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On Homogeneous Cubic Equation with Four Unknowns $x^3 + y^3 = 13zw^2$

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ABSTRACT: This paper is devoted to obtain non-zero distinct integer solutions to homogeneous cubic equation with four unknowns represented by $x^3 + y^3 = 13zw^2$ along with few observations.

KEYWORDS:
homogeneous cubic, cubic with four unknowns, integer solutions

NOTATIONS

- Polygonal number of rank *n* with size *m*

$$t_{m,n} = n\left[1 + \frac{(n-1)(m-2)}{2}\right]$$

I. INTRODUCTION

The cubic Diophantine equations are rich in variety and offer an unlimited field for research [1,3,4]. For an extensive review of various problems, one may refer [2,5-24]. This paper concerns with another interesting cubic Diophantine equation with four unknowns $x^3 + y^3 = 13zw^2$ for determining its infinitely many non-zero integral solutions.

II. METHOD OF ANALYSIS

The homogeneous cubic equation with four unknowns to be solved is

$$x^3 + y^3 = 13zw^2 \tag{1}$$

Introduction of the linear transformations

$$x = u + v, y = u - v, z = 2u, u \neq v \tag{2}$$

in (1) leads to

$$u^2 + 3v^2 = 13w^2 \tag{3}$$

We present below different methods of solving (3) and thus obtain different patterns of integral solutions to (1).

Pattern-I

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Let $w = a^2 + 3b^2$ (4)

where a and b are non-zero integers.

Write 13 as $13 = (1 + i2\sqrt{3})(1 - i2\sqrt{3})$ (5)

Using (4), (5) in (3) and applying the method of factorization, define

$$(u + i\sqrt{3}v) = (1 + i2\sqrt{3})(a + i\sqrt{3}b)^2 \quad (6)$$

Equating the real and imaginary parts, we have

$$\left.\begin{array}{l} u = a^2 - 12ab - 3b^2 \\ v = 2a^2 + 2ab - 6b^2 \end{array}\right\} \quad (7)$$

Using (7) and (2), the values of x, y and z are given by

$$\left.\begin{array}{l} x = x(a,b) = 3a^2 - 10ab - 9b^2 \\ y = y(a,b) = -a^2 - 14ab + 3b^2 \\ z = z(a,b) = 2a^2 - 24ab - 6b^2 \end{array}\right\} \quad (8)$$

Thus (4) and (8) represent the non-zero integer solutions to (1).

Observations:

1. $x(a,1) - y(a,1) - t_{10,a} = 7a - 12$
2. $y(1,b) + z(1,b) + t_{8,a} = 1 - 40b$
3. $x(a,1) - w(a,1) - t_{6,a} \equiv 0 \pmod{3}$
4. $y(1,b) - w(1,b) \equiv 0 \pmod{2}$
5. $z(a,1) - w(a,1) - t_{4,a} = -(24a + 9)$

Note:1 One may represent 13 as the product of complex conjugates as shown below:

$$13 = \frac{(7 + i\sqrt{3})(7 - i\sqrt{3})}{4} \quad (9)$$

$$13 = \frac{(5 + i3\sqrt{3})(5 - i3\sqrt{3})}{4} \quad (10)$$

$$13 = \frac{(25 + i2\sqrt{3})(25 - i2\sqrt{3})}{49} \quad (11)$$

Following the procedure as above, the corresponding integer solutions to (1) along with respective observations are presented below:

Set: I

Solutions of (1) using (9):

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$$\left.\begin{array}{l} x = x(a,b) = 4a^2 + 4ab - 12b^2 \\ y = y(a,b) = 3a^2 - 10ab - 9b^2 \\ z = z(a,b) = 7a^2 - 6ab - 21b^2 \\ w = a^2 + 3b^2 \end{array}\right\}$$

Observations:

1. $x(1,b) + w(1,b) + t_{20,b} = 5 - 4b$
2. $y(a,1) + w(a,1) - t_{10,a} = -(7a+6)$
3. $z(a,1) - w(a,1) - t_{14,a} = -(a+24)$
4. $x(1,b) - z(1,b) + w(1,b) - t_{26,b} = 21b - 2$
5. $y(a,1) - z(a,1) + t_{10,a} = 12 - 7a$

Set: II

Solutions of (1) using (10):

$$\left.\begin{array}{l} x = x(a,b) = 4a^2 - 4ab - 12b^2 \\ y = y(a,b) = a^2 - 14ab - 3b^2 \\ z = z(a,b) = 5a^2 - 18ab - 15b^2 \\ w = a^2 + 3b^2 \end{array}\right\}$$

Observations:

1. $x(a,1) - w(a,1) - t_{8,a} = -(2a + 15)$
2. $y(1,b) - w(1.b) + t_{14,b} = -19b$
3. $.x(a,1) + y(a,1) - t_{12,a} = -14a - 15$
4. $y(1,b) - z(1,b) - t_{26,b} = 15b - 4$
5. $x(a,1) - z(a,1) - t_{20,a} = -14a - 27$

Set: III

Solutions of (1) using (11):

$$\left.\begin{array}{l} x = x(A,B) = 189A^2 + 266AB - 567B^2 \\ y = y(A,B) = 161A^2 - 434AB - 483B^2 \\ z = z(A,B) = 350A^2 - 168AB - 1050B^2 \\ w = w(A,B) = 49(A^2 + 3B^2) \end{array}\right\}$$

Observations:

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$$1.x(A,1)-w(A,1)-14t_{22,A}=392A$$
$$2.y(1,B)+w(1,B)+16t_{44,B}=210-754B$$
$$3.z(A,1)-w(A,1)-43t_{16,A}\equiv 0(\bmod 3)$$
$$4.x(1,B)-y(1,B)+4t_{44,B}=620B+28$$

Pattern-II

Write (3) as

$$u^2+3v^2=13w^2*1 \qquad (12)$$

Write 1 as

$$1=\left(\frac{(1+i\sqrt{3})(1-i\sqrt{3})}{4}\right) \qquad (13)$$

Using (4), (5),(13) in (12) and applying the method of factorization, define

$$(u+i\sqrt{3}v)=(1+i2\sqrt{3})(a+i\sqrt{3}b)^2\left(\frac{1+i\sqrt{3}}{2}\right)$$

from which we have

$$\left.\begin{aligned} u&=\frac{1}{2}(-5a^2-18ab+15b^2)\\ v&=\frac{1}{2}(3a^2-10ab-9b^2)\end{aligned}\right\} \qquad (14)$$

Using (14) and (2), the values of x,y and z are given by

$$\left.\begin{aligned} x&=x(a,b)=-a^2-14ab+3b^2\\ y&=y(a,b)=-4a^2-4ab+12b^2\\ z&=z(a,b)=-5a^2-18ab+15b^2\end{aligned}\right\} \qquad (15)$$

Thus (4) and (15) represent the non-zero integer solutions to (1).

Observations:

$$1.x(a,1)-w(a,1)+t_{6,a}\equiv 0(\bmod 5)$$
$$2.y(1,b)-z(1,b)+t_{8,b}=1+12b$$
$$3.x(a,1)-y(a,1)-t_{8,a}=-(8a+9)$$
$$4.z(1,b)-w(1,b)-t_{26,b}=-(7b+6)$$
$$5.x(a,1)+y(a,1)-z(a,1)-w(a,1)+t_{4,a}\equiv 0(\bmod 3)$$

Note:2 One may represent 1 on the R.H.S. of (12) as below:

$$1=\left(\frac{(1+i4\sqrt{3})(1-i4\sqrt{3})}{49}\right) \qquad (16)$$

$$1=\left(\frac{(11+i4\sqrt{3})(11-i4\sqrt{3})}{169}\right) \qquad (17)$$

$$1=\left(\frac{\left(13+i3\sqrt{3}\right)\left(13-i3\sqrt{3}\right)}{196}\right) \tag{18}$$

Following the procedure presented as above, for simplicity and brevity, we present below the integer solutions to (1) obtained from (12) for other representations of 13 and 1 on the R.H.S. of (12):

Set: IV

Solutions of (1) using (5) and (16):

$$\left.\begin{aligned} x &= x(A,B)=\left(-119A^2-574AB+357B^2\right)\\ y &= y(A,B)=\left(-203A^2+70AB+609B^2\right)\\ z &= z(A,B)=\left(-322A^2-504AB+966B^2\right)\\ w &= w(A,B)=49(A^2+3B^2)\end{aligned}\right\}$$

Observations:

1. $x(A,1)-y(A,1)-12t_{16,A}=-548A-252$
2. $y(1,B)-z(1,B)+51t_{16,B}=268B+119$
3. $y(A,1)+z(A,1)-w(A,1)+14t_{84,A}=1428-994A$
4. $x(1,B)+w(1,B)-63t_{18,B}=-(133B+70)$
5. $z(1,B)-w(1,B)-91t_{20,B}=224B-371$

Set: V

Solutions of (1) using (5) and (17):

$$\left.\begin{aligned} x &= x(a,b)=a^2-14ab-3b^2\\ y &= y(a,b)=-3a^2-10ab+9b^2\\ z &= z(a,b)=-2a^2-24ab+6b^2\\ w &= a^2+3b^2\end{aligned}\right\}$$

Observations:

1. $x(1,b)+w(1,b)=2-14b$
2. $x(a,1)+y(a,1)-w(a,1)+t_{8,a}=3-26a$
3. $y(1,b)-z(1,b)-w(1,b)=14b-2$
4. $y(a,1)+w(a,1)+t_{6,a}=12-11a$

Set: VI

Solutions of (1) using (5) and (18):

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$$\left.\begin{aligned} x &= x(A,B) = \left(336A^2 - 2576AB - 1008B^2\right) \\ y &= y(A,B) = \left(-476A^2 - 2296AB + 1428B^2\right) \\ z &= z(A,B) = \left(-140A^2 - 4872AB + 420B^2\right) \\ w &= w(A,B) = 196(A^2 + 3B^2) \end{aligned}\right\}$$

Observations:

1. $x(1,B) - w(1,B) + 133t_{26,B} = -4039B + 140$
2. $y(A,1) + z(A,1) - w(A,1) + 29t_{58,A} = 1260 - 7951A$
3. $y(1,B) - w(1,B) - 40t_{44,B} = -(1496B + 672)$
4. $x(A,1) - z(A,1) - 28t_{36,A} = 2744A - 1428$

Set: VII

Solutions of (1) using (9) and (18):

$$\left.\begin{aligned} x &= x(A,B) = \left(3248A^2 - 1120AB - 9744B^2\right) \\ y &= y(A,B) = \left(1344A^2 - 10304AB - 4032B^2\right) \\ z &= z(A,B) = \left(4592A^2 - 11424AB - 13776B^2\right) \\ w &= w(A,B) = 784(A^2 + 3B^2) \end{aligned}\right\}$$

Set: VIII

Solutions of (1) using (10) and (16):

$$\left.\begin{aligned} x &= x(A,B) = \left(-112A^2 - 2800AB - 336B^2\right) \\ y &= y(A,B) = \left(-756A^2 - 1064AB + 2268B^2\right) \\ z &= z(A,B) = \left(-868A^2 - 3864AB + 2604B^2\right) \\ w &= w(A,B) = 196(A^2 + 3B^2) \end{aligned}\right\}$$

Observations:

1. $x(1,B) + w(1,B) - 84t_{24,B} = 28(3 - 70B)$
2. $z(A,1) + w(A,1) + 32t_{44,A} = 3192 - 4504A$
3. $x(A,1) - y(A,1) - 28t_{48,A} = -(1120A + 1932)$
4. $y(1,B) - z(1,B) + w(1,B) - 28t_{20,B} = 308 + 3024B$

Set: IX

Solutions of (1) using (10) and (17):

$$\left.\begin{aligned} x = x(A,B) &= \left(1872A^2 - 7280AB - 5616B^2\right) \\ y = y(A,B) &= \left(-884A^2 - 9256AB + 2652B^2\right) \\ z = z(A,B) &= \left(988A^2 - 16536AB - 2964B^2\right) \\ w = w(A,B) &= 676(A^2 + 3B^2) \end{aligned}\right\}$$

Set: X

Solutions of (1) using (10) and (18):

$$\left.\begin{aligned} x = x(A,B) &= \left(2576A^2 - 6944AB - 7728B^2\right) \\ y = y(A,B) &= \left(-448A^2 - 11200AB + 1344B^2\right) \\ z = z(A,B) &= \left(2128A^2 - 18144AB - 6384B^2\right) \\ w = w(A,B) &= 784(A^2 + 3B^2) \end{aligned}\right\}$$

Set: XI

Solutions of (1) using (11) and (13):

$$\left.\begin{aligned} x = x(A,B) &= \left(644A^2 - 1736AB - 1932B^2\right) \\ y = y(A,B) &= \left(-112A^2 - 2800AB + 336B^2\right) \\ z = z(A,B) &= \left(532A^2 - 4536AB - 1596B^2\right) \\ w = w(A,B) &= 196(A^2 + 3B^2) \end{aligned}\right\}$$

Observations:

1. $x(A,1) - w(A,1) - 105t_{18,A} = -(1001A + 1344)$
2. $z(1,B) - y(1,B) + 84t_{48,B} = 644 - 3584B$
3. $y(A,1) + w(A,1) - 12t_{16,A} = 924 - 2728A$
4. $z(1,B) - x(1,B) - 21t_{34,B} = -112 - 2485B$

Set: XII

Solutions of (1) using (11) and (16):

$$\left.\begin{aligned} x = x(A,B) &= \left(5047A^2 - 29890AB - 15141B^2\right) \\ y = y(A,B) &= \left(-4949A^2 - 30086AB + 14847B^2\right) \\ z = z(A,B) &= \left(98A^2 - 59976AB - 294B^2\right) \\ w = w(A,B) &= 2401(A^2 + 3B^2) \end{aligned}\right\}$$

Set: XIII

Solutions of (1) using (11) and (17):

$$\left.\begin{aligned} x &= x(A,B) = \left(33943A^2 - 20930AB - 101829B^2\right) \\ y &= y(A,B) = \left(11739A^2 - 112294AB - 35217B^2\right) \\ z &= z(A,B) = \left(45682A^2 - 133224AB - 137046B^2\right) \\ w &= w(A,B) = 8281(A^2 + 3B^2) \end{aligned}\right\}$$

Set: XIV

Solutions of (1) using (11) and (18):

$$\left.\begin{aligned} x &= x(A,B) = \left(39984A^2 + 784AB - 119952B^2\right) \\ y &= y(A,B) = \left(20188A^2 - 119560AB - 60564B^2\right) \\ z &= z(A,B) = \left(60172A^2 - 118776AB - 180516B^2\right) \\ w &= w(A,B) = 9604(A^2 + 3B^2) \end{aligned}\right\}$$

Pattern-III

We can write (3) in the form of ratio as

$$\frac{u+w}{3(2w+v)} = \frac{2w-v}{u-w} = \frac{\alpha}{\beta}$$

The above equation is equivalent to the double equations

$$\beta u - 3\alpha v + (\beta - 6\alpha)w = 0$$

$$\alpha u + \beta v - (\alpha + 2\beta)w = 0$$

Applying the method of cross multiplication, we get

$$\left.\begin{aligned} u &= 3\alpha^2 + 12\alpha\beta - \beta^2 \\ v &= -6\alpha^2 + 2\alpha\beta + 2\beta^2 \end{aligned}\right\} \quad (19)$$

$$w = 3\alpha^2 + \beta^2 \quad (20)$$

Using (19) in (2), we have

$$\left.\begin{aligned} x(\alpha,\beta) &= -3\alpha^2 + 14\alpha\beta + \beta^2 \\ y(\alpha,\beta) &= 9\alpha^2 + 10\alpha\beta - 3\beta^2 \\ z(\alpha,\beta) &= 6\alpha^2 + 24\alpha\beta - 2\beta^2 \end{aligned}\right\} \quad (21)$$

Thus, (20) and (21) give a set of integer solutions to (1).

Note :3 One may also write (3) in the form of ratio as

$$\frac{u+w}{(2w+v)} = \frac{3(2w-v)}{u-w} = \frac{\alpha}{\beta}, \beta \neq 0$$

Following the above procedure, a different set of solutions to (1) are obtained.

Pattern-IV

Rewrite (3) as $3v^2 = 13w^2 - u^2$ (22)

Introducing the linear transformations

$$u = X + 13T, w = X + T, v = 2V \quad (23)$$

in (22) , it leads to

$$X^2 = V^2 + 13T^2 \quad (24)$$

which is satisfied by

$$T = 2rs, V = 13r^2 - s^2, X = 13r^2 + s^2$$

Substituting the above values of X and T in (23), we have

$$u = 13r^2 + s^2 + 26rs, w = 13r^2 + s^2 + 2rs, v = 2(13r^2 - s^2)$$

Substituting the values of u and v in (2) , the corresponding non-zero integer values of x,y,z satisfying (1) are given by

$$x = x(r,s) = 39r^2 - s^2 + 26rs$$
$$y = y(r,s) = -13r^2 + 3s^2 + 26rs$$
$$z = z(r,s) = 26r^2 + 2s^2 + 52\text{rs}$$

Note: 4

The linear transformation (23) can also be taken as

$$u = X - 13T, w = X - T, v = 2V$$

By following the procedure as above we get a different choice of integer solutions to (1).

Pattern-V

Equation (24) can be written in the form $X^2 - V^2 = 13T^2$

which is equivalent to the system of double equation as shown below:

System		**1**	**2**	**3**
$X + V$	$=$	$13T^2$	T^2	$13T$
$X - V$	$=$	1	13	T

Solving these equations, we obtain the values of X and T.

In View of (23) and (2), the corresponding non-zero distinct integer solutions of (1) are

System:1	**System:2**	**System:3**
$x = 78t^2 + 104t + 32$	$x = 6t^2 + 32t + 8$	$x = 32t$
$y = -26t^2 + 8$	$y = -2t^2 + 24t + 32$	$y = 8t$
$z = 52t^2 + 104t + 40$	$z = 4t^2 + 56t + 40$	$z = 40t$
$w = 26t^2 + 28t + 8$	$w = 2t^2 + 4t + 8$	$w = 8t$

REMARKABLE OBSERVATIONS:

If the non zero integer triplet (u_0, v_0, w_0) is any solution of (3), then each of the following three quadruple of integers based on $u_0, v_0$and $w_0$also satisfies (1).

Quadruple:1 (x_n, y_n, z_n, w_n)

$$x_n = [2^{n-1} + 2(-2)^{n-2}]u_0 + [-2^{n-1} + 6(-2)^{n-2}]v_0$$

$$y_n = 2^n u_0 - 2^n v_0$$

$$z_n = 2\left[[3.2^{n-2} + (-2)^{n-2}]u_0 + [-3.2^{n-2} + 3(-2)^{n-2}]v_0\right]$$

$$w_n = 2^n w_0$$

Quadruple:2 (x_n, y_n, z_n, w_n)

$$x_n = [13.2^{n-2} - 9(-2)^{n-2}]u_0 + 39[(-2^{n-2}) + (-2)^{n-2}]w_0 + 2^n v_0$$

$$y_n = [13.2^{n-2} - 9(-2)^{n-2}]u_0 + 39[(-2^{n-2}) + (-2)^{n-2}]w_0 - 2^n v_0$$

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$$z_n = 2\{[13.2^{n-2} - 9(-2)^{n-2}]u_0 + 39[(-2^{n-2}) + (-2)^{n-2}]w_0\}$$

$$w_n = 3[2^{n-2} - (-2)^{n-2}]u_0 + [-9(2^{n-2}) + 13(-2)^{n-2}]w_0$$

Quadruple:3 (x_n, y_n, z_n, w_n)

$$x_n = 5^n u_0 + \frac{1}{2}\{[13.5^{n-1} - 3(-5)^{n-1}]v_0 + 13[-5^{n-1} + (-5)^{n-1}]w_0\}$$

$$y_n = 5^n u_0 - \frac{1}{2}\{[13.5^{n-1} - 3(-5)^{n-1}]v_0 + 13[-5^{n-1} + (-5)^{n-1}]w_0\}$$

$$z_n = 2.5^n u_0$$

$$w_n = \frac{1}{2}\{3[5^{n-1} - (-5)^{n-1}]v_0 + [-3.5^{n-1} + 13(-5)^{n-1}]w_0\}$$

III. CONCLUSION

In this paper, we have made an attempt to determine different patterns of non-zero distinct integer solutions to the homogeneous cubic equation with four unknowns. As the cubic equations are rich in variety, one may search for other forms of cubic equations with multi-variables to obtain their corresponding solutions .

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