International Journal of Advances in Engineering and Management (IJAEM)
Volume 7, Issue 01 Jan. 2025, pp: 588-596 www.ijaem.net ISSN: 2395-5252

Optimal InventoryManagement for Deteriorating and Ameliorating Items with Stochastic Demand and Time-varying Holding Cost.

Dr. Biswaranjan Mandal
Associate Professor of Mathematics
Acharya Jagadish Chandra Bose College, Kolkata, West Bengal, India

Date of Submission: 20-01-2025 Date of Acceptance: 30-01-2025

ABSTRACT:
The present paper deals with an optimal inventory management for deteriorating and ameliorating items with stochasticdemand in which shortages are allowed withfully backlogged. The holding cost follows time dependent. Here deterioration is time-varying and the environment of amelioration followedby a two-parameter Weibull (Swedish engineer Wallodi Weibull) distribution to describe the different life spans effectively by utilizing the changes of the parameters. The demand pattern is assumed to be linearly dependent on to time with a stochastic error. The model is minimized to the total average cost by finding optimal values. The developed model is illustrated by a numerical example and finally the sensitivity analysis for the optimal solutions towards the changes in the values of key parameters has been presented.

Key Words: Inventory management, Weibull deteriorating items, ameliorating items, stochastic demand, holding cost and shortages.
Subject classification: AMS Classification No. 90B05

I. INTRODUCTION:

It is seen widely that many items like fruits, flowers, green vegetables, dairy products, radioactive substances etc deteriorate over time. Several researchers have addressed the importance of the deterioration phenomenon in their field of applications; as a result, many inventory models with deteriorating items have been developed. Even seen that in inventory management, "amelioration" refers to a situation where the value or quality of an item increases over time, essentially meaning the product becomes more valuable as it ages, unlike typical inventory where items tend to decrease in value due to factors like deterioration or obsolescence; this can lead to a potential increase in price compared to its original purchase cost. The effect of ameliorating items like broiler, ducks, pigs etc. in the poultry farm, highbred fishes in berry (pond) is increasing gradually in the inventory management system. It is a natural phenomenon observing in much life stock models. A few researchers have focused on ameliorating system. Hwang [1] developed an inventory model for ameliorating items only. Again Hwang [2] added to a stock model for both ameliorating and deteriorating things independently. Mallick et al. [7] has considered a creation inventory model for both ameliorating and deteriorating items. Many other researchers like Moon et al [3], L-Q ji[4], Valliathal et al[5], Nodoust [6],Dr. Biswaranjan Mandal [8] are mentioned a few. In this paper, we consider the model where the deterioration is time-varying and the environment of Amelioration followed by Weibull Distribution to describe the different life spans effectively by utilizing the changes of parameters.

The assumption of constant demand rate may not be always appropriate for many inventory theories. For examples, milk items, vegetables, fruits, cosmetics etc have a negative impact on demand due to loss of confidence of consumers on the quality of such products for their age of inventory.Also we noticed that the demand of seasonal foods and garments is highly dependent on time. So it can be concluded that demand for items varies with respect to time. However, in order to match with real-life criteria, many authors have developed new types of inventory models with a variable demand rate. Also the acceptance of some constant demand rate is not reasonable for many inventory items such as electronic goods, fashionable garments, tasty foods, volatile liquids etc , as they fluctuate in the demand rate. As a result, linear trended dependent rateon to time with a stochastic error has a prominent role in inventory control system. Researchers like U Dave et al [9], T K Datta et al [10], J T Teng et al [11], Biswaranjan[12], M Mallick et al [13] A Kundu [14] are mentioned a few.

In a classical inventory management system, it is assumed that holding cost is fixed as constant. But in reality, inventory is stored up to meet the demand of the customers or for future usage. So variable holding cost is attracting most of the related researchers 'attention because maintaining inventory is very much crucial. Holding cost is also high and dependent on time for many deteriorating items like fruits, vegetables, medicine etc. Therefore time-varying holding cost is utmost important in inventory management system. In this matter, few researchers like M Goh [15], A Roy [16], V K Mishra [17], Tripathi et al [18], M Sharma [19] are noteworthy.

The word shortage means a state or situation in which the needed items cannot be obtained in sufficient amounts or totally absent. It has a great importance for many models, especially when delay in payment is considered. When a shortage occurs but the company offers delay in payment, it can gain more orders from the customers. So shortages have an important role on optimization in inventory theory.

For these sort of situations and facts, efforts have been made to develop an inventory model for time-varying deteriorating items and Weibull distributed ameliorating itemshavinglinear time dependent demand rate with a stochastic error in which shortages are allowed. The holding cost follows time dependent.The model is minimized to the total average cost by finding optimal values. The developed model is illustrated by a numerical example and finally the sensitivity analysis for the optimal solutions towards the changes in the values of key parameters has been presented.

II. ASSUMPTIONS AND NOTATIONS:

2.1 Assumptions:

The present inventory model is developed on the basis of the following assumptions

i. Lead time is zero.
ii. Replenishment rate is infinite but size is finite.
iii. The time horizon is finite.
iv. There is no repair of deteriorated items occurring during the cycle.
v. Rate of deterioration follows time dependent.
vi. The amelioration rate is Weibull distributed in nature.
vii. The demand rate is a linear function of time with stochastic error.
viii. Holding cost follows time-varying.
ix. Shortages are allowed and completely backlogged.

2.2 Notations:

The following notations are used in the proposed model:

i. Q : On hand inventory at time t.
ii. The time-varying deterioration rate is given by

$$\theta(t)=\theta_o t, 0\le\theta_o <<1.$$

. iii. A(t) is the amelioration rate following Weibull distributed

$$A(t)=\alpha\beta t^{\beta-1}, 0\le\alpha<<1, \beta\ge 1,$$

where α is the shape parameter and β is the scale parameter.

iv. t_1 : The time length in which the stock is completely diminished
v. T : The fixed length of each production cycle.
vi. d_c : The deterioration cost per unit item.
vii. a_c : The cost of amelioration per unit item.
viii. c_s : The shortage cost per unit item.
ix. h(t) : Holding cost $h(t)=kt$, $k>0$.
x. D(t) : Demand rate D(t) = $a+bt+\varepsilon$, where a,b>0 so that the demand is positive throughout the demand, ε (stochastic error). Here the shape of the demand curve is deterministic while the scaling parameter representing the size of the market is random. From practical stand point if "a" is large relative to the variance of ε, unbounded probability distribution such as the normal distribution provides adequate approximation. We assume that F(.) and f(.) represent the cumulative distribution and probability density function of ε, respectively having mean μ and standard deviation δ.
xi. q(t) : The level of inventory

$$q(t)=\left|\begin{matrix} q_1(t), 0\le t\le t_1 \\ q_2(t), t_1\le t\le T\end{matrix}\right\rangle$$

xii. ATC : Average total cost per unit time.
xiii. <ATC> : Expected average total coat per unit time.

III. FORMULATION AND SOLUTION:

In this model, we consider the variation of the inventory level during the period $[0,T]$. The inventory level is depleted only due to demand and deterioration and ultimately falls to zero at t $=t_1$. The shortages occur during timeperiod $[t_1$, T] which are completelybacklogged. The differential equations pertaining to the above situations are given by

$$\frac{dq_1(t)}{dt}+(\theta(t)-A(t))q_1(t)=-D(t), 0\le t\le t_1 \quad (3.1)$$

$$\text{And } \frac{dq_2(t)}{dt}=-D(t), t_1\le t\le T \quad (3.2)$$

The initial conditionsare $q_1(0)=Q$, $q_1(t_1)=0$ and $q_2(t_1)=0$ (3.3)

Putting the values of $\theta(t)=\theta_0 t, 0\le\theta_0<<1,\ A(t)=\alpha\beta t^{\beta-1}, 0\le\alpha<<1, \beta\ge 1$ and $D(t)=a+bt+\varepsilon, a,b>0$, we get the following equations

$$\frac{dq_1(t)}{dt}+(\theta_0 t-\alpha\beta t^{\beta-1})q_1(t)=-[a+bt+\varepsilon], 0\le t\le t_1 \quad (3.4)$$

$$\text{And } \frac{dq_2(t)}{dt}=-(a+bt+\varepsilon), t_1\le t\le T \quad (3.5)$$

Since $\theta_0(0\le\theta_0<<1)$ and $\alpha(0<\alpha<<1)$, we ignore the terms $O(\theta_0{}^2)$ and $O(\alpha^2)$, thenthe solutions of the equations (3.4) and (3.5)using (3.3) are given by the following

$$q_1(t)=Q(1-\frac{\theta_0}{2}t^2+\alpha t^\beta)-\{(a+\varepsilon)t+\frac{b}{2}t^2-\frac{\theta_0}{3}(a+\varepsilon)t^3-\frac{b\theta_0}{8}t^4$$
$$+\frac{\alpha\beta}{\beta+1}(a+\varepsilon)t^{\beta+1}+\frac{b\alpha\beta}{2(\beta+2)}t^{\beta+2}\},, 0\le t\le t_1 \quad (3.6)$$

$$\text{And } q_2(t)=(a+\varepsilon)(t_1-t)+\frac{b}{2}(t_1^2-t^2),\ t_1\le t\le T \quad (3.7)$$

Since $q_1(0)=Q$, we get the following expression of on-hand inventoryfrom the equation (3.6), neglecting second and higher order powers of θ_0 and α,

$$Q=(a+\varepsilon)t_1+\frac{b}{2}t_1^2+\frac{(a+\varepsilon)\theta_0}{6}t_1^3+\frac{b\theta_0}{8}t_1^4-\frac{(a+\varepsilon)\alpha}{\beta+1}t_1^{\beta+1}-\frac{b\alpha}{\beta+2}t_1^{\beta+2} \quad (3.8)$$

IV. COST COMPONENTS:

The total costover the period [0, T] consists of the following cost components:

- Holding costfor carrying inventory**(HC)**over the period [0,T]

HC = $[\int_0^{t_1} ktq_1(t)dt]$

Putting the values of $q_1(t)$ from (3.6), and integrating and then substituting the value of Q from (3.6), we get the following expression after neglecting second and higher order powers of θ_0 and α,

International Journal of Advances in Engineering and Management (IJAEM)
Volume 7, Issue 01 Jan. 2025, pp: 588-596 www.ijaem.net ISSN: 2395-5252

$$HC= k[Q(\frac{1}{2}t_1^2-\frac{\theta_0}{8}t_1^4+\frac{\alpha}{\beta+2}t_1^{\beta+2})-\{\frac{a+\varepsilon}{3}t_1^3+\frac{b}{8}t_1^4-\frac{(a+\varepsilon)\theta_0}{15}t_1^5-\frac{b\theta_0}{48}t_1^6$$
$$+\frac{(a+\varepsilon)\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3}+\frac{b\alpha\beta}{2(\beta+2)(\beta+4)}t_1^{\beta+4}\}] \quad (4.1)$$

- Cost due to deterioration **(CD)** over the period [0,T]

$$CD = d_c\int_0^{t_1}\theta_0 t q_1(t)dt$$

Putting the value $q_1(t)$ from (3.6) and then integrating and neglecting second and higher order powers of θ_0 and α ,we get the following

$$CD = d_c\theta_0[Q(\frac{1}{2}t_1^2-\frac{\theta_0}{8}t_1^4+\frac{\alpha}{\beta+2}t_1^{\beta+2})-\{\frac{a+\varepsilon}{3}t_1^3+\frac{b}{8}t_1^4-\frac{(a+\varepsilon)\theta_0}{15}t_1^5-\frac{b\theta_0}{48}t_1^6$$
$$+\frac{(a+\varepsilon)\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3}+\frac{b\alpha\beta}{2(\beta+2)(\beta+4)}t_1^{\beta+4}\}] \quad (4.2)$$

- The amelioration cost **(AMC)** over the period [0,T]

$$AMC = a_c\int_0^{t_1}\alpha\beta t^{\beta-1}q_1(t)dt$$

Putting the value $q_1(t)$ from (3.4) and then integrating and neglecting second and higher order powers of θ_0 and α , we get the following

$$AMC = a_c\alpha[\frac{a+\varepsilon}{\beta+1}t_1^{\beta+1}+\frac{b}{\beta+2}t_1^{\beta+2}] \quad (4.3)$$

- Cost due to shortage **(CS)** over the period [0,T]

$$CS = -c_s\int_{t_1}^{T}q_2(t)dt$$

Putting the value $q_2(t)$ from (3.7), and then integrating we get the following

$$CS= c_s[\frac{a+\varepsilon}{2}(T^2-2Tt_1+t_1^2)+\frac{b}{6}(T^3-3Tt_1^2+2t_1^3)] \quad (4.4)$$

Thus the average total costper unit time of the system during the cycle [0,T] will be

$$ATC(t_1) = \frac{1}{T}\textbf{[HC+CD+AMC+CS]}$$

$$= (k+d_c\theta_0)[Q(\frac{1}{2}t_1^2-\frac{\theta_0}{8}t_1^4+\frac{\alpha}{\beta+2}t_1^{\beta+2})-\{\frac{a+\varepsilon}{3}t_1^3+\frac{b}{8}t_1^4-\frac{(a+\varepsilon)\theta_0}{15}t_1^5-\frac{b\theta_0}{48}t_1^6$$
$$+\frac{(a+\varepsilon)\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3}+\frac{b\alpha\beta}{2(\beta+2)(\beta+4)}t_1^{\beta+4}\}] + a_c\alpha[\frac{a+\varepsilon}{\beta+1}t_1^{\beta+1}+\frac{b}{\beta+2}t_1^{\beta+2}]$$

$$+\ c_s[\frac{a+\varepsilon}{2}(T^2-2Tt_1+t_1^2)+\frac{b}{6}(T^3-3Tt_1{}^2+2t_1^3)] \quad (4.5)$$

Let us suppose that

$$f(\varepsilon)=\frac{1}{k_1-k_0},\ k_1\le\varepsilon\le k_0$$

= 0, elsewhere

Where (μ,σ) are mean and standard deviation.

Therefore the expected average total cost (EATC) is given by

$$\text{EATC} = <ATC(t_1)> = \frac{1}{T}[$$

$$(k+d_c\theta_0)[Q(\frac{1}{2}t_1^2-\frac{\theta_0}{8}t_1^4+\frac{\alpha}{\beta+2}t_1^{\beta+2})-\{\frac{a+\mu}{3}t_1^3+\frac{b}{8}t_1^4-\frac{(a+\mu)\theta_0}{15}t_1^5-\frac{b\theta_0}{48}t_1^6$$

$$+\frac{(a+\mu)\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3}+\frac{b\alpha\beta}{2(\beta+2)(\beta+4)}t_1^{\beta+4}\}]\ +\ a_c\alpha[\frac{a+\mu}{\beta+1}t_1^{\beta+1}+\frac{b}{\beta+2}t_1^{\beta+2}]$$

$$+c_s[\frac{a+\mu}{2}(T^2-2Tt_1+t_1^2)+\frac{b}{6}(T^3-3Tt_1{}^2+2t_1^3)]] \quad (4.6)$$

To minimize the cost, the necessary condition is $\frac{d<ATC(t_1)>}{dt_1}=0$

This gives

$$(k+d_c\theta_0)\{\frac{a+\mu}{2}t_1^2+\frac{b}{2}t_1^3+\frac{(a+\mu)\theta_0}{8}t_1^4+\frac{b\theta_0}{8}t_1^5+\frac{(a+\mu)\alpha\beta}{\beta+1}t_1^{\beta+2}+\frac{b\alpha\beta}{2(\beta+2)}t_1^{\beta+3}\}$$

$$+\ a_c\alpha\{(a+\mu)t_1^{\beta}+bt_1^{\beta+1}\}\ +\ c_s\{(a+\mu+bt_1)(t_1-T)\}=0 \quad (4.7)$$

For minimum, the sufficient condition $\frac{d^2<ATC(t_1)>}{dt_1{}^2}>0$ would be satisfied.

Let $t_1=t_1^*$ be the optimum value of t_1.

The optimal expected values $<Q^*>$ of Q and $<ATC^*>$ of <ATC> are obtained from the expressions (3.8) and (4.6) by putting the value $t_1=t_1^*$.

V. SOME SPECIAL CASES:

(a). **Absence of deterioration :**

If the deterioration of items is switched off i.e. $\theta_0=0$, then the expressions (3.8) and (4.6) of on-hand inventory(Q) and expected average total costper unit time (<ATC(t_1)>) during the period [0,T] become

$$Q=(a+\varepsilon)t_1+\frac{b}{2}t_1^2-\frac{(a+\varepsilon)\alpha}{\beta+1}t_1^{\beta+1}-\frac{b\alpha}{\beta+2}t_1^{\beta+2} \quad (5.1)$$

$$\text{And} <ATC(t_1)> = \frac{1}{T}[k[Q(\frac{1}{2}t_1^2+\frac{\alpha}{\beta+2}t_1^{\beta+2})-\{\frac{a+\mu}{3}t_1^3+\frac{b}{8}t_1^4+\frac{(a+\mu)\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3}+\frac{b\alpha\beta}{2(\beta+2)(\beta+4)}t_1^{\beta+4}\}]$$

$$+\ a_c\alpha[\frac{a+\mu}{\beta+1}t_1^{\beta+1}+\frac{b}{\beta+2}t_1^{\beta+2}]\ +c_s[\frac{a+\mu}{2}(T^2-2Tt_1+t_1^2)+\frac{b}{6}(T^3-3Tt_1{}^2+2t_1^3)]] \quad (5.2)$$

The equation (4.7) becomes

$$k\{\frac{a+\mu}{2}t_1^2+\frac{b}{2}t_1^3+\frac{(a+\mu)\alpha\beta}{\beta+1}t_1^{\beta+2}+\frac{b\alpha\beta}{2(\beta+2)}t_1^{\beta+3}\} + a_c\alpha\{(a+\mu)t_1^{\beta}+bt_1^{\beta+1}\} + c_s\{(a+\mu+bt_1)(t_1-T)\} = 0 \quad (5.3)$$

This gives the optimum valueof t_1.

(b). **Absence of amelioration:**

If the amelioration environment is not considered i.e. $\alpha=0$, then the expressions (3.8) and (4.8) of on-hand inventory (Q) and expected average total cost per unit time (<ATC(t_1)>) during the period [0,T] become

$$Q=(a+\varepsilon)t_1+\frac{b}{2}t_1^2+\frac{(a+\varepsilon)\theta_0}{6}t_1^3+\frac{b\theta_0}{8}t_1^4 \quad (5.4)$$

And $$<ATC(t_1)>=\frac{1}{T}[(k+d_c\theta_0)[Q(\frac{1}{2}t_1^2-\frac{\theta_0}{8}t_1^4)-\{\frac{a+\mu}{3}t_1^3+\frac{b}{8}t_1^4-\frac{(a+\mu)\theta_0}{15}t_1^5-\frac{b\theta_0}{48}t_1^6]$$
$$+c_s[\frac{a+\mu}{2}(T^2-2Tt_1+t_1^2)+\frac{b}{6}(T^3-3Tt_1{}^2+2t_1^3)]] \quad (5.5)$$

The equation (4.7) becomes

$$(k+d_c\theta_0)\{\frac{a+\mu}{2}t_1^2+\frac{b}{2}t_1^3+\frac{(a+\mu)\theta_0}{8}t_1^4+\frac{b\theta_0}{8}t_1^5\} + c_s\{(a+\mu+bt_1)(t_1-T)\} = 0 \quad (5.6)$$

This gives the optimum value of t_1.

VI. NUMERICAL ANALYSIS:

To exemplify the above model numerically, let the values of parameters be as follows:

a= 100; b = 0.5; $\theta_0=0.001$ α = 0.002; β = 2; k = 20; d_c = $9 per unit; a_c = $7 per unit; c_s = $10 per unitand T = 1 year

Also we assume that a uniformly-distributed random demand component exhibited an error span of u= 20 with $[k_0,k_1]$=[10,20], and a mean μ =20.

Solving the equation (4.7) with the help of computer using the above values of parameters, we find the following optimum outputs

t_1^* = 0.62year;$<Q^*>$= 75.03 units and $<ATC^*>$ = Rs.89.87

It is checked that this solution satisfies the sufficient condition for optimality.

For Special Cases:

Nature of deterioration	t_1^*	$<Q^*>$	$<ATC^*>$
Absence of deterioration	0.62	75.03	89.82
Absence of amelioration	0.63	75.10	89.55

VII. SENSITIVITY ANALYSISAND DISCUSSION:

We now study the effects of changes in the system a = 100; b = 0.5; $\theta_0=0.001$ α = 0.002; β = 2; k = 20; d_c = $9 per unit; a_c = $7 per unit; c_s = $10 per unit and μ =20on the optimumexpected ordering quantity ($<Q^*>$) and expected average total cost per unit time (<ATC(t_1)>) in the present inventory model. The sensitivity analysis is performed by changing each of the parameters by – 50%, – 20%, +20% and +50%, taking one parameter at a time and keeping remaining parameters unchanged. The results are furnished in table A.

International Journal of Advances in Engineering and Management (IJAEM)
Volume 7, Issue 01 Jan. 2025, pp: 588-596 www.ijaem.net ISSN: 2395-5252

Table A: Effect of changes in the parameters on the model

Changing parameter	% change in the system parameter	% change in $<Q^*>$	% change in $<ATC^*>$
a	-50 -20 +20 +50	-41.61 -16.64 16.64 41.61	- 41.44 - 16.62 16.62 41.55
b	-50 -20 +20 +50	- 0.07 - 0.03 0.03 0.07	- 0.14 -0.06 0.06 0.14
θ_0	-50 -20 +20 +50	0.005 0.002 -0.002 -0.005	- 0.02 - 0.01 0.01 0.02
α	-50 -20 +20 +50	0.05 0.02 -0.02 -0.05	- 0.18 - 0.07 0.07 0.18
β	-50 -20 +20 +50	-0.07 -0.02 0.02 0.04	0.32 0.10 - 0.08 - 0.16
k	-50 -20 +20 +50	20.02 6.67 - 5.46 - 12.01	- 48.29 - 18.57 17.03 39.62
, d_c	-50 -20 +20 +50	0.006 0.002 -0.002 - 0.006	- 0.02 - 0.01 0.01 0.02
a_c	-50 -20 +20 +50	0.02 0.01 -0.01 - 0.02	- 0.14 - 0.06 0.06 0.14
c_s	-50 -20 +20 +50	- 20.02 - 6.68 6.23 12.04	-13.30 - 2.24 2.25 14.10
μ	-50 -20 +20 +50	-8.32 -3.33 3.33 8.32	- 8.31 - 3.32 3.32 8.31

Analyzing the results of table A, the following observations may be made:

(i) The optimum expected ordering quantity $<Q^*>$ increase or decrease with the increase or decrease in the values of the system parameters a; b; β; c_s and μ. On the other hand $<Q^*>$ increase or decrease with the decrease or increase in the values of the system parameters θ_0, α; k; d_c and a_c. The results obtained show that $<Q^*>$ is very highly sensitive to changes in the value of parameters a, k and c_s; moderate sensitive to the changes of parameter μ, and less sensitive to the changes of parameters b, θ_0, α; β; d_c and a_c.

(ii) The optimum expected average total cost $<ATC^*>$ increase or decrease with the increase or decrease in the values of the system

International Journal of Advances in Engineering and Management (IJAEM)
Volume 7, Issue 01 Jan. 2025, pp: 588-596 www.ijaem.net ISSN: 2395-5252

parametersa, b, θ_0, α ,k, d_c, a_c, c_s and μ. On the other hand $< ATC^* >$ increase or decrease with the decrease or increase in the values of the system parameter β. The results obtained show that $< ATC^* >$ is very highly sensitive to changes in the value of parameters a and k , moderate sensitive for c_s and μ, and less sensitive to the changes of parameters b , θ_0, α , β, d_c and a_c.

From the above analysis, it is seen thata and k are very sensitive parameters in the sense that any error in the estimation of these parameters result in significant errors in the optimal cost solution. Hence estimation of these parameters needs adequate attention.

Scope of future work:

In the present paper, we have discussed an inventory management model for time dependent deterioration and a two-parameter Weibull distribution amelioration with stochastic demand with fully backlogged shortages. The holding cost follows time dependent. The demand pattern is assumed to be linearly dependent on to time with a stochastic error. The model is minimized to the total average cost by finding optimal values. Eventually, a researcher can extend this model considering cubic demand under stochastic behaviour along with salvage value and permissible delay in payments.

REFERENCES:

[1]. H.S. Hwang, **"**A study of an inventory model for items with Weibull ameliorating", Computers and Industrial Engineering, 33, 1997, pp.701-704.

[2]. H.S**.** Hwang, **"**A stochastic set-covering location model for both ameliorating and deteriorating items", Computers and Industrial Engineering, 46, 2004, pp.313-319.

[3]. I. Moon, B. C. Giri and B. Ko, "Economic order quantity models for ameliorating/deteriorating items under inflation and time discounting",European Journal of Operational Research, 162(3), 2005, pp. 773–785.

[4]. L.-Q. Ji, "The influences of inflation and time-value of money on an EOQ model for both ameliorating and deteriorating items with partial backlogging", in Proceedings of the 2008 International Conference on Wireless Communications, Networking and Mobile Computing, WiCOM, Dalian, China, 2008.

[5]. M. Valliathal and R. Uthayakumar, "The production-inventory problem for ameliorating/deteriorating items with non-linear shortage cost under inflation and time discounting", Applied Mathematical Sciences, 4(5-8), 2010, pp. 289–304.

[6]. S., Nodoust A. Mirzazadeh and G. Weber, " An evidential reasoning approach for production modeling with deteriorating and ameliorating items", Operational Research, DOI:10.1007/s12351-017-0313-x ,2017.

[7]. M. Mallick, S. Mishra, U. K. Mishra and S.K. Paikray, "Optimal inventory control for ameliorating, deteriorating items under time varying demand condition", Journal of Social Science Research, 3(1), 2018, pp.166-173.

[8]. Dr. Biswaranjan Mandal, "An Inventory Management System for Deteriorating and Ameliorating items with a Linear Trended in Demand and Partial Backlogging", Global Journal of Pure and Applied Mathematics, ISSN 0973-9750, 16(6), 2020, pp. 759-770.

[9]. U Dave and L K Patel, " (T, S_j) policy inventory model for deteriorating items with time proportional demand", Journal of the Operational Research Society, 32, 1981, pp. 137 – 142.

[10]. T. K. Dutta and A.K.," Pal A note on a replenishment policy for an inventory model with linear trend in demand and shortages", J. Operational Research Society, 43, 1992, pp. 993-1001

[11]. J T Teng, H J Chang, C Y Dye and C H Hung, " An optimal replenishment policy for deteriorating items with time-varying demand and partial backlogging", Operations Research Letters, 30(6), 2002, pp. 387-393.

[12]. Biswaranjan Mandal: An inventory model for random deteriorating items with a linear trended in demand and partial backlogging", Res. J. Business Mgmt. and Accounting, 2(3), 2012, 2010, pp.48-52.

[13]. Mallick M., Mishra S., Mishra U. K. and Paikray S.K. : Optimal inventory control for ameliorating, deteriorating items under time varying demand condition, Journal of Social Science Research, 3(1), 2018, pp.166-173.

[14]. A Kundu, " An inventory model for deteriorating items with price dependent stochastic demand and time-varying holding cost", JETIR, 11(9), 2024, pp 44-51.

[15]. M Goh, " EOQ model with general demand and with holding cost function", European J of Operational Research, 73, 1994, pp. 50-54.

[16]. A Roy, " An inventory model for deteriorating items with price dependent demand and time-varying holding cost", Advanced Modelling and Optimization, 10, 2008, pp. 25-37.

[17]. V K Mishra, L S Singh and R Kumar, " An inventory model for deteriorating items with time-dependent demand and time-varying holding cost under partial backlogging", J Indus Eng, 2013, pp. 1-10.

[18]. R. P. Tripathi , D. Singh and A Surbhi, "Inventory models for stock-dependent demand and time varying holding cost under different trade credits", Yugoslav Journal of Operations Research, DOI: https://doi.org/10.2298/YJOR160317018T , 2017, pp. 1-13.

[19]. Dr. Mamta Sharma, "A Study on Ordering Policy with Time-Varying Demand and Holding Cost for Deteriorating Items with Shortages", International Journal for Research in Applied Science & Engineering Technology, 10, 2022, pp. 4398-4407.

[20]. T Whitin, " The theory of Inventory Management", Princeton University Press, New Jersey, 2nd edition.