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Steady Flow of Chemically Reacting Temperature Dependent Viscosity Fluid through a Porous Vertical Surface

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ABSTRACT:The motivation of present research paper is to analyze the Steady Flow of Chemically Reacting Temperature Dependent Fluid Flows through a Porous Vertical Surface analytical technique for temperature equation and numerical technique for velocity field, under which consistency and convergence criterion exist. The results show increase velocity with increases as thermal Grashof number increases, while velocity decreases with increase in viscosity and suction parameters. It is also established that there exist a linear relationship between the thermal buoyancy and the fluid velocity. The temperature on the other hand, is a decreasing function of suction parameter or the spatial variable.

KEYWORDS: Newtonian fluids; non-Newtonian fluid; chemically reacting flow;combustion; Arrhenius kinetics; incompressible Navier-Stokes equations;

I. INTRODUCTION

It is common knowledge that chemical reactions occur more rapidly at higher temperatures. Everyone knows that milk turns sour much more rapidly if stored at room temperature rather than in a refrigerator, butter goes rancid more quickly in the summer than in the winter, and eggs hard-boil more quickly at sea level than in the mountains. A chemically reacting flow is a fluid flow in which a chemical reaction is also occurring. Such flows occur in a wide range of fields including combustion, chemical engineering, biology, and pollution abatement. For most liquids, the viscosity decreases with temperature and increases with pressure. For gases, it increases with both temperature and pressure [1]. Broadly, higher is the viscosity of a substance, more resistance it presents to flow (and hence more difficult to pump!). Ever since the formulation of the equations of continuity (mass) and momentum (Cauchy, Navier-Stokes), the fluid dynamics of Newtonian fluids has come a long way during the past 300 or so years, albeit significant challenges especially in the field of turbulence and multi-phase flows still remain [2].It is appropriate to mention here that it has long been a matter of debate and discussion in the literature whether a true yield stress exists or not, example, see the paper of [3] and the review of [4] for different viewpoints on this matter. Evidently, the answer to the question whether a substance has a yield stress or not seems to be closely related to the choice of a time scale of observation. In spite of this fundamental difficulty, the notion of an apparent yield stress is of considerable value in the context of engineering applications, especially for product development and design in food, pharmaceutical and healthcare sectors [5] and [6]. Of the time-independent fluids, this sub-class has generated very little interest and hence very few reliable data are available. Indeed, until up to about early 1980s, this type of flow behavior was considered to be rare, but, however, with the recent growing interest in the handling and processing of systems with high solids loadings, it is no longer so, this is explained in the works of [7], [8], [9], for instance. Exothermic and endothermic chemical reactions and its application to processes of ignition and combustions has been a subject of intense study by scientists. In a study carried out by [10], a three-component model of a system is considered which includes exothermic oxidation and endothermic evaporation process. They report that, the full system can be approximated and the safe and dangerous regions of parameter space can be identified by assuming a slow rate of consumption of fuel and oxygen. Significant research effort has been expended in seeking a similar expression for σ for non-Newtonian fluids which should be able not only to predict shear-dependent viscosity, yield stress, visco-elastic effects in shear and extensional flows, rheopexy and thixotropy but should also satisfy the requirements of frame indifference, material objectivity, etc. [11]. [12] - [15] amongst others gives critical appraisals of the current state of the art and useful guidelines for the selection of an appropriate expression for σ(constitutive equation) which are available in the literature.

Therefore, if one were able to develop an appropriate constitutive equation and/or to choose one from the existing selection, it is possible to set up the governing differential equations together with suitable boundary conditions, albeit there are situations in which the prescription of boundary conditions is also far from obvious, particularly in flows with a free surface, slip etc. [16] investigate the influence of chemical reaction and the combined effects of internal heat generation and a convective boundary condition on the laminar boundary layer MHD heat and mass transfer flow over a moving vertical flat plate. The effects of physical parameters on the velocity, temperature, and concentration profiles are illustrated graphically. For most elementary reactions, the rearrangement of atoms in going from reactants to products via a transition state proceeds through the movements of atomic nuclei that experience a potential energy field that is generated by the rapid motions of the electrons in the system. On this potential energy surface there will be a path of minimum energy expenditure for the reaction to proceed from reactants to products (reaction coordinate). The low energy positions of reactants and products on the potential energy surface will be separated by a higher energy region. The highest energy along the minimum energy pathway in going from reactants to products defines the transition state [17]. Finally, experimentalists also confront similar challenges both in terms of material characterization (rheometry) as well as in terms of the interpretation and representation of data using dimensionless groups, e.g. see [18] and [19] for rheometry.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Following [19] the physical processes involved in a reacting flow are: (1) the fluid dynamics, (2) the thermodynamics, and (3) the chemical reactions. The fluid dynamics process is the balance between the temporal evolution and the spatial convection of the flow properties due to conservation of mass, momentum, and energy. We also assume the existence of a partition $\partial\Omega = \Gamma_1 \cup \Gamma_0 \cup \Gamma_w$ corresponding to the portions of the boundary where fluid flows into the domain, portions where fluid flows out, and the walls of the container, respectively. On all of$\partial\Omega$, the fluid flow u will be specified by Dirichlet boundary data; however, the boundary conditions for T and $Y_i$will vary across the partition [19]. Thus the equations governing the process in terms ofvelocity and Temperature fields is expressed as

$$\rho\left(\frac{\partial u}{\partial t}+v_0\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\rho g\beta(T-T_0)\quad(1)$$

$$\rho c_p\left(\frac{\partial T}{\partial t}+v_0\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)(2)$$

The appropriate initial and boundary conditions are

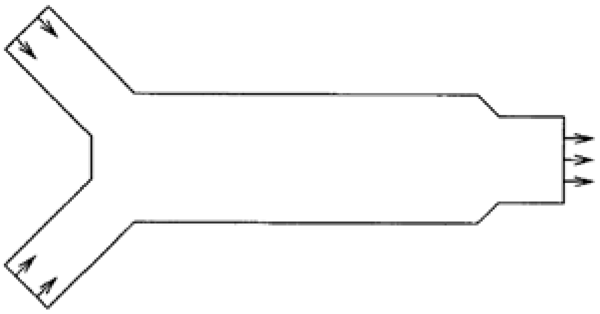

Fig.1. Domain Ω configuration

$$\left.\begin{array}{l} t \le 0:\ u = 0 \quad T = T_0 \qquad \forall y \\ t > 0: \begin{cases} u = U_0 \ \ T = T_1 \ \ y = 0 \\ u = 0 \ \ T = T_0 \ \ y \to \infty \end{cases} \end{array}\right\} \qquad (3)$$

Where all the variables and parameters have their usual meaning.

III. METHOD OF SOLUTION

The viscosity is define as $\mu = e^{-E/RT}$ (see [20])
Using the following dimensionless quantities

$$\frac{E(T - T_0)}{RT_0^2} = \theta, \frac{u}{U_0} = \phi, t = \frac{\rho U_0^2}{\mu_0} t', y = \frac{1}{h} y'$$

Equations (1) and (2) becomes

$$\frac{\partial}{\partial t}\emptyset(x,y) - s\frac{\partial}{\partial y}\emptyset(x,y) = \frac{\partial}{\partial y}\left(e^{\frac{\lambda\,(y,t)}{1+\varepsilon\theta\,(y,t)}}\right)\frac{\partial}{\partial y}\emptyset(y,t) + \mathrm{Grt}\,\theta(x,t)(4)$$

$$\frac{\partial\theta(y,t)}{\partial t} - s\frac{\partial}{\partial y}\emptyset(y,t) = \frac{1}{\Pr}\frac{\partial^2}{\partial y^2}\theta(x,t) \quad (5)$$

Where

$$s = \frac{v_0}{U_0}, U_0 = \frac{\mu_0}{\rho h}, \mathrm{Grt} = \frac{g\beta\mu_0 RT_0^2}{\epsilon T_0}, \Pr = \frac{\mu c_p}{k},$$

The boundary condition (3) also becomes

$$\left.\begin{array}{l} t \le 0:\ \phi = 0, \quad \theta = 0 \qquad \forall y \\ t > 0: \begin{cases} \phi = 1, \quad \ \ \theta = 1 \ \ y = 0 \\ \ \phi \to 0 \ \ \theta \to 0 \ \ y \to \infty \end{cases} \end{array}\right\} (6)$$

For steady state, we let
$\emptyset = f(y), \theta = g(y)$, equation (4) and
(5) becomes
For steady state, we let$\emptyset = f(y), \theta = g(y)$, equation (4) and (5) becomes

$$-s\frac{d}{dy}f(y) = \frac{d}{dy}\left(e^{\frac{\lambda g(y)}{1+\epsilon g(y)}}\frac{d}{dy}f(y)\right) + \mathrm{Grt}g(y) \qquad (7)$$

$$-s\frac{d}{dy}g(y) = \frac{1}{pr}\frac{d^2}{dy^2}g(y) \qquad (8)$$

$\theta(0) = 1, \theta(y) \to 0 \text{as } y \to \infty$ (9)

Solving (8) together with boundary conditions (9) we have,

$$g(y) = e^{-prsy} \qquad (10)$$

Using (10) in (7)

$$-s\frac{d}{dy}f(y) = \frac{d}{dy}\left(e^{\frac{\lambda g\,(y)}{1+\epsilon g\,(y)}}\frac{d}{dy}f(y)\right) + Grte^{-prsy} \qquad (11)$$

As always in modern CFD methods, the performance of numerical schemes employed will be greatly impacted by the theoretical models. The theoretical model must be accurate such that the underlying non-linear physics of flow motions will be faithfully described [21]. Consequently, numerical inaccuracy and oscillations occur near the flame front.
Equation (11) is highly nonlinear, thus we employed a numerical method implemented by Maple. The strategies used for solving this problem are;

1. Specification of an initial approximation to (11) by using *approxsoln* (for the first case).
2. Determine a "nearby" problem, and use continuation and *mincont* to obtain an accurate initial profile.
3. Attempt an initial solution with a greater number of points than the default by using *initmesh*.

The dsolve command with the *numeric or type = numeric* option on a real-valued two-point boundary value problem (BVP), is employed to finds a numerical solution for (11).

This type of problem (IBVP) is automatically detected by dsolve, we then give the optional equation $method = bvp[traprich]$ which is used to indicates the solver $traprich$ be used. This submethod, traprich is a trapezoid method that use Richardson extrapolation enhancement ([22], [23] and [24]. Plot and plot3d methods are used to compute/or view the solution of our input PDE returned by pdsolve/numeric module. In addition, the settings method is configured to query/set certain parameters of the solution process [25]. The odeplot function plots or animates one or more solution curves (either 2-D or 3-D) obtained from the output (dsn) of a call to dsolve/numeric [26]. All methods involving the computation of solution values allow specification of multiple functions of the dependent and independent variables along with options specific to each function. This allows multiple plots to be displayed with the same command using different options for each. The plot method supports the following calling sequences $\mathrm{plot(depspec, t = numeric, x = range, options)}$.

IV. RESULTS AND DISCUSSION

For the purpose of discussing the effect of various parameters on the reaction behaviour, calculations have been carried out for different values ofs, λ and Grt and for fixed values of Pr and ϵ. In order to point out the effects of these parameters on flow characteristic, to be realistic, the value of Prandtl number is chosen to be $\mathrm{Pr} = 0.71$ which represents air at temperature $25^{o}\mathrm{C}$ and one atmospheric pressure. All parameters are primarily chosen as follows: $s = 0.1, \mathrm{Grt} = 0.5$ and $\lambda = 0.1$for a fixed value of $\mathrm{Pr} = 0.71$ and $\epsilon = 0.05$unless otherwise stated.

The velocity distribution for chemically reacting fluid flow are displayed in Figures (2) to (4). In Figure 2, we displayed the velocity distribution for various values of thermal buoyancy (Grt, thermal Grashof number). It could be seen that velocity increases as thermal Grashof number increases$(\mathrm{Grt} > 0)$. A reverse flow is observed when the thermal buoyance is opposing the flow $(\mathrm{Grt} < 0)$. We noted that, at time $t = 5$ unit when Grt is increased from 0.0 to 0.1 unit, the velocity increases by 64%. Further increase in Grt by 50% increases the velocity by 38.4%. While another 50% increment in thermal buoyancy result in 42.7% increase in reacting fluid velocity. It could then be said that there is a linear relationship between the thermal buoyancy and the reacting fluid velocity. Also, we observed that the momentum boundary thickens as Grashof number increases with occurrence of a pick in the profiles when $\mathrm{Grt} \geq 2$ while other parameters are kept constant. This picks indicate that maximum velocity occurs in the body of the fluid and not on the surface. The effect of viscosity parameter is shown in figure 3. In this figure, we observed that increase in viscosity parameter brings about reduction in the fluid flow. It is deduced at $t = 3.5$ unit that when viscosity is increased from $\lambda = 0.0$to $\lambda = 0.5$, the velocity drops by 26.32%, further increase from $\lambda = 0.5$ to $\lambda = 1.5$ the velocity drop further by 26.9% and increasing the viscosity by 25% reduces the velocity by 15.71%. The differences in velocities with respect to viscosity parameter continuously decreases when y>5 until beyond y=13 when it is noticed that differences becomes insignificant. Similar effect is observed in varying suction parameter as shown in figure 4, where we observed that increase in suction parameter brings about reduction in the fluid flow. It is also deduced that at $t = 3.5$ unit that when suction parameter is increased by about 33% the velocity drops by 37.5%, further increase 50% the velocity drop further by 35.6% and increasing the suction by 25% reduces the velocity by 60%. The boundary layer decreases with increase in suction.

From the close – form solution of temperature field (equation (11)), temperature distribution for the problem considered here is a decreasing function of Prandtl number, suction parameter or the spatial variable.

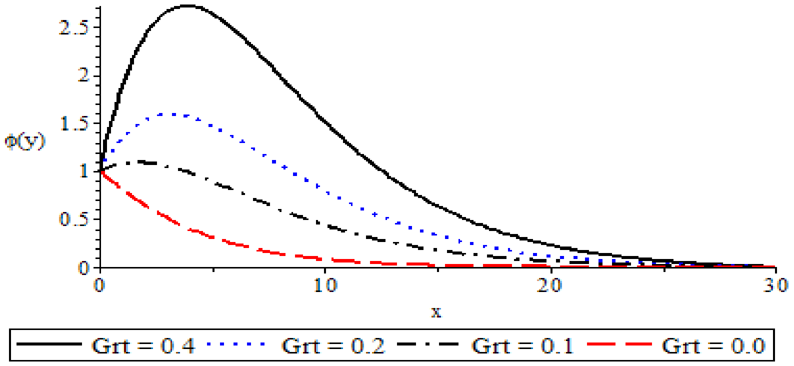

Fig.2 Velocity distribution for various values of Thermal Bouyancy (Grt)

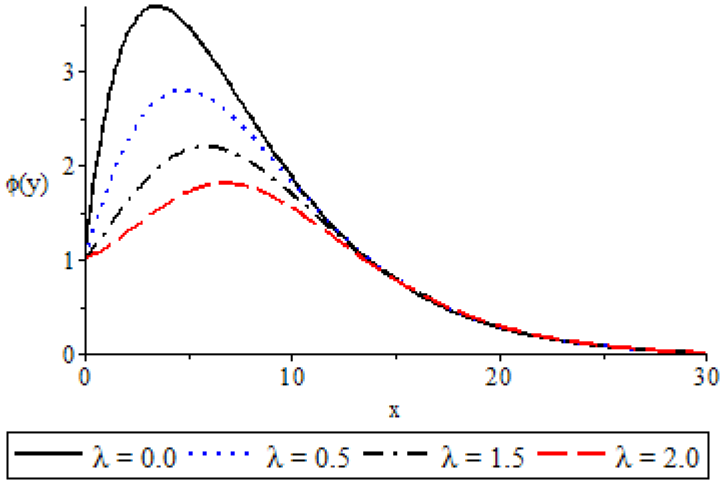

Fig. 3 Velocity distribution for various values of Viscoucity Parameter (λ)

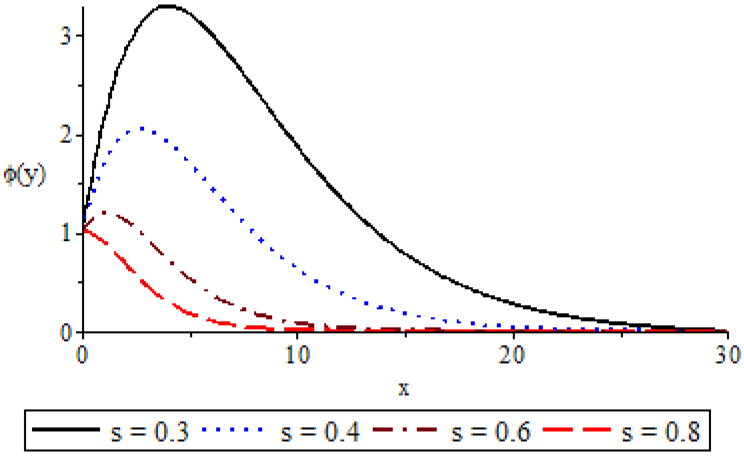

Fig.4 Velocity distribution for various values of suction Parameter (s)

V. CONCLUSION

The motivation of present research paper is to analyze the Steady Flow of Chemically Reacting Temperature Dependent Fluid Flows through a Porous Vertical Surface analytical technique for temperature equation and numerical technique for velocity field, under which consistency and convergence criterion exist. The theoretical non-linear model for velocity has been derived by employing the Cauchy momentum equation. From our analysis, the following deductions were made:

- that velocity increases as thermal Grashof number increases
- that there is a linear relationship between the thermal buoyancy and the fluid velocity
- that the momentum boundary thickens as Grashof number increases
- that increase in viscosity parameter brings about reduction in the fluid flow
- that increase in suction parameter brings about reduction in the fluid flow
- that he boundary layer decreases with increase in suction
- that temperature is a decreasing function of suction parameter or the spatial variable

NOMENCLATURE

u	dimensional velocity field	t
p	Pressure	R_0
T	dimensional temperature field	Y_i
y	Spatial coordinate	C_i
k	thermal conductivity	θ
T_0	free stream temperature	$\emptyset$
A_j	are the frequency factors	Pr
m_i	molecular weight of chemical	Le
c_p	specific heat at constant	ρ
E_j	the activation energies	0
C_i	Concentration for $i-$th	w
T_1	surface temperature	ϵ
$f_0(T)$	the forcing term from buoyancy	μ

t	Time
R_0	universal gas
Y_i	Mass fraction of the chemical species i
C_i	are the concentrations
θ	dimensionless temperature
$\emptyset$	dimensionless fuel Concentration
Pr	Prandtl number
Le	Lewis number
ρ	density
$\circ$	Initial condition
w	Condition at wall
ϵ	$0 < \epsilon \ll 1$
μ	Dynamic viscousity

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