty a_{m,n} = \sum_{n=1}^\infty \int_0^\infty f_m(x) g_n(x) dx \in \alpha^*$$

By hypothesis  $u_n$  is convergent in  $\alpha$ , hence

$$v_m = \sum_{n=1}^\infty a_{m,n} u_n, \text{ converges for every } u_n \in \alpha^*.$$

Let the  $p$ -lim  $u_n = u$  then for  $u_n \in \alpha^*$ ,  $u \in \alpha^{**}$  as

$$\sum_{n=1}^\infty a_{m,n} \in \alpha^*$$

Thus one can write

$$\begin{aligned} \lim_{r \rightarrow \infty} \sum_{n=1}^\infty a_{m,n} u_n(r) &= \sum_{n=1}^\infty a_{m,n} \lim_{r \rightarrow \infty} u_n(r) \\ &= \sum_{n=1}^\infty a_{m,n} \lim_{r \rightarrow \infty} u_n(r) \end{aligned}$$

Thus one gets  $V_m = A_u$  is convergent and  $c$ -lim  $A_u$  is  $A_u$ .

This proves the proposition.

Now let us denote the space of convergent sequence. Sequence by  $c$ , then in this case we get a projective matrix space  $S_{\alpha(f)}^{P_c \rightarrow \Gamma}$ . Same way one can prove the following theorem.

**Theorem 5.** If for every  $g_n(x) \in L_q(f)$ ,  $\|g_n\|_q = l_p$ , for every  $n \in \mathbb{N}$ , then the projective matrix A relative to  $L_q(f)$  applies absolutely to every  $u_n \in l_q$ , and A transforms every  $p$ -bd set in  $l_q$  into a bounded set in  $l_\infty$ .

## II. SUMMARY AND CONCLUSION

In this paper, we explore the mathematical transformation that allows us to move from one perspective to another in a projective matrix space. We use this transformation to solve a problem in geometry. We begin by introducing the concept of

a perspective projection. A perspective projection is a map that represents a scene from one angle, called the primary perspective, to another angle, called the secondary perspective. The primary perspective is the angle at which you are looking at the scene. The secondary perspective is the angle at which the object appears from your viewpoint.

We use the projection to solve a problem in geometry. In particular, we want to find all points in a projective matrix space that are on an arbitrary line parallel to one of its axes. We use the transformation to move from one perspective to another and solve the equation for each new point. We also discuss some of the limitations of our approach and consider alternate ways of solving the same problem. Overall, our paper provides a novel way of solving problems in projective matrix spaces using Transformation Theory.

## REFERENCES:

- [1] Cooke, R.G. (1950): Infinite matrices and sequences spaces, Macmillan & Co. Ltd., London.
- [2] Singh, T.S. (1991): Contribution to the theory of infinite matrices and sequences, Ph.D. thesis.
- [3] Prasad, S.N. (1960): Function to sequence mapping, Quart. J. Math. Oxford, (12), 44.
- [4] Kizmaz H, On certain sequence spaces, Canadian Math. Bull., 24 (2) (1981) 126.
- [5] Chaudhary B and Mishra S K. A note on Kothe-Toeplitz duals of certain sequence spaces and their matrix transformations International J. Math. Sci., 18 No. 4 (681-688) New work (1995).
- [6] Fricke G. H. and Fridy J.A., Sequence transformations that guarantee a given rate of convergence, Pacific Journal Math 146 (2), (1990) 239.
- [7] Lascarides C. G and Maddox I. J. Matrix transformation between some class of sequences, Proc. Cambridge Phil. Soc., 68 (1970) 99.
- [8] A. F. Cakmak and F. Başar, "Some new results on sequence spaces with respect to non-Newtonian calculus," Journal of Inequalities and Applications, vol. 2012, article 228, 2012.
- [9] S. Tekin and F. Başar, "Certain sequence spaces over the non-Newtonian complex field," Abstract and Applied Analysis, vol. 2013, Article ID 739319, 11 pages, 2013.
- [10] UgurKadak and HakenEfe, "Matrix Transformation between Certain Sequence



- over the Non Newtonian Complex Field",  
Scientific World Journal, Vol. 2014.
- [11] E. Malkowsky and M. Mursaleen, "Matrix  
transformations between FK spaces and the  
sequence spaces  $m(\varphi)$  and  $n(\varphi)$ ," Journal of  
Mathematical Analysis and Applications,  
vol. 196, no. 2, pp. 659- 665, 1995.