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Solving Fuzzy Transportation Problem Using Hungarian Method

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ABSTRACT:In this paper we are handling with a fuzzy assignment problem. The main objective of an assignment problem is to designate n jobs to n workers. The fuzzy considered here is a triangular fuzzy. In this paper we use Hungarian method for solving the given FAP. We have used a numerical example which incorporates a fuzzy number.

KEYWORDS: Assignment problem, Triangular fuzzy number, Fuzzy numbers, Hungarian method, defuzzification.

I. INTRODUCTION:

An assignment problem is a special type of linear programming problem that deals with assigning the activities such as jobs or task or products to their respective facilities like men, machine, laborers, etc. in such a way that the total time / total cost involved in assignment is minimized and the total sale or profit incurred is maximized and it also ensures that the total satisfaction of the group is also maximized. Tremendous number of efforts have been spent to make significant advances on the development of numerous methodologies and their applications to various transportation problems. Fuzzy assignment problems have received great attention in recent years due to its efficiency. The first fuzzy assignment problem was solved by Lin and Wen (2004), by using fuzzy interval number costs by using a labelling algorithm. Later, Isabel's and Uthra (2012), Presented an assignment problem with fuzzy costs represented by linguistic variables which are replaced by triangular fuzzy numbers. The coefficients of FAP problems are assumed to be known. In practice, the coefficients of fuzzy transportation problems (few or all) are not exact due to the errors of measurement or vary with market conditions …etc. In this paper, the proposed the fuzzy assignment problem is solved by the Hungarian method,where the fuzzy assignment problem has been converted into crisp one by using graded mean integration method and Hungarian assignment has been applied to find an optimal solution.

II. PRELIMINARIES:

In this section, the basic definitions are presented for a Fuzzy Sets [8, 12, 4]. Let X denote a universal set. A fuzzy subset $\tilde{A}$ of X is defined by its membership function $\mu A\ (x) : X \rightarrow [0,1]$ A fuzzy set $\tilde{A}$ can be characterized as a set of ordered pairs of x elements and grade $\mu A(x)$. It is written as: $\tilde{A} = \{(, \mu A\ (x)) : x \epsilon X\ , \mu A\ (x) \epsilon [0,1]\ \}$.

2.1 TRIANGULAR FUZZY NUMBER (TFNS):

A fuzzy number R is said to be a triangular fuzzy number (TFN) if its membership function aˆ: R −→ [0, 1] has the following characteristics:

$\mu a\hat{}(x) = (x - a1)/(a2 - a1)$ if $a1 \le x \le a2$ $(a3 - x)/(a3 - a2)$

if $a2 \le x \le a3$ 0

We can write this triangular fuzzy number as aˆ = (a1, a2, a3). We use F(R) to denote the set of all triangular fuzzy numbers,also if m = a2, represents the modal value or midpoint, α = (a2 −a1) represents the left spread and β = (a3 −a2) represents the right spread of the triangular fuzzy number aˆ = (a1, a2, a3) then the triangular fuzzy number aˆ can be represented by the triple aˆ = (α, m, β,) i.e., aˆ = (a1, a2, a3) = (α, m, β)

2.2 DEFUZZIFICATION:

Defuzzification is the process of finding a singleton value or a crisp value from an aggregated fuzzy which represents the average value of the TFNs.

2.3 GRADED MEAN INTEGRATION METHOD:

Graded mean integration method maps the set of all fuzzy numbers to a set of real numbers is defined as

$$R(\widetilde{A}) = \frac{a_1 + 4a_2 + a_3}{6}$$

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III. ASSIGNMENT PROBLEM FORMULATION

3.1ASSIGNMENT PROBLEMS

The assignment problem can be represented in the form of a n × n cost matrix [Cij] of real numbers as given in the following table :

Jobs -> Person ↓	1	2	3	….j….	N
1	C11	C12	C13	Cij	C1n
2	C21	C22	C23	C2j	C2n
-	-	-	-	-	-
-	-	-	-	-	-
i	Ci1	Ci2	Ci3	C1j	Cin
-	-	-	-	-	-
N	Cn1	Cn2	Cn3	cNj	cnn

Mathematically, an assignment problem can be stated as,

$(AP): MinZ = \sum \sum cijj{=}nj{=}1\ i{=}ni{=}1\ xij$

$Subjectto$:

$\sum xiji{=}ni{=}1 = 1\ , j = 1,2, \dots , n,$

$\sum xijj{=}nj{=}1 = 1\ , i = 1,2, \dots ,$ n,

Where $xij = 1,\ if theith\ personassignmentthejth\ job,$

$0,\ otherwise$

This is the decision variable denoting the assignment of a person i to job j .

3.2 FUZZY ASSIGNMENT PROBLEM

The generalized fuzzy assignment problem can be denoted in the form of n x n fuzzy matrix C˜ij as given in the table below:

Jobs -> Person ↓	1	2	3	….j….	N
1	C11	C12	C13	Cij	C1n
2	C21	C22	C23	C2j	C2n
-	-	-	-	-	-
-	-	-	-	-	-
i	Ci1	Ci2	Ci3	C1j	Cin
-	-	-	-	-	-
N	Cn1	Cn2	Cn3	cNj	cnn

The goal of this is to effectively assign the j th job to the ith resource (i.e., all jobs to available resources)

IV. ALGORITHMS

4.1. HUNGARIAN ASSIGNMENT ALGORITHM

Various steps for obtaining an optimal solution by this method can be summarized as follows:

Step 1: Check if the number of rows and columns are equal. If they are not equal, then a dummy row or column must be added with zero cost elements to balance the matrix.

Step 2: Find the smallest cost element in each row of the matrix and subtract this smallest element from each element in that row. Then there will be at least one zero in each row of this new obtained matrix which can be called as the first cost reduced matrix.

Step 3: In this reduced cost matrix, find the smallest cost element in each column and then subtract this element from each element in that column. As a result, there will be at least one zero in each row and column of what we call as the second reduced cost matrix.

Step 4: Now to determine an optimum assignment for the given problem the following steps can be followed:

(i) Examine the rows such that a row with exactly one zero each is found. circle around the zero element as assigned to that cell and cross out all other zeros from that column. Continue this method until all the rows have been examined. If there are more than one zero in any row, then skip that row and pass on to the next one.

(ii) Repeat the same procedure for the columns of the reduced cost matrix. If there is no single zero in any row or column of the reduced matrix, then arbitrarily choose a row or column having the minimum number of zeroes. Arbitrarily, choose zero in the row or column and cross the remaining zeroes in that row or column. Repeat steps (i) and (ii) until all zeros are either assigned or crossed out.

Step 5: An optimal assignment is found for theproblem, if the number of assigned cells equals the number of rows (and columns). If a zero cell is arbitrarily chosen, there can be another alternate optimal solution. If no optimum is, then we can proceed to the next step.

Step 6: Draw the minimum number of horizontal and / or vertical lines through all the zeros as follow:

(i) Mark (X) to those rows where no assignment has been made.
(ii) Mark (X) to those columns which have zeros in the marked rows.
(iii) Mark (X) rows (not already marked) which have assignments in marked columns.
(iv) The process may be repeated until no more rows or columns can be checked.
(v) Draw straight lines through all the unmarked rows and marked columns.

Step 7: If the minimum number of lines passing through all the zeros in the matrix is equal to the number of rows or columns, then the optimum solution is obtained by an arbitrary allocation in the positions of the zeroes not crossed in previous steps. Otherwise, we can move to the next step.

Step 8: Revise the cost matrix as follows:

(i) Choose the smallest one from those which is not covered by any line and then subtract this element from all the uncrossed elements and add the same element to the elements which are at the point of intersection of two lines.
(ii) Other elements crossed by the lines remain unchanged.

Step 9: Go to Step 4 and repeat the procedure till an optimum solution is attained.

4.2. ALGORITHM TO SOLVE FUZZY ASSIGNMENT PROBLEM

Step 1: First we look whether the given cost matrix of a fuzzy assignment problem is a balanced one or not. If not, we have to change this unbalanced assignment problem into a balanced one by adding dummy row/ column and provide the values as zero. If it is a balanced one (i.e., number of people are equal to the number of jobs) then go to the next step.

Step 2.: Now defuzzify the fuzzy cost matrix by using graded mean integration method.

Step 3: Apply Hungarian transportation algorithm to determine the best combination for producing the minimized total costs, where each machine/person should be assigned to only one job.

V. NUMERICAL EXAMPLE:

Consider a fuzzy assignment problem with each row representing seven persons and each column representing seven jobs with Assignment cost. The cost matrix $(\tilde{C}\, ij)n\times n$ is given whose elements are triangular fuzzy numbers as follows in the table

JOBS→ PERSONS ↓	Job1	Job2	Job3	Job4	Job5	Job6	Job7
Person1	(11,12,13)	(5,6,13)	(7,9,11)	(3,10,11)	(3,8,13)	(12,14,16)	(7,9,11)
Person2	(7,9,11)	(9,10,11)	(5,8,11)	(8,10,12)	(5,6,7)	(9,11,13)	(8,10,11)
Person3	(3,9,15)	(7,8,13)	(6,7,9)	(4,5,12)	(4,6,12)	(8,15,18)	(3,4,17)
Person4	(5,6,13)	(13,17,21)	(5,7,13)	(8,10,16)	(8,15,18)	(7,11,15)	(3,9,13)
Person5	(6,8,10)	(6,9,12)	(9,11,13)	(11,13,15)	(8,9,10)	(6,8,12)	(4,8,12)
Person6	(12,14,18)	(9,13,17)	(4,6,10)	(2,4,18)	(3,8,17)	(6,12,14)	(7,8,13)
Person7	(4,7,10)	(7,8,13)	(5,6,7)	(1,8,13)	(1,8,13)	(5,12,19)	(7,8,17)

The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum.

Solution:

To solve the given problem we have to convert the fuzzy into a crisp assignment problem with the graded mean integration method

$$R(c11)=\frac{a_1+4a_2+a_3}{6}$$

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$= \frac{11+4(12)+13}{6}$

$= 12$

$R(c12)= \frac{5+4(6)+13}{6}$

$=7$

$R(c13)=\frac{7+4(9)+11}{6}$

$=9$

Similarly, applying this to all the values, we get the following table:

JOBS→ PERSONS↓	Job1	Job2	Job3	Job4	Job5	Job6	Job7
Person1	12	7	9	9	8	14	9
Person2	9	10	8	10	6	11	9.8
Person3	9	9	7.1	6	7	14	7
Person4	7	17	8	11	14	11	8.6
Person5	8	9	11	13	9	8.3	8
Person6	14	13	6.3	7	9	11	9
Person7	7	9	6	7.6	10	12	10

To solve the resulting assignment problem, we use Hungarian method:
Let $piandqj$ denote row $iand$ column j,
Step 1: First determine the minimum in each column of the matrix.
Step 2. From the cost matrix resulting from Step 1 , subtract the obtained value from each column.
The minimum values for each column are:

Q1=7 Q4=6 Q7=7
Q2=7 Q5=6
Q3=6 Q6=8.3

JOBS→ PERSONS↓	Job1	Job2	Job3	Job4	Job5	Job6	Job7
Person1	5	0	3	3	2	5.7	2
Person2	2	3	2	4	0	2.7	2.8
Person3	2	2	1.1	0	1	5.7	0
Person4	0	10	2	5	8	2.7	1.6
Person5	1	2	5	7	3	0	1
Person6	6.7	5.7	0	0.7	2.7	2.4	1.7
Person7	0	2	0	1.6	4	3.7	3

Step 3. From the cost matrix resulting from Step 2, we have to identify the minimum value of each row and subtract it from all the entries of their respective row of the above table.
The row minimum is:

P1=0 P4=0 P7=0
P2=0 P5=0
P3=0 P6=0

JOBS→ PERSONS↓	Job1	Job2	Job3	Job4	Job5	Job6	Job7
Person1	5	0	3	2.3	2	5.7	1.3
Person2	2	3	2	3.3	0	2.7	2.1
Person3	2.7	2.7	1.8	0	1.7	6.4	0
Person4	0	10	2	4.3	8	2.7	0.9
Person5	1	2	5	6.3	3	0	0.3
Person6	6.7	5.7	0	0	2.7	2.4	1
Person7	0.7	2.7	0.7	1.6	4.7	4.4	3

JOBS→ PERSONS↓	Job1	Job2	Job3	Job4	Job5	Job6	Job7
Person1	5	0	2.3	2.3	2	5.7	1.3
Person2	2	3	1.3	3.3	0	2.7	2.1
Person3	3.4	3.4	1.1	0.7	2.4	7.1	0.7
Person4	0	10	1.3	4.3	8	2.7	0.9
Person5	1	2	4.3	6.3	3	0	0.3
Person6	7.4	6.4	0	0.7	3.4	3.1	1.7
Person7	0.7	2.7	0	1.6	4.7	4.4	3

Step 4: Now to identify the optimal solution which is considered as the feasible assignment for the cost matrix and this is found by associating with the zero elements of the table

JOBS→ PERSONS↓	Job1	Job2	Job3	Job4	Job5	Job6	Job7
Person1	5	0	3	2.3	2	5.7	1.3
Person2	2.7	3	2	3.3	0	2.7	2.1
Person3	3.4	2.7	1.1	0 ×	2.4	6.4	0
Person4	0	9.3	1.3	3.6	8	2	0.2
Person5	1.7	2	5	6.3	3	0	0.3
Person6	8.1	6.4	0.7	0	3.4	2.4	1
Person7	0.7	2	0	0.9	4.7	3.7	2.3

The optimal solution for the assignment problem is as follows:
person 1→ $J2$,
person 2→ $J5$,
person 3→ $J7$,
person 4→ $J1$,
person 5→ $J6$,
person 6 → $J4$,
person 7 → $J3$.

VI. CONCLUSION

In this paper, the assignment costs are described as imprecise triangular fuzzy numbers. The given fuzzy assignment problem has been transformed into a crisp assignment problem using graded mean integration and has been solved by using Hungarian method. By comparing the results of the method proposed in here and other existing methods, we can see that the proposed method has given greater results than the existing methods.

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